2024Call4.

(1) **Q1**

EMBEDDED ANSWERS penalty 0.10

If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$(x+1)\log x = \boxed{\mathbf{a}} + \boxed{\mathbf{b}}(x-1) + \boxed{\mathbf{c}}(x-1)^2 + \boxed{\frac{\mathbf{d}}{\mathbf{e}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

a:	
Numerical marked out of 1	
0 🗸	
b:	
Numerical marked out of 1	
2 🗸	
<u>c</u> :	
Numerical marked out of 1	
0 🗸	
d:	
Numerical marked out of 1	
1 🗸	
e:	
Numerical marked out of 2	
6 🗸	

$$(x-1)\sqrt{x+3} = \boxed{g} + \boxed{h}(x-1) + \frac{\boxed{i}}{\boxed{j}}(x-1)^2 + \frac{\boxed{k}}{\boxed{1}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

g:	
Numerical marked out of 1	
0 🗸	
h:	

Numerical marked out of 1
2 ✓
i:
Numerical marked out of 1
1 🗸
j :
NUMERICAL marked out of 1
4 ✓
k:
Numerical marked out of 1
-1 ✓
1:
Numerical marked out of 1
64 🗸
For various $\alpha, \beta \in \mathbb{R}$, study the limit:
$\lim_{x \to 1} \frac{(x+1)\log x + (x-1)\sqrt{x+3} + \alpha(x-1) + \beta(x-1)^2}{(x-1)^3}.$
$\frac{1}{x \to 1} \frac{1}{(x-1)^3}$
This limit converges for $\alpha = p$, $\beta = q$.
This limit converges for $\alpha = \boxed{p}, \beta = \boxed{q}$. \boxed{p} :
p:
P: Numerical marked out of 6
P: Numerical marked out of 6 -4 ✓
p: Numerical marked out of 6 -4 ✓ q:
P: Numerical marked out of 6 -4 ✓ Q: Numerical marked out of 3 -1 ✓ r:
P: Numerical marked out of 6 -4 ✓ Q: Numerical marked out of 3 -1 ✓ r: Numerical marked out of 3
P: NUMERICAL marked out of 6 -4 Q: NUMERICAL marked out of 3 -1 r: NUMERICAL marked out of 3
P: Numerical marked out of 6 -4 ✓ Q: Numerical marked out of 3 -1 ✓ r: Numerical marked out of 3
P: NUMERICAL marked out of 6 -4 ✓ Q: NUMERICAL marked out of 3 -1 ✓ r: NUMERICAL marked out of 3 4 ✓ In that case, the limit is w. V:
P: Numerical marked out of 6 -4 Q: Numerical marked out of 3 -1 r: Numerical marked out of 3 4 In that case, the limit is v V: Numerical marked out of 3
P: Numerical marked out of 6 -4 Q: Numerical marked out of 3 -1 T: Numerical marked out of 3 4 In that case, the limit is W V: Numerical marked out of 3
P: Numerical marked out of 6 -4 Q: Numerical marked out of 3 -1 r: Numerical marked out of 3 4 In that case, the limit is w. V: Numerical marked out of 3 29 W:
P: Numerical marked out of 6 -4 Q: Numerical marked out of 3 -1 T: Numerical marked out of 3 4 In that case, the limit is W V: Numerical marked out of 3

