Lomov Regularization in Degenerate Differential Equations.

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Particular case of E boundary problem (Keldysh classification) for irregularly degenerate elliptic operator in rectangle $D = [0, 1] \times [0, b]$ is considered.

$$\begin{cases} y^2 u''_{yy} + u''_{xx} + c(y)u'_y - a(y)u = f(x, y), \\ u(0, y) = u(1, y) = u(x, b) = 0, |u(x, 0)| < +\infty. \end{cases} \tag{1}$$

Coefficients $a(y), c(y)$ and right part $f(x, y)$ are analytic functions of complex variable y in circle $|y| < R, R > b$, $a(y), c(y) \geq 0, c(0) = 0$. Built the formal solution of the problem in a form of decomposition to series by eigen functions

$$\sum_{k=1}^{+\infty} (\eta_k(y) + (y/b)^{r_k} \varphi_k(y)) \sin \pi k x, \tag{2}$$

$$r_k = \frac{1 - c'(0) + \sqrt{(c'(0) - 1)^2 + 4a(0) + 4\pi^2 k^2}}{2}.$$

Function $\varphi_k(y)$ and $\eta_k(y)$ are analytic in the same area where coefficients are. Proven the next

Theorem. *Let* $c'(0), a(0) \in \mathbb{Q}, f \in A(\{(x, y) \in \mathbb{C}^2 : |x - 0.5| < R', |y| < R\}), R' > 0.5$. *Then series (2) are defined, for all* $(x, y) \in D$ *converges uniformly to a continuous function, which has continuous second derivatives inside* D*, this function is the only classical solution of problem (1).*