2024Call5.

(1) **Q1**

$\boxed{\text{EMBEDDED ANSWERS}}$ $\boxed{\text{penalty 0.10}}$

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\exp(-3x) = \boxed{a} + \boxed{b}x + \frac{\boxed{c}}{\boxed{d}}x^2 + \frac{\boxed{e}}{\boxed{f}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{a}$:

$\boxed{\text{NUMERICAL}}$ $\boxed{\text{marked out of 1}}$

1 ✓	

$\boxed{b}$:

$\boxed{\text{NUMERICAL}}$ $\boxed{\text{marked out of 1}}$

-3 ✓	

$\boxed{c}$:

$\boxed{\text{NUMERICAL}}$ $\boxed{\text{marked out of 1}}$

9 ✓	

$\boxed{d}$:

$\boxed{\text{NUMERICAL}}$ $\boxed{\text{marked out of 1}}$

2 ✓	

$\boxed{e}$:

$\boxed{\text{NUMERICAL}}$ $\boxed{\text{marked out of 1}}$

-9 ✓	

$\boxed{f}$:

$\boxed{\text{NUMERICAL}}$ $\boxed{\text{marked out of 1}}$

2 ✓	

$$(x+4)^{\frac{3}{2}} = \boxed{h} + \boxed{i}x + \frac{\boxed{j}}{\boxed{k}}x^2 + \frac{\boxed{l}}{\boxed{m}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{h}$:

NUMERICAL | marked out of 1

8 ✓	

i:

NUMERICAL | marked out of 1

3 ✓	

j:

NUMERICAL | marked out of 1

3 ✓	

k:

NUMERICAL | marked out of 1

16 ✓	

l:

NUMERICAL | marked out of 1

-1 ✓	

m:

NUMERICAL | marked out of 1

128 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 0} \frac{\exp(-3x) + (x+4)^{\frac{3}{2}} + \alpha + \beta x^2}{x^3}.$$

This limit converges for $\alpha = \boxed{s}$, $\beta = \frac{\boxed{t}}{\boxed{u}}$.

s:

NUMERICAL | marked out of 6

-9 ✓	

t:

NUMERICAL | marked out of 3

-75 ✓	

u:

NUMERICAL | marked out of 3

16 ✓	

In that case, the limit is $\frac{\boxed{v}}{\boxed{w}}$.

v:

NUMERICAL | marked out of 3

-577 ✓	

w:

NUMERICAL | marked out of 3

128 ✓	

(2) **Q2**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Determine $r = \frac{1}{\sqrt{\boxed{\text{a}}}}, \theta = \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}}$ such that $re^{i\theta} = -\frac{1}{2} + i\frac{1}{2}$ and $0 \leq \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}} < 2\pi$.

a:

NUMERICAL | marked out of 4

2 ✓	

b:

NUMERICAL | marked out of 2

3 ✓	

c:

NUMERICAL | marked out of 2

4 ✓	

Compute $\lim_{n\to\infty}(-\frac{1}{2} + i\frac{1}{2})^n = \boxed{\text{e}}$.

e:

NUMERICAL | marked out of 8

0 ✓	

Consider the series $\sum_{n=0}^{\infty} \frac{2^n+1}{n!} z^n$.

Calculate the partial sum $\sum_{n=0}^{2} \frac{2^n+1}{n!} z^n = \frac{\boxed{\text{j}}}{\boxed{\text{k}}} + i\boxed{\text{l}}$ with $z = i$.

j:

NUMERICAL | marked out of 2

-1 ✓	

k:

NUMERICAL | marked out of 2

2 ✓	

$\boxed{\text{l}}$:

NUMERICAL | marked out of 4

3 ✓	

In order to use the ratio test for $z \in \mathbb{R}$, we put $a_n = \frac{2^n+1}{n!}|z|^n$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{\text{p}}.$$

$\boxed{\text{p}}$:

NUMERICAL | marked out of 8

0 ✓	

Choose all z for which the series converges.

MULTIPLE CHOICE | marked out of 8 | Multiple answers allowed

- -4 (9.0909%)
- -3 (9.0909%)
- -2 (9.0909%)
- -1 (9.0909%)
- -0.5 (9.0909%)
- 0 (9.0909%)
- 0.5 (9.0909%)
- 1 (9.0909%)
- 2 (9.0909%)
- 3 (9.0909%)
- 4 (9.0909%)

(3) **Q3**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function on its natural domain

$$f(x) = \frac{xe^x}{1-x}.$$

Choose all asymptotes of $f(x)$.

MULTIPLE CHOICE | marked out of 4 | Multiple answers allowed | Shuffle

- $y = -\pi$ (-100%)

- $y = -\frac{\pi}{2}$ (-100%)
- $y = -\frac{\pi}{4}$ (-100%)
- $y = 0$ (50%)
- $y = \frac{\pi}{4}$ (-100%)
- $y = \frac{\pi}{2}$ (-100%)
- $y = e$ (-100%)
- $x = -1$ (-100%)
- $x = -\frac{1}{e}$ (-100%)
- $x = 0$ (-100%)
- $x = 1$ (50%)
- $x = \frac{1}{e}$ (-100%)
- $y = x$ (-100%)
- $y = -x$ (-100%)
- $y = ex$ (-100%)

The function $f(x)$ has $\boxed{\text{a}}$ stationary point(s) in the domain

$\boxed{\text{a}}$:

NUMERICAL | marked out of 4

2 (-100%)	

Among the stationary point(s), there is a local maximum at

$$x = \frac{\boxed{\text{b}}}{\boxed{\text{c}}} + \frac{\sqrt{\boxed{\text{d}}}}{\boxed{\text{e}}}.$$

$\boxed{\text{b}}$:

NUMERICAL | marked out of 2

1 (-100%)	

$\boxed{\text{c}}$:

NUMERICAL | marked out of 2

2 (-100%)	

$\boxed{\text{d}}$:

NUMERICAL | marked out of 2

5 (-100%)	

$\boxed{\text{e}}$:

NUMERICAL | marked out of 2

2 (-100%)	

Choose the behaviour of $f(x)$ in the interval $(1, \infty)$.