(2) Q1

EMBEDDED ANSWERS penalty 0.10

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

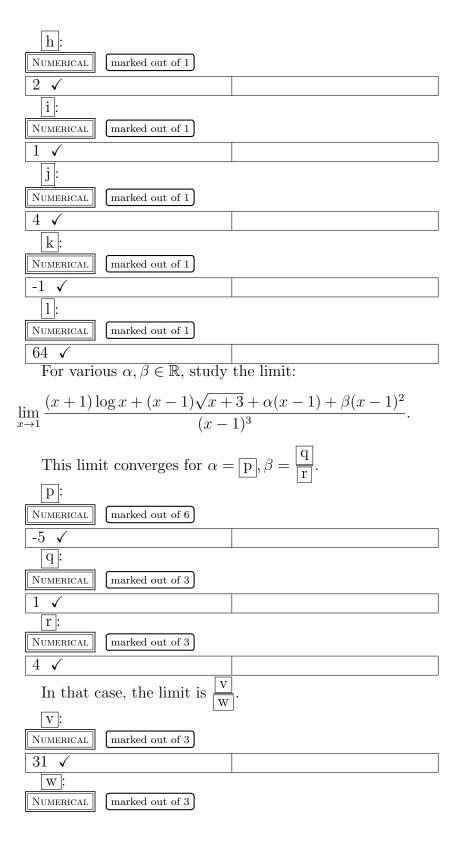
$$(x+2)\log x = \boxed{a} + \boxed{b}(x-1) + \boxed{c}{\boxed{d}}(x-1)^2 + \boxed{e}{\boxed{f}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

[a]:
NUMERICAL marked out of 1
0 🗸
b:
NUMERICAL marked out of 1
3 ✓
<u>C</u> :
NUMERICAL marked out of 1
-1 ✓
d:
NUMERICAL marked out of 1
2 🗸
e:
NUMERICAL marked out of 1
1 🗸
f :
NUMERICAL marked out of 1
2 🗸

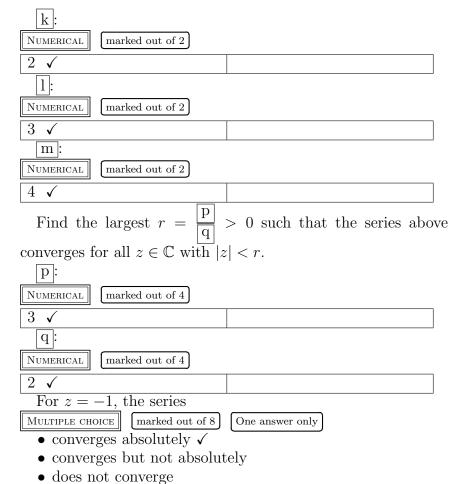
$$(x-1)\sqrt{x+3} = \boxed{g} + \boxed{h}(x-1) + \boxed{\frac{1}{\boxed{j}}}(x-1)^2 + \boxed{\frac{k}{\boxed{1}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

g:
Numerical marked out of 1

0 ✓



	64 🗸				
)	$\mathbf{Q}2$				
	EMBEDDED ANSWERS penalty 0.10				
If not specified otherwise, fill in the blanks with integers (pe					
	sibly 0 or negative). A fraction should be reduced (for ex-				
	ample, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the				
	answer boxes (such as $\begin{bmatrix} a \\ b \end{bmatrix}$) have ambiguity, the negative sign				
	should be put on the numerator (for example $\frac{-1}{2}$ is accepted				
	but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.				
	Determine $r = \frac{\boxed{a}}{\boxed{b}}, \theta = \frac{\boxed{c}\pi}{\boxed{d}}$ such that $re^{i\theta} = \frac{1}{2\sqrt{2}} + i\frac{1}{2\sqrt{2}}$ and				
	$0 \le \frac{ \mathbf{c} ^{\pi}}{ \mathbf{d} } < 2\pi.$				
	a:				
	NUMERICAL marked out of 2				
	1				
	<u>b</u> :				
	Numerical marked out of 2				
	2 🗸				
	c:				
	Numerical marked out of 2				
	1 🗸				
	d:				
	Numerical marked out of 2				
	4 \(\)				
	Compute $\lim_{n\to\infty} \left(\frac{1}{2\sqrt{2}} + i\frac{1}{2\sqrt{2}}\right)^n = \boxed{e}$.				
	<u>e</u> :				
	Numerical marked out of 8				
	0 🗸				
	Consider the series $\sum_{n=0}^{\infty} \frac{2^n+1}{3^n+1} z^n$.				
	Calculate the partial sum $\sum_{n=0}^{2} \frac{2^{n}+1}{3^{n}+1} z^{n} = \frac{\boxed{j}}{\boxed{k}} + i \frac{\boxed{l}}{\boxed{m}}$ with				
	z = i.				
	<u> </u>				
	Numerical marked out of 2				
	1 🗸				



