

2024Call4.

(1) Q1

EMBEDDED ANSWERS

penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$(x+1) \log x = \boxed{a} + \boxed{b}(x-1) + \boxed{c}(x-1)^2 + \frac{\boxed{d}}{\boxed{e}}(x-1)^3 + o((x-1)^3) \text{ as } x \rightarrow 1.$$

\boxed{a} :

NUMERICAL

marked out of 1

0 ✓

\boxed{b} :

NUMERICAL

marked out of 1

2 ✓

\boxed{c} :

NUMERICAL

marked out of 1

0 ✓

\boxed{d} :

NUMERICAL

marked out of 1

1 ✓

\boxed{e} :

NUMERICAL

marked out of 2

6 ✓

$$(x-1)\sqrt{x+3} = \boxed{g} + \boxed{h}(x-1) + \frac{\boxed{i}}{\boxed{j}}(x-1)^2 + \frac{\boxed{k}}{\boxed{l}}(x-1)^3 + o((x-1)^3) \text{ as } x \rightarrow 1.$$

\boxed{g} :

NUMERICAL

marked out of 1

0 ✓

\boxed{h} :

NUMERICAL	marked out of 1
2 ✓	
i:	
NUMERICAL	marked out of 1
1 ✓	
j:	
NUMERICAL	marked out of 1
4 ✓	
k:	
NUMERICAL	marked out of 1
-1 ✓	
l:	
NUMERICAL	marked out of 1
64 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \rightarrow 1} \frac{(x+1) \log x + (x-1)\sqrt{x+3} + \alpha(x-1) + \beta(x-1)^2}{(x-1)^3}.$$

This limit converges for $\alpha = \boxed{p}, \beta = \frac{\boxed{q}}{\boxed{r}}$.

p:	
NUMERICAL	marked out of 6
-4 ✓	
q:	
NUMERICAL	marked out of 3
-1 ✓	
r:	
NUMERICAL	marked out of 3
4 ✓	

In that case, the limit is $\frac{\boxed{v}}{\boxed{w}}$.

v:	
NUMERICAL	marked out of 3
29 ✓	
w:	
NUMERICAL	marked out of 3
192 ✓	

(2) Q1

EMBEDDED ANSWERS

penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$(x+2)\log x = \boxed{a} + \boxed{b}(x-1) + \frac{\boxed{c}}{\boxed{d}}(x-1)^2 + \frac{\boxed{e}}{\boxed{f}}(x-1)^3 + o((x-1)^3) \text{ as } x \rightarrow 1.$$

a:

NUMERICAL

marked out of 1

0 ✓

b:

NUMERICAL

marked out of 1

3 ✓

c:

NUMERICAL

marked out of 1

-1 ✓

d:

NUMERICAL

marked out of 1

2 ✓

e:

NUMERICAL

marked out of 1

1 ✓

f:

NUMERICAL

marked out of 1

2 ✓

$$(x-1)\sqrt{x+3} = \boxed{g} + \boxed{h}(x-1) + \frac{\boxed{i}}{\boxed{j}}(x-1)^2 + \frac{\boxed{k}}{\boxed{l}}(x-1)^3 + o((x-1)^3) \text{ as } x \rightarrow 1.$$

g:

NUMERICAL

marked out of 1

0 ✓

h:

NUMERICAL

marked out of 1

2 ✓

i:

NUMERICAL

marked out of 1

1 ✓

j:

NUMERICAL

marked out of 1

4 ✓

k:

NUMERICAL

marked out of 1

-1 ✓

l:

NUMERICAL

marked out of 1

64 ✓

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \rightarrow 1} \frac{(x+1) \log x + (x-1)\sqrt{x+3} + \alpha(x-1) + \beta(x-1)^2}{(x-1)^3}.$$

This limit converges for $\alpha = \frac{p}{q}, \beta = \frac{q}{r}$.

p:

NUMERICAL

marked out of 6

-5 ✓

q:

NUMERICAL

marked out of 3

1 ✓

r:

NUMERICAL

marked out of 3

4 ✓

In that case, the limit is $\frac{v}{w}$.

v:

NUMERICAL

marked out of 3

31 ✓

w:

NUMERICAL

marked out of 3

64 ✓	
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(3) Q2

EMBEDDED ANSWERS

penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Determine $r = \frac{\boxed{a}}{\boxed{b}}$, $\theta = \frac{\boxed{c}}{\boxed{d}}\pi$ such that $re^{i\theta} = \frac{1}{2\sqrt{2}} + i\frac{1}{2\sqrt{2}}$ and

$$0 \leq \frac{\boxed{c}}{\boxed{d}}\pi < 2\pi.$$

 \boxed{a} :

NUMERICAL

marked out of 2

1 ✓	
-----	--

 \boxed{b} :

NUMERICAL

marked out of 2

2 ✓	
-----	--

 \boxed{c} :

NUMERICAL

marked out of 2

1 ✓	
-----	--

 \boxed{d} :

NUMERICAL

marked out of 2

4 ✓	
-----	--

Compute $\lim_{n \rightarrow \infty} \left(\frac{1}{2\sqrt{2}} + i\frac{1}{2\sqrt{2}}\right)^n = \boxed{e}$.

