Problem 11748
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Proposed by C. Lupu (USA) and T. Lupu (Romania).

Is there a sequence $a_1, a_2, \ldots$ *of positive real numbers such that* $\sum_{n=1}^{\infty} 1/a_n$ *converges, and* $\prod_{k=1}^{n} a_k < n^n$ *for all* n*?*

Solution proposed by Roberto Tauraso, Dipartimento di Matematica, Università di Roma "Tor Vergata", via della Ricerca Scientifica, 00133 Roma, Italy.

No, such sequence does not exist, otherwise, by Carleman's inequality, we would have the following contradiction

$$+\infty = \sum_{n=1}^{\infty} \frac{1}{n} \leq \sum_{n=1}^{\infty} \frac{1}{\left(\prod_{k=1}^{n} a_k\right)^{1/n}} \leq e \sum_{n=1}^{\infty} \frac{1}{a_n} < +\infty$$

□