(4) **Q2**

Embedded answers penalty 0.10

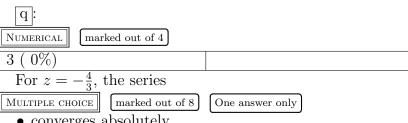
If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Determine $r = \frac{\mathbf{a}}{\mathbf{b}}, \theta = \frac{\mathbf{C}\pi}{\mathbf{d}}$ such that $re^{i\theta} = \frac{1}{\sqrt{3}} + i\frac{1}{3}$ and $0 \le \frac{\mathbf{C}\pi}{\mathbf{d}} < 2\pi$.

a:

Numerical marked out of 2

2 (0%)	
b:	
Numerical marked out of 2	
3 (0%)	
C:	
Numerical marked out of 2	
1 (0%)	
d:	
Numerical [marked out of 2]	
6 (0%)	
Compute $\lim_{n\to\infty} (\frac{1}{\sqrt{3}} + i\frac{1}{3})^n$	= $[e]$.
e:	
NUMERICAL marked out of 8	
0 (0%)	11
Consider the series $\sum_{n=0}^{\infty} \frac{3^n}{4^n}$.	$\frac{+1}{+1}z^n$.
Calculate the partial sum	$\sum_{n=0}^{2} \frac{3^{n}+1}{4^{n}+1} z^{n} = \frac{\left \mathbf{j} \right }{\left \mathbf{k} \right } + i \frac{\left \mathbf{l} \right }{\left \mathbf{m} \right } \text{ with }$
z = i.	
<u>j</u> :	
Numerical marked out of 2	
7 (0%)	
k:	
Numerical marked out of 2	
17 (0%)	
1:	
Numerical marked out of 2	
4 (0%)	
<u>m</u> :	
Numerical marked out of 2	
5 (0%)	
Find the largest $r = \frac{ \mathbf{p} }{ \mathbf{q} }$	> 0 such that the series above
converges for all $z \in \mathbb{C}$ with $ z $	< r.
p:	
Numerical marked out of 4	
4 (0%)	



- converges absolutely
- converges but not absolutely
- does not converge ✓

(5) **Q3**

penalty 0.10 Embedded answers

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|a|}{|b|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{(x+1)(e^x + 3)}{(e^x + 1)}$$

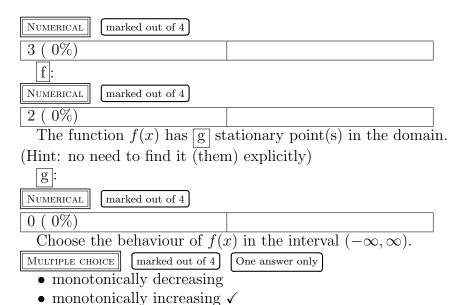
This function has two oblique asymptotes. They are y =[a]x + [b], [c]x + [d] with [a] < [c].

a : Numericalmarked out of 1 1 (0%) b : Numerical marked out of 11 (0%)c: Numerical marked out of 1 3(0%)d: Numerical marked out of 1 3 (0%)

One has

$$f'(0) = \frac{\boxed{e}}{\boxed{f}}.$$

e:



• neither decreasing nor increasing (6) Q3

EMBEDDED ANSWERS penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{(x+1)(e^x - 2)}{(e^x + 1)}$$

This function has two oblique asymptotes. They are $y = \boxed{ax + b}, \boxed{cx + d}$ with $\boxed{a} < \boxed{c}$.

| a | x + | b | C | x + | d | WITH | a | < | C |.
| a |:
| NUMERICAL | marked out of 1

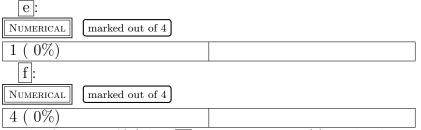
| -2 (0%) |
| b |:
| NUMERICAL | marked out of 1

| -2 (0%) |
| c |:

Numerical marked out of 1

1 (0%)	
d:	
Numerical marked out of 1	
1 (0%)	
One has	

$$f'(0) = \frac{\boxed{e}}{\boxed{f}}.$$



The function f(x) has \boxed{g} stationary point(s) in the domain. (Hint: no need to find it (them) explicitly)

Numerical marked out of 4

Choose the behaviour of f(x) in the interval $(-\infty, \infty)$.