 \boxed{e} :

NUMERICAL

marked out of 8

0 ✓	
-----	--

Consider the series $\sum_{n=0}^{\infty} \frac{2^n+1}{3^n+1} z^n$.

Calculate the partial sum $\sum_{n=0}^2 \frac{2^n+1}{3^n+1} z^n = \frac{\boxed{j}}{\boxed{k}} + i \frac{\boxed{l}}{\boxed{m}}$ with

 $z = i$. \boxed{j} :

NUMERICAL

marked out of 2

1 ✓	
-----	--

k :	
NUMERICAL	marked out of 2
2 ✓	
l :	
NUMERICAL	marked out of 2
3 ✓	
m :	
NUMERICAL	marked out of 2
4 ✓	

Find the largest $r = \frac{p}{q} > 0$ such that the series above converges for all $z \in \mathbb{C}$ with $|z| < r$.

p :	
NUMERICAL	marked out of 4
3 ✓	
q :	
NUMERICAL	marked out of 4
2 ✓	

For $z = -1$, the series

MULTIPLE CHOICE	marked out of 8	One answer only
<ul style="list-style-type: none"> converges absolutely ✓ converges but not absolutely does not converge 		

(4) Q2

EMBEDDED ANSWERS	penalty 0.10
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If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Determine $r = \frac{a}{b}$, $\theta = \frac{c}{d}\pi$ such that $re^{i\theta} = \frac{1}{\sqrt{3}} + i\frac{1}{3}$ and $0 \leq \frac{c}{d}\pi < 2\pi$.

a :	
NUMERICAL	marked out of 2

2 (0%)	
---------	--

b:

NUMERICAL	marked out of 2
-----------	-----------------

3 (0%)	
---------	--

c:

NUMERICAL	marked out of 2
-----------	-----------------

1 (0%)	
---------	--

d:

NUMERICAL	marked out of 2
-----------	-----------------

6 (0%)	
---------	--

Compute $\lim_{n \rightarrow \infty} (\frac{1}{\sqrt{3}} + i\frac{1}{3})^n = \boxed{\text{e}}$.

e:

NUMERICAL	marked out of 8
-----------	-----------------

0 (0%)	
---------	--

Consider the series $\sum_{n=0}^{\infty} \frac{3^n+1}{4^n+1} z^n$.

Calculate the partial sum $\sum_{n=0}^2 \frac{3^n+1}{4^n+1} z^n = \frac{\boxed{\text{j}}}{\boxed{\text{k}}} + i \frac{\boxed{\text{l}}}{\boxed{\text{m}}}$ with

$z = i$.

j:

NUMERICAL	marked out of 2
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7 (0%)	
---------	--

k:

NUMERICAL	marked out of 2
-----------	-----------------

17 (0%)	
----------	--

l:

NUMERICAL	marked out of 2
-----------	-----------------

4 (0%)	
---------	--

m:

NUMERICAL	marked out of 2
-----------	-----------------

5 (0%)	
---------	--

Find the largest $r = \frac{\boxed{\text{p}}}{\boxed{\text{q}}} > 0$ such that the series above converges for all $z \in \mathbb{C}$ with $|z| < r$.

p:

NUMERICAL	marked out of 4
-----------	-----------------

4 (0%)	
---------	--

q:

NUMERICAL

marked out of 4

3 (0%)

For $z = -\frac{4}{3}$, the series

MULTIPLE CHOICE

marked out of 8

One answer only

- converges absolutely
- converges but not absolutely
- does not converge ✓

(5) Q3

EMBEDDED ANSWERS

penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{(x+1)(e^x+3)}{(e^x+1)}$$

This function has two oblique asymptotes. They are $y = \boxed{a}x + \boxed{b}$, $\boxed{c}x + \boxed{d}$ with $\boxed{a} < \boxed{c}$.

a:

NUMERICAL

marked out of 1

1 (0%)

b:

NUMERICAL

marked out of 1

1 (0%)

c:

NUMERICAL

marked out of 1

3 (0%)

d:

NUMERICAL

marked out of 1

3 (0%)

One has

$$f'(0) = \frac{\boxed{e}}{\boxed{f}}.$$

e:

NUMERICAL marked out of 4

3 (0%)

f:

NUMERICAL marked out of 4

2 (0%)

The function $f(x)$ has stationary point(s) in the domain.
(Hint: no need to find it (them) explicitly)

g:

NUMERICAL marked out of 4

0 (0%)

Choose the behaviour of $f(x)$ in the interval $(-\infty, \infty)$.

