**Problem 11802**
(American Mathematical Monthly, Vol.121, October 2014)

Proposed by I. Mezö (China).

*Let* $H_{n,2} = \sum_{k=1}^{n} 1/k^2$ *and let* $D_n = n! \sum_{k=0}^{n} (-1)^k/k!$. *Prove that*

$$\sum_{n=1}^{\infty} \frac{(-1)^n H_{n,2}}{n!} = \frac{\pi^2}{6e} - \sum_{n=0}^{\infty} \frac{D_n}{n!(n+1)^2}.$$

Solution proposed by Roberto Tauraso, Dipartimento di Matematica, Università di Roma "Tor Vergata", via della Ricerca Scientifica, 00133 Roma, Italy.

$$\begin{aligned}
\sum_{n=1}^{\infty} \frac{(-1)^n H_{n,2}}{n!} &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \sum_{k=1}^{n} \frac{1}{k^2} \\
&= \sum_{k=1}^{\infty} \frac{1}{k^2} \sum_{n=k}^{\infty} \frac{(-1)^n}{n!} \\
&= \sum_{k=1}^{\infty} \frac{1}{k^2} \left( \frac{1}{e} - \sum_{n=0}^{k-1} \frac{(-1)^n}{n!} \right) \\
&= \frac{\pi^2}{6e} - \sum_{k=1}^{\infty} \frac{1}{k^2} \sum_{n=0}^{k-1} \frac{(-1)^n}{n!} \\
&= \frac{\pi^2}{6e} - \sum_{k=0}^{\infty} \frac{1}{(k+1)^2} \sum_{n=0}^{k} \frac{(-1)^n}{n!} \\
&= \frac{\pi^2}{6e} - \sum_{k=0}^{\infty} \frac{D_k}{k!(k+1)^2}.
\end{aligned}$$

□