Prove that there exists a real number a > 0 such that $[a^n] - n$ is even for every positive integer n. Here [x] denotes the integer part of x, that is, the biggest integer not exceeding x.