**Problem 12145**
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Proposed by T. Amdeberhan and V. Moll (USA).

*Prove*

$$\int_0^\infty \frac{\cos(t)\sin(\sqrt{1+t^2})}{\sqrt{1+t^2}}\,dt = \frac{\pi}{4}.$$

Solution proposed by Roberto Tauraso, Dipartimento di Matematica, Università di Roma "Tor Vergata", via della Ricerca Scientifica, 00133 Roma, Italy.

*Solution.* By letting $t = \sinh(x)$, by using the Werner's formula

$$2\cos(\alpha)\sin(\beta) = \sin(\alpha+\beta) - \sin(\alpha-\beta),$$

and finally by letting $s = e^{\pm x}$, we have

$$\begin{aligned}
\int_0^\infty \frac{\cos(t)\sin(\sqrt{1+t^2})}{\sqrt{1+t^2}}\,dt &= \int_0^\infty \cos(\sinh(x))\sin(\cosh(x))\,dx \\
&= \frac{1}{2}\int_0^\infty (\sin(\sinh(x)+\cosh(x)) - \sin(\sinh(x)-\cosh(x)))\,dt \\
&= \frac{1}{2}\int_0^\infty \big(\sin(e^x) + \sin(e^{-x}))\big)\,dx \\
&= \frac{1}{2}\int_1^\infty \frac{\sin(s)}{s}\,ds + \frac{1}{2}\int_0^1 \frac{\sin(s)}{s}\,ds \\
&= \frac{1}{2}\int_0^\infty \frac{\sin(s)}{s}\,ds = \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4}.
\end{aligned}$$

□