Wightman fields for 2d CFT and integrable perturbation
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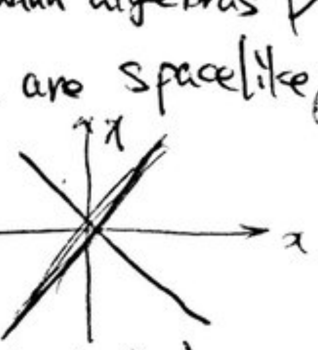
1 Mathematical approaches to Quantum Field Theory.

QFT: Quantum physics (Hilbert space, operators) with infinite degrees of freedom. Axiomatic approaches + examples.

- Gårding-Wightman: unbounded operator-valued distribution Φ.
  $[\Phi(f), \Phi(g)] = 0$ if supp f and supp g are spacelike.
- Araki-Haag-Kastler: net of von Neumann algebras $\{\mathcal{A}(O)\}$
  $[\mathcal{A}(O_1), \mathcal{A}(O_2)] = \{0\}$ if O_1 and O_2 are spacelike

$\mathcal{A}(O) = \{e^{i\Phi(f)} : \mathrm{supp}\, f \subset O\}''$



2. 2d Conformal Field Theories

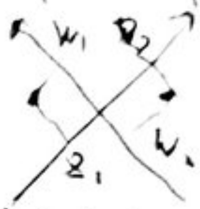
A special class of 2d QFT.
with conformal covariance: $\mathrm{Diff}(\mathbb{R}) \times \mathrm{Diff}(\mathbb{R})$ symmetry.
$\Rightarrow$ chiral components (fixed points w.r.t. $\mathrm{Diff}(\mathbb{R}) \times \mathbb{Z}_2$, $\mathbb{Z}_2 \times \mathrm{Diff}(\mathbb{R})$).
QFT on $\mathbb{R}$ (S^1) conformal net on S^1.
cf. Tachikawa, Carpi, Gui, Tener, Xu.
Representation of a conformal net $\to$ Jones-Wassermann subfactor
2-interval subfactor, complete rationality (Kawahigashi-Longo-Müger)
Extension $\mathcal{A} \subset \mathcal{B}$. $\mathcal{A}(I) \subset \mathcal{B}(I)$.
Kawahigashi-Longo: classification of extensions of the Virasoro nets Vir_c, $c<1$.
2d extensions $\mathrm{Vir}_c \otimes \mathrm{Vir}_c \subset \mathcal{B}$. cf. Longo-Rehren

3. Wightman fields

cf. Henriques-Tener
Carpi-Kawahigashi-Longo-Weiner: VOA $\Longleftrightarrow$ conformal net on S^1
$\Downarrow$
under some technical conditions. (quasi)primary fields
cf. Tener.
2d Wightman fields?

4. Example: $U(1)$-current / Heisenberg alg / massless free field.

Lie alg $\{J_n, I\}$, $[J_m, J_n] = m\delta_{m+n} I$. $\Rightarrow$ vacuum rep.
For $f \in C^\infty(S^1)$. $f(e^{i\theta}) = \sum \hat{f}_n e^{in\theta}$, $J(f) = \sum \hat{f}_n J_n$ $\mathcal{H}_0$
J is a Wightman field on S^1. $\mathcal{A}_{U(1)}(I) = \{e^{iJ(f)}, \operatorname{supp} f \subset I\}''$.
For $\alpha \in \mathbb{R}$, there is a representation $\mathcal{H}_\alpha$ of $\mathcal{A}_{U(1)}$.
given by $J_0 = \alpha$. $\alpha = 0$ is the vacuum rep.
There are intertwining operators $Y_{\alpha,s}$: $\mathcal{H}_\beta \to \mathcal{H}_{\beta+\alpha}$
$[\hat{J}_m, Y_{\alpha,s}] = \alpha Y_{\alpha, m+s}$.
Fix $\alpha_0 \in \mathbb{R}$. Put $\tilde{\mathcal{H}} = \bigoplus_{j \in \mathbb{Z}} \mathcal{H}_{j\alpha}$.



Braiding : $Y_\alpha(z) Y_\alpha(w) = e^{+2\pi i \alpha^2} Y_\alpha(w) Y_\alpha(z)$
if $\arg z > \arg w$. Define $\tilde{Y}_\alpha(z,w) = Y_\alpha(z) \otimes Y_\alpha(w)$
$\alpha \in \mathbb{Z}\alpha_0$.

<u>Thm</u> (Adamo-Giorgetti-T.) $\tilde{Y}_\alpha(z,w)$ is a 2d conformal Wightman field (cf. Rehren).

5. Towards massive deformation
<u>cf.</u> Constructive QFT of ϕ^4_2 (Glimm-Jaffe).
start with the massive free field at $\lambda = 0$.
modify the dynamics (Hamiltonian) by adding
$\int :\phi(x)^4: dx$.
start with a 2d CFT $\tilde{Y}_\alpha(z,w)$ $|\alpha| < \frac{1}{\sqrt{2}}$.
modify the ~~Ham~~ dynamics by $\tilde{Y}_\alpha(z,w)$ (cf. Zamolodchikov)

$\leftarrow$ de Sitter space $\curvearrowleft$ Lorentz group. $\hat{L}_0, \hat{L}_{-1}, \hat{L}_1$

<u>Thm</u> (Jäkel-T.) $L^{\alpha}_m = \hat{L}_m + \lambda \hat{Y}_{\alpha,m}$, $m = 1, -1$, $L^{\alpha}_0 = \hat{L}_0$.
$\{L^\alpha_m\}$ satisfy the Lorentz relations weakly. $\lambda \in \mathbb{R}$

6. Outlook.
domain problem. Euclidean approach?
Work in progress with Adamo, Moriwaki
Full VOA + X $\overset{?}{\Rightarrow}$ Conformal Osterwalder-Schrader