MULTIPLE CHOICE marked out of 4 One answer only

- monotonically decreasing
- ullet monotonically increasing
- \bullet neither decreasing nor increasing \checkmark

$(7) \mathbf{Q4}$

1(0%)

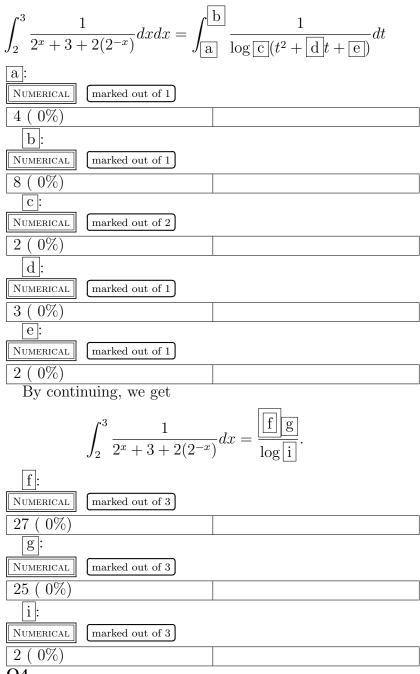
EMBEDDED ANSWERS penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{2}^{3} \frac{1}{2^{x} + 3 + 2(2^{-x})} dx.$$

Let us change the variables $2^x = t$. Complete the formula



 $(8) \mathbf{Q4}$

Embedded answers penalty 0.10

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the

answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{1}^{2} \frac{1}{2^{x} + 5 + 6(2^{-x})} dx.$$

Let us change the variables $2^x = t$. Complete the formula

$$\int_{1}^{2} \frac{1}{2^{x}+5+6(2^{-x})} dx dx = \int_{\boxed{a}}^{\boxed{b}} \frac{1}{\log \boxed{c}(t^{2}+\boxed{d}t+\boxed{e})} dt$$

$$\boxed{a}:$$

$$\boxed{\text{NUMERICAL}} \quad \text{marked out of 1}$$

$$\boxed{2 (0\%)}$$

$$\boxed{b}:$$

$$\boxed{\text{NUMERICAL}} \quad \text{marked out of 1}$$

$$\boxed{4 (0\%)}$$

$$\boxed{c}:$$

$$\boxed{\text{NUMERICAL}} \quad \text{marked out of 2}$$

$$\boxed{2 (0\%)}$$

$$\boxed{d}:$$

$$\boxed{\text{NUMERICAL}} \quad \text{marked out of 1}$$

$$\boxed{5 (0\%)}$$

$$\boxed{e}:$$

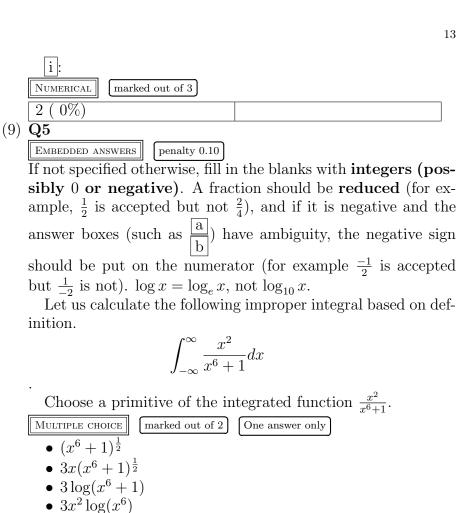
$$\boxed{\text{NUMERICAL}} \quad \text{marked out of 1}$$

$$\boxed{6 (0\%)}$$

By continuing, we get

$$\int_{1}^{2} \frac{1}{2^{x} + 5 + 6(2^{-x})} dx = \frac{\boxed{\mathbf{f}}}{\log \boxed{\mathbf{i}}}.$$

f:		
Numerical	marked out of 3	
15 (0%)		
g:		
Numerical	marked out of 3	
14 (0%)		



