2024Call3.

(1) **Q1**

EMBEDDED ANSWERS [penalty 0.10]

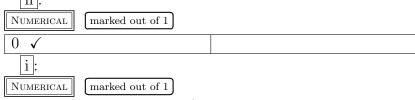
If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$(x^{2}+1)^{\frac{3}{2}} = [a] + [b]x + \frac{c}{d}x^{2} + [e]x^{3} + o(x^{3}) \text{ as } x \to 0.$$



$$x\log(1+3x) = \boxed{\mathbf{h}} + \boxed{\mathbf{i}}x + \boxed{\mathbf{j}}x^2 + \frac{\boxed{\mathbf{k}}}{\boxed{\mathbf{l}}}x^3 + o(x^3) \text{ as } x \to 0.$$



j :
NUMERICAL marked out of 2
k:
NUMERICAL marked out of 2
-9 V
NUMERICAL marked out of 2
For various $\alpha, \beta \in \mathbb{R}$, study the limit:
$\lim_{x \to 0} \frac{(x^2 + 1)^{\frac{3}{2}} + x \log(1 + 3x) + \alpha + \beta x^2}{x^2 \sin(x)}.$
$\lim_{x \to 0} \frac{1}{x^2 \sin(x)}.$
This limit converges for $\alpha = [\underline{s}], \beta = [\underline{t}]$.
s:
NUMERICAL marked out of 8
-1 🗸
t:
NUMERICAL marked out of 4
$-9 \checkmark$
NUMERICAL marked out of 4
V
In that case, the limit is \boxed{W} .
V:
NUMERICAL marked out of 4
-9 🗸
W:
NUMERICAL marked out of 4
$2 \checkmark$
Q1
Q1 EMBEDDED ANSWERS penalty 0.10

EMBEDDED ANSWERS [penalty 0.10] If not specified otherwise, fill in the blanks with **integers (pos-sibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the

answer boxes (such as $\begin{bmatrix} a \\ b \end{bmatrix}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$. Complete the formulae.

$$(x^{2}+1)^{\frac{4}{3}} = \boxed{a} + \boxed{b}x + \frac{\boxed{c}}{\boxed{d}}x^{2} + \boxed{e}x^{3} + o(x^{3}) \text{ as } x \to 0.$$

a :	
NUMERICAL marked out of 2	
1 🗸	
b:	
NUMERICAL marked out of 1	
0 🗸	
<u>C</u> :	
NUMERICAL marked out of 2	
4 🗸	
d	
NUMERICAL marked out of 2	
3 🗸	
e:	
NUMERICAL marked out of 1	
0 🗸	

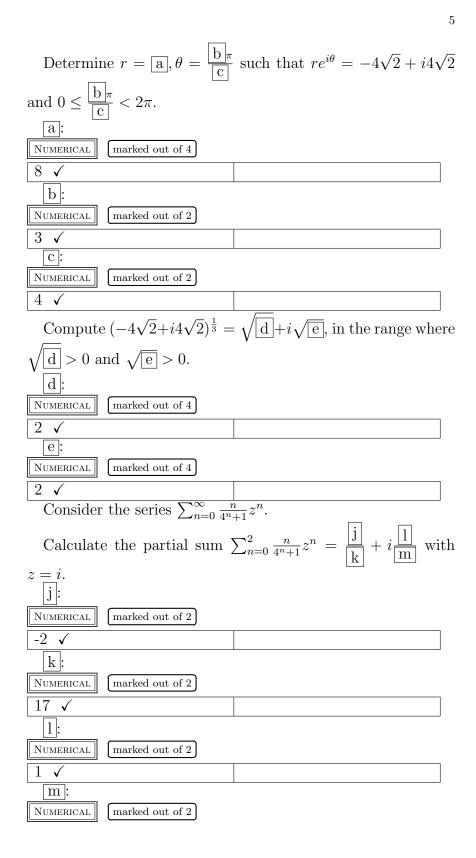
$$x \log(1+5x) = \boxed{h} + \boxed{i}x + \boxed{j}x^2 + \frac{\boxed{k}}{\boxed{l}}x^3 + o(x^3) \text{ as } x \to 0.$$

h :	
NUMERICAL marked out of 1	
0 🗸	
i:	
NUMERICAL marked out of 1	
0 🗸	
j:	
NUMERICAL marked out of 2	
5 🗸	
k:	

NUMERICAL marked out of 2
-25 🗸
NUMERICAL marked out of 2
$2 \checkmark$
For various $\alpha, \beta \in \mathbb{R}$, study the limit:
$\lim_{x \to 0} \frac{(x^2 + 1)^{\frac{4}{3}} + x \log(1 + 5x) + \alpha + \beta x^2}{x^2 \sin(x)}.$
$\lim_{x \to 0} \frac{1}{x^2 \sin(x)}.$
This limit converges for $\alpha = [s], \beta = [t]$.
NUMERICAL marked out of 8
NUMERICAL marked out of 4
-19 ✓
NUMERICAL marked out of 4
In that case, the limit is $\begin{bmatrix} v \\ w \end{bmatrix}$.
V:
NUMERICAL marked out of 4
-25 🗸
W:
NUMERICAL marked out of 4
2 🗸
$(3) \mathbf{Q2}$

Embedded answers penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.



