Abstract: Multigrid methods based on high order tensor product B-Splines for the valuation of American options with stochastic volatility and their Greeks

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The valuation of an American option with the Heston stochastic volatility model leads to a free boundary problem in terms of a two-dimensional parabolic partial differential equation with a diffusion, convection and reaction term depending on the price of the underlying asset and its volatility. We formulate this problem as a parabolic variational inequality on a closed convex set and show the existence and uniqueness of the solution with suitable initial and boundary conditions. To determine optimal risk strategies, one is not only interested in the solution of the variational inequality but specifically in the pointwise derivatives of the solution up to order two in space, the so-called Greeks. A particular difficulty arises in the a priori unknown free boundary, the optimal exercise price of the option.

This variational inequality is discretized as follows. In time, we employ the standard Crank-Nicolson method. To enable an accurate pointwise approximation of the partial derivatives in space, we discretise the variational inequality by tensor product B-splines of high order. Since often the initial conditions are given as piecewise linear continuous functions, we approximate these with B-splines with coinciding knots at the points where the initial condition is not differentiable. Furthermore, an improvement of the approximations of the spatial derivatives in the initial time steps is achieved by employing Rannacher timestepping. For solving the nonsymmetric discretised variational inequality in each time step and determining the derivative of the solution, we develop a monotone multigrid method for high order B-splines (with possibly coinciding knots) together with a projected iterative scheme. To do so, we construct restriction operators and monotone coarse grid approximations for tensor product B-splines of arbitrary order (with coinciding knots).

Corresponding numerical experiments confirm that initial errors are damped fast when using Rannacher timestepping instead of the Crank-Nicolson method. We demonstrate that we achieve fast convergence rates of the monotone multigrid method and highly accurate approximations of the Greeks.