MULTIPLE CHOICE marked out of 4 One answer only

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(6) Q3

EMBEDDED ANSWERS penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{(x+1)(e^x - 2)}{(e^x + 1)}$$

This function has two oblique asymptotes. They are $y = \text{a}x + \text{b}$, $\text{c}x + \text{d}$ with $\text{a} < \text{c}$.

a:

NUMERICAL marked out of 1

-2 (0%)

b:

NUMERICAL marked out of 1

-2 (0%)

c:

NUMERICAL marked out of 1

1 (0%)	
---------	--

d

:

NUMERICAL	marked out of 1
-----------	-----------------

1 (0%)	
---------	--

One has

$$f'(0) = \frac{\boxed{e}}{\boxed{f}}.$$

e

:

NUMERICAL	marked out of 4
-----------	-----------------

1 (0%)	
---------	--

f

:

NUMERICAL	marked out of 4
-----------	-----------------

4 (0%)	
---------	--

The function $f(x)$ has \boxed{g} stationary point(s) in the domain.
(Hint: no need to find it (them) explicitly)

g

:

NUMERICAL	marked out of 4
-----------	-----------------

1 (0%)	
---------	--

Choose the behaviour of $f(x)$ in the interval $(-\infty, \infty)$.

MULTIPLE CHOICE	marked out of 4	One answer only
-----------------	-----------------	-----------------

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(7) Q4

EMBEDDED ANSWERS	penalty 0.10
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If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the

answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign

should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_2^3 \frac{1}{2^x + 3 + 2(2^{-x})} dx.$$

Let us change the variables $2^x = t$. Complete the formula

$$\int_2^3 \frac{1}{2^x + 3 + 2(2^{-x})} dx = \int_{\boxed{a}}^{\boxed{b}} \frac{1}{\log \boxed{c} (t^2 + \boxed{d}t + \boxed{e})} dt$$

\boxed{a} :

NUMERICAL

marked out of 1

4 (0%)

\boxed{b} :

NUMERICAL

marked out of 1

8 (0%)

\boxed{c} :

NUMERICAL

marked out of 2

2 (0%)

\boxed{d} :

NUMERICAL

marked out of 1

3 (0%)

\boxed{e} :

NUMERICAL

marked out of 1

2 (0%)

By continuing, we get

$$\int_2^3 \frac{1}{2^x + 3 + 2(2^{-x})} dx = \frac{\boxed{f} \boxed{g}}{\log \boxed{i}}.$$

\boxed{f} :

NUMERICAL

marked out of 3

27 (0%)

\boxed{g} :

NUMERICAL

marked out of 3

25 (0%)

\boxed{i} :

NUMERICAL

marked out of 3

2 (0%)

(8) Q4

EMBEDDED ANSWERS

penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the

answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_1^2 \frac{1}{2^x + 5 + 6(2^{-x})} dx.$$

Let us change the variables $2^x = t$. Complete the formula

$$\int_1^2 \frac{1}{2^x + 5 + 6(2^{-x})} dx = \int_{\boxed{a}}^{\boxed{b}} \frac{1}{\log \boxed{c} (t^2 + \boxed{d}t + \boxed{e})} dt$$

\boxed{a} :

NUMERICAL

marked out of 1

2 (0%)

\boxed{b} :

NUMERICAL

marked out of 1

4 (0%)

\boxed{c} :

NUMERICAL

marked out of 2

2 (0%)

\boxed{d} :

NUMERICAL

marked out of 1

5 (0%)

\boxed{e} :

NUMERICAL

marked out of 1

6 (0%)

By continuing, we get

$$\int_1^2 \frac{1}{2^x + 5 + 6(2^{-x})} dx = \frac{\frac{\boxed{f}}{\boxed{g}}}{\log \boxed{i}}.$$

\boxed{f} :

NUMERICAL

marked out of 3

15 (0%)

\boxed{g} :

NUMERICAL

marked out of 3

14 (0%)

i:

NUMERICAL

marked out of 3

2 (0%)

(9) Q5

EMBEDDED ANSWERS

penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral based on definition.

$$\int_{-\infty}^{\infty} \frac{x^2}{x^6 + 1} dx$$

Choose a primitive of the integrated function $\frac{x^2}{x^6+1}$.

