Analisi Matematica
Foglio di esercizi n. 10

1. Calcolare
$$\lim_{x\to 0} \frac{(\log(1-x)-x)^2 - 2\sin(2x^2+x^3)}{e^{-x^2}-\cos(\sqrt{2}x)}.$$

2. Calcolare i seguenti integrali:

a. $\displaystyle\int_0^2 \frac{3x^3+2x^2}{\sqrt{4-x^2}}\,dx$ **b.** $\displaystyle\int_0^1 \frac{\log(x+3)}{\sqrt{x}}\,dx$

c. $\displaystyle\int_0^{+\infty} \frac{x+5}{(x+3)(x^2+3x+2)}\,dx$ **d.** $\displaystyle\int_0^{+\infty} \frac{1}{\sqrt{x}+x^2}\,dx$

e. $\displaystyle\int_0^{\pi/4} \tan(x)\tan(2x)\,dx$ **f.** $\displaystyle\int_0^{+\infty} \frac{1}{\sqrt{e^{4x}-1}}\,dx$

g. $\displaystyle\int_0^1 \frac{\arccos(x)}{\sqrt{1-x}}\,dx$ **h.** $\displaystyle\int_0^{+\infty} e^{-\lfloor x\rfloor}\,dx$

3. Discutere la convergenza dei seguenti integrali impropri al variare del parametro $a\in\mathbb{R}$:

a. $\displaystyle\int_0^{\pi/2} \frac{\sqrt{\sin(x)(1-\sin(x))}}{\tan^a(x)\cos^2(x)}\,dx$ **b.** $\displaystyle\int_{\pi/4}^{\pi/2} \frac{\tan(x)}{(\tan^2(x)-1)^a}\,dx$

c. $\displaystyle\int_0^3 \frac{\sqrt{x+2}\log(x+2)}{(6+x-x^2)^a}\,dx$ **d.** $\displaystyle\int_0^1 \frac{|\sin(ax)-\log(1+2x)|}{x^2(\sqrt{4+x}+\sqrt{x}-2)}\,dx$

4. Calcolare la somma della serie:

a. $\displaystyle\sum_{k=1}^{\infty} \frac{3\cdot 4^k - 2\cdot 5^k}{20^{k-1}}.$ **b.** $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2-1}.$

5. Studiare il comportamento delle seguenti serie:

a. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^{1+\frac{1}{k}}}$ **b.** $\displaystyle\sum_{k=1}^{\infty} \frac{\sin(1/k)}{\log(k^2+k+1)}$

c. $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k^2+1}-k}{e^{-k}+\log^2(k)}$ **d.** $\displaystyle\sum_{k=1}^{\infty} k^k e^{-k^2}$

e. $\displaystyle\sum_{k=0}^{\infty} \cos(\pi k)\arctan(1/k)$ **f.** $\displaystyle\sum_{k=1}^{\infty} \left(\sqrt{k^3+1}-\sqrt{k^3-1}\right)$

g. $\displaystyle\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$ **h.** $\displaystyle\sum_{k=1}^{\infty} \frac{1}{2^{\log(k)}}$

6. Risolvere i seguenti problemi:

a. sapendo che $\sum_{k=0}^{\infty}\frac{1}{k!}=e$, calcolare $\sum_{k=0}^{\infty}\frac{k}{k!}$ e $\sum_{k=0}^{\infty}\frac{k^2}{k!}$;

b. fare un esempio di una funzione f positiva e derivabile in $(0,+\infty)$ tale che $\lim_{x\to 0^+} f(x) = +\infty$, $\lim_{x\to+\infty} f(x) = 0$ e $\int_0^{+\infty} f(x)\,dx$ sia convergente.