Problems

1. Find the asymptotic for the number of 1×2 -tilings of $4 \times n$ table.

2. Suppose that $a_1 < a_2 < \ldots < a_n$ and $b_1 < b_2 < \ldots < b_n$ are natural numbers. Prove that determinant of the matrix $M = (m_{ij})$, where m_{ij} is the binomial coefficient $\binom{a_i}{b_j}$, is always nonnegative and that det M = 0 if and only if $a_i < b_i$ for some *i*.

- **3.** Find the spectrum of the complete bipartite graph $K_{m,n}$ on m+n vertices.
- 4. Compute the Hodge-Helmholtz decomposition for some triangulation of the torus.