MULTIPLE CHOICE

marked out of 2

One answer only

- $(x^6 + 1)^{\frac{1}{2}}$
- $3x(x^6 + 1)^{\frac{1}{2}}$
- $3 \log(x^6 + 1)$
- $3x^2 \log(x^6)$
- $1/(x^6 + 1)^2$
- $x^2/(x^6 + 1)^2$
- $\arctan(x^3)/3$ ✓
- $3 \arctan(x^3 + 1)$

Calculate $\int_{-\infty}^{\infty} \frac{x^2}{x^6+1} dx = \frac{a}{b} \pi$

a:

NUMERICAL

marked out of 1

1 (0%)

b:

NUMERICAL

marked out of 1

3 (0%)

Let $s > 0$. Choose the value of s for which the improper integral $\int_0^{\infty} \frac{x^2}{(x^6+1)^s} dx$ converges.

MULTIPLE CHOICE

marked out of 2

Multiple answers allowed

- 0.1 (−100%)

- 0.2 (−100%)
- 0.3 (−100%)
- 0.4 (−100%)
- 0.5 (−100%)
- 0.6 (12.5%)
- 0.7 (12.5%)
- 0.8 (12.5%)
- 0.9 (12.5%)
- 1 (12.5%)
- 1.5 (12.5%)
- 2 (12.5%)
- 3 (12.5%)

Consider the following three improper integrals.

$$(1) \int_1^{\infty} x^{100} e^{-x} dx, \quad (2) \int_0^{\infty} x^{101} e^{-x} dx, \quad (3) \int_0^{\infty} e^{-100x} dx, \quad (4) \int_0^{\infty} e^{-x} dx.$$

Give the correct order. $(\boxed{c}) < (\boxed{d}) < (\boxed{e}) < (\boxed{f})$.

\boxed{c} :

NUMERICAL	marked out of 1
3 (13%)	

\boxed{d} :

NUMERICAL	marked out of 1
4 (13%)	

\boxed{e} :

NUMERICAL	marked out of 1
1 (13%)	

\boxed{f} :

NUMERICAL	marked out of 1
2 (13%)	

(10) **Q5**

EMBEDDED ANSWERS	penalty 0.10
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If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral based on definition.

$$\int_{-\infty}^{\infty} \frac{x^3}{x^8 + 1} dx$$

Choose a primitive of the integrated function $\frac{x^3}{x^8+1}$.

MULTIPLE CHOICE

marked out of 2

One answer only

- $(x^8 + 1)^{\frac{1}{2}}$
- $4x(x^8 + 1)^{\frac{1}{2}}$
- $4\log(x^8 + 1)$
- $4x^2\log(x^8)$
- $\arctan(x^4)/4$ ✓
- $4\arctan(x^4 + 1)$
- $1/(x^8 + 1)^2$
- $x^3/(x^8 + 1)^2$

Calculate $\int_{-\infty}^{\infty} \frac{x^3}{x^8+1} dx =$

:

NUMERICAL

marked out of 2

0 (0%)

Let $s > 0$. Choose the value of s for which the improper integral $\int_0^{\infty} \frac{x}{(x^8+1)^s} dx$ converges.

MULTIPLE CHOICE

marked out of 2

Multiple answers allowed

- 0.1 (−100%)
- 0.2 (−100%)
- 0.3 (9.09119%)
- 0.4 (9.09119%)
- 0.5 (9.09119%)
- 0.6 (9.09119%)
- 0.7 (9.09119%)
- 0.8 (9.09119%)
- 0.9 (9.09119%)
- 1 (9.09119%)
- 1.5 (9.09119%)
- 2 (9.09119%)
- 3 (9.09119%)

Consider the following three improper integrals.

$$(1) \int_0^{\infty} e^{-x} dx, \quad (2) \int_0^{\infty} e^{-100x} dx, \quad (3) \int_0^{\infty} x^{101} e^{-x} dx, \quad (4) \int_1^{\infty} x^{100} e^{-x} dx.$$

Give the correct order. $(\boxed{c}) < (\boxed{d}) < (\boxed{e}) < (\boxed{f})$.

c:	
NUMERICAL	marked out of 1
2 (9%)	
d:	
NUMERICAL	marked out of 1
1 (9%)	
e:	
NUMERICAL	marked out of 1
4 (9%)	
f:	
NUMERICAL	marked out of 1
3 (9%)	

Total of marks: 230