MULTIPLE CHOICE | marked out of 4 | One answer only | Shuffle

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(4) **Q4**

Embedded answers | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^{\frac{\pi}{2}} \cos^2(x) \sin\left(x + \frac{\pi}{6}\right) dx.$$

Complete the formula

$$\sin\left(x + \frac{\pi}{6}\right) = \frac{\sqrt{\boxed{\text{a}}}}{\boxed{\text{b}}} \sin x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}} \cos x.$$

$\boxed{\text{a}}$:

Numerical | marked out of 2

3 (0%)	

$\boxed{\text{b}}$:

Numerical | marked out of 2

2 (0%)	

$\boxed{\text{c}}$:

Numerical | marked out of 2

1 (0%)	

$\boxed{\text{d}}$:

Numerical | marked out of 2

2 (0%)	

Choose a primitive of $\cos^2(x)\sin(x)$.

Multiple choice | marked out of 8 | One answer only | Shuffle

- $\frac{1}{3}\cos^3(x)\sin(x)$
- $-\frac{1}{3}\cos^3(x)\sin(x)$
- $\frac{1}{3}\sin^3(x)$
- $-\frac{1}{3}\cos^3(x)$ ✓
- $\frac{1}{2}\cos^3(\sin(x))$
- $-\frac{1}{2}\cos^3(\sin(x))$
- $\frac{1}{2}\sin^3(\cos(x))$
- $-\frac{1}{2}\sin^3(\cos(x))$

Choose a primitive of $\cos^3(x)$.

MULTIPLE CHOICE | marked out of 8 | One answer only | Shuffle

- $-\frac{1}{4}\cos^4(x)$
- $\frac{1}{4}\sin^4(x)$
- $\sin(x)+\frac{1}{3}\cos^3(x)$
- $\cos(x)-\frac{1}{3}\cos^3(x)$
- $-\cos(x)+\frac{1}{3}\cos^3(x)$
- $\sin(x)-\frac{1}{3}\sin^3(x)$ ✓
- $x-\frac{1}{3}\sin^3(x)$
- $x-\frac{1}{3}\cos^3(x)$
- $x+\frac{1}{4}\sin^4(x)$
- $x-\frac{1}{4}\cos^4(x)$

By continuing, we get

$$\int_0^{\frac{\pi}{2}} \cos^2(x)\sin\left(x+\frac{\pi}{6}\right)dx = \frac{\boxed{e}}{\boxed{f}} + \frac{\sqrt{\boxed{g}}}{\boxed{h}}$$

.

$\boxed{e}$:

NUMERICAL | marked out of 4

1 (0%)	

$\boxed{f}$:

NUMERICAL | marked out of 4

3 (0%)	

$\boxed{g}$:

NUMERICAL | marked out of 4

3 (0%)	

$\boxed{h}$:

NUMERICAL | marked out of 4

6 (0%)	

(5) **Q5**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = 2\exp(y(x))\cos(x^2)x.$$

MULTIPLE CHOICE | marked out of 2 | One answer only

- $y(x) = \exp(\cos(x^2)) + C$
- $y(x) = \log(\cos(x^2)) + C$
- $y(x) = \exp(-\cos(x^2) + C)$
- $y(x) = -\log(\cos(x^2) + C)$
- $y(x) = \exp(-\cos(x^2)) + C$
- $y(x) = -\log(\sin(x^2)) + C$
- $y(x) = -\log(-\sin(x^2) + C)$ ✓
- $y(x) = \exp(-\sin(x^2)) + C$

Determine $C = \boxed{a}$ with the initial condition $y(\sqrt{\frac{\pi}{2}}) = 0$

$\boxed{a}$:

NUMERICAL | marked out of 2

2 (0%)	

Choose the general solution of the following differential equation.

$$y''(x) - 5y'(x) + 6y(x) = 0.$$

MULTIPLE CHOICE | marked out of 2 | One answer only

- $y(x) = C_1\exp(2x) + C_2\exp(-3x)$
- $y(x) = C_1\exp(-x) + C_2\exp(6x)$
- $y(x) = C_1\exp(2x) + C_2\exp(3x)$ ✓
- $y(x) = C_1\exp(-6x) + C_2\exp(-x)$
- $y(x) = C_1\sin(-3x) + C_2\cos(2x)$
- $y(x) = C_1\sin(-2x) + C_2\cos(3x)$
- $y(x) = C_1\sin(x) + C_2\cos(-6x)$
- $y(x) = C_1\sin(6x) + C_2\cos(x)$

Find a solution $y(x)$ such that $y(0) = 1$ and $y'(0) = 0$. $C_1 = \boxed{b}, C_2 = \boxed{c}$.

$\boxed{b}$:

NUMERICAL | marked out of 1

3 (0%)	

$\boxed{c}$:

NUMERICAL | marked out of 1

-2 (0%)	

For these values of C_1, C_2, find a value $A = \boxed{d}$ such that $\lim_{x\to\infty} y(x)/\exp(Ax) = -2$.

$\boxed{d}$:

NUMERICAL marked out of 2

3 (0%)	

Total of marks: 140