Low complexity matrix projections preserving actions on vectors

Stefano Cipolla

In this lecture we fully solve a problem originally introduced in [1]:

Problem 1 *Given a symmetric matrix $B \in \mathbb{R}^{n\times n}$ and a vector $\mathbf{v} \in \mathbb{R}^n$, find a low complexity unitary matrix L such that defining $\mathcal{L} = sd\, L$, we have*

$$\mathcal{L}_B \mathbf{v} = B\mathbf{v}, \tag{1}$$

being $\mathrm{sd}\, L := \{Ld(\mathbf{z})L^H \ : \ \mathbf{z} \in \mathbb{C}^n\}$ and $\mathcal{L}_B$ such that

$$||\mathcal{L}_B - B||_F \leq ||X - B||_F, \ \ \forall\, X \ \in \ \mathcal{L}.$$

Actually, by using the Arnoldi procedure for Block-Krylov subspaces, it is possible to solve the above problem in the more general case where $\mathbf{v}$ is replaced by a matrix $V \in \mathbb{R}^{n\times r}$ (see [2] for more details).

References

[1] S. Cipolla, C. Di Fiore, F. Tudisco, and P. Zellini. Adaptive matrix algebras in unconstrained minimization. *Linear Algebra Appl.*, 471:544 – 568, 2015.

[2] S Cipolla, C Di Fiore, and P Zellini. Low complexity matrix projections preserving actions on vectors. *Submitted for publication*, 2018.