• $1/(x^6+1)^2$ • $x^2/(x^6+1)^2$

a : Numerical

1 (0%)b : Numerical

• $\arctan(x^3)/3$ \checkmark • $3\arctan(x^3+1)$

Calculate $\int_{-\infty}^{\infty} \frac{x^2}{x^6+1} dx = \frac{\boxed{a}\pi}{\boxed{b}}$

marked out of 1

marked out of 1

integral $\int_0^\infty \frac{x^2}{(x^6+1)^s} dx$ converges. MULTIPLE CHOICE marked out of 2

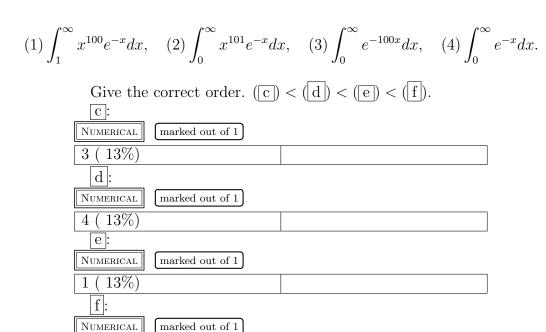
• 0.1 (-100%)

Let s > 0. Choose the value of s for which the improper

Multiple answers allowed

- 0.2 (-100%)
- 0.3 (-100%)
- 0.4 (−100%)
- 0.5 (-100%)
- 0.6 (12.5%)
- 0.7 (12.5%)
- 0.8 (12.5%)
- 0.9 (12.5%)
- 1 (12.5%)
- 1.5 (12.5%)
- 2 (12.5%)
- 3 (12.5%)

Consider the following three improper integrals.



$(10) \ \underline{\mathbf{Q5}}$

2 (13%)

Embedded answers penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral based on definition.

$$\int_{-\infty}^{\infty} \frac{x^3}{x^8 + 1} dx$$

Choose a primitive of the integrated function $\frac{x^3}{x^8+1}$.

MULTIPLE CHOICE marked out of 2 One answer only

- $(x^8+1)^{\frac{1}{2}}$
- $4x(x^8+1)^{\frac{1}{2}}$
- $4\log(x^8+1)$
- $4x^2 \log(x^8)$
- $\arctan(x^4)/4$
- $4\arctan(x^4+1)$

• $1/(x^8 + 1)^2$ • $x^3/(x^8 + 1)^2$ Calculate $\int_{-\infty}^{\infty} \frac{x^3}{x^8 + 1} dx = \boxed{a}$

Numerical marked out of 2

0 (0%)

Let s > 0. Choose the value of s for which the improper integral $\int_0^\infty \frac{x}{(x^8+1)^s} dx$ converges.

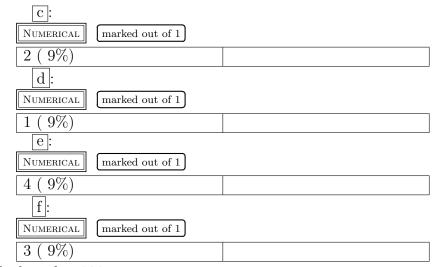
MULTIPLE CHOICE marked out of 2 Multiple answers allowed

- 0.1 (-100%)
- 0.2 (-100%)
- 0.3 (9.09119%)
- 0.4 (9.09119%)
- 0.5 (9.09119%)
- 0.6 (9.09119%)
- 0.7 (9.09119%)
- 0.8 (9.09119%)
- 0.9 (9.09119%)
- 1 (9.09119%)
- 1.5 (9.09119%)
- 2 (9.09119%)
- 3 (9.09119%)

Consider the following three improper integrals.

$$(1) \int_0^\infty e^{-x} dx, \quad (2) \int_0^\infty e^{-100x} dx, \quad (3) \int_0^\infty x^{101} e^{-x} dx, \quad (4) \int_1^\infty x^{100} e^{-x} dx.$$

Give the correct order. $(\boxed{c}) < (\boxed{d}) < (\boxed{e}) < (\boxed{f})$.



Total of marks: 230