$5 \checkmark$
Find the largest $r = [p] > 0$ such that the series above con-
verges for all $z \in \mathbb{C}$ with $ z < r$.
p:
NUMERICAL marked out of 8
$4 \checkmark$
For $z = -4$, the series
MULTIPLE CHOICE marked out of 8 One answer only
• converges absolutely
• converges but not absolutely
• does not converge \checkmark
$(4) \mathbf{Q2}$
EMBEDDED ANSWERS penalty 0.10
If not specified otherwise, fill in the blanks with integers (pos-
sibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the
answer boxes (such as $\frac{ \mathbf{a} }{ \mathbf{b} }$) have ambiguity, the negative sign
should be put on the numerator (for example $\frac{-1}{2}$ is accepted
but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.
Determine $r = \boxed{a}\sqrt{2}, \theta = \frac{\boxed{b}_{\pi}}{\boxed{c}}$ such that $re^{i\theta} = -4 - 4i$ and
$0 \le \frac{b_{\pi}}{c} < 2\pi.$
NUMERICAL marked out of 4
4 (0%)
b:
NUMERICAL (marked out of 2)
5 (0%)
C:
NUMERICAL marked out of 2
4 (0%)
Compute $(-4 - 4i)^{\frac{1}{5}} = d + ie$, in the range where $d > 0$
and $e > 0$.
<u>d</u> :
NUMERICAL marked out of 4

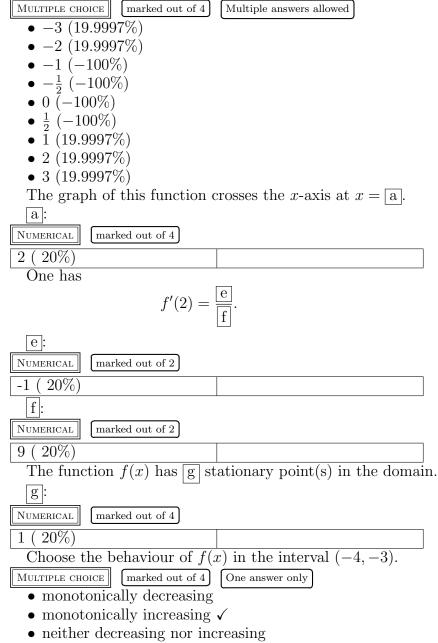
	e:
	NUMERICAL marked out of 4
	Consider the series $\sum_{n=0}^{\infty} \frac{n}{3^n+1} z^n$.
	Calculate the partial sum $\sum_{n=0}^{2} \frac{n}{3^{n}+1} z^{n} = \frac{j}{k} + i \frac{j}{m}$ with
	z = i.
	j:
	NUMERICAL marked out of 2
	-1 (0%)
	k :
	NUMERICAL marked out of 2
	NUMERICAL marked out of 2
	m:
	NUMERICAL marked out of 2
	Find the largest $r = [p] > 0$ such that the series above con-
	verges for all $z \in \mathbb{C}$ with $ z < r$.
	p:
	NUMERICAL marked out of 8
	For $z = -3$, the series
	MULTIPLE CHOICE marked out of 8 One answer only
	• converges absolutely
	• converges but not absolutely
(5)	• does not converge ✓ Q3
(0)	EMBEDDED ANSWERS penalty 0.10
	If not specified otherwise, fill in the blanks with integers (pos-
	sibly 0 or negative). A fraction should be reduced (for ex-

If not specified otherwise, fill in the blanks with **integers (pos-sibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as \boxed{a}) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function for $x \in \mathbb{R}$

$$f(x) = \log \frac{2x^2 + 1}{2x^2 + x - 1}$$

The function $f(x)$ is not defined on the whole real line l	$\mathbb{R}.$
Choose all the points that are in the natural domain of $f(x)$).



(6) **Q3**

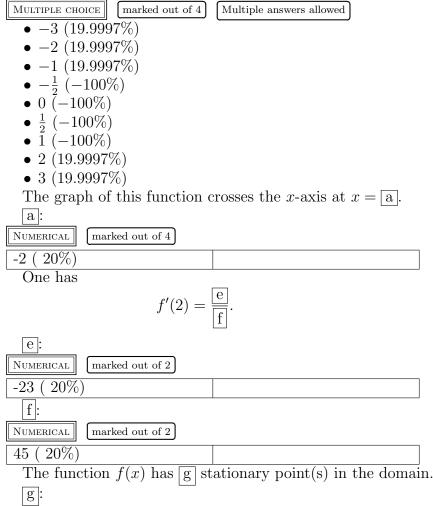
EMBEDDED ANSWERS penalty 0.10	Embedded answers	penalty 0.10
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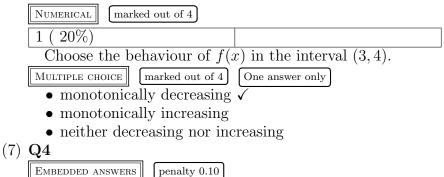
If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function for $x \in \mathbb{R}$

$$f(x) = \log \frac{2x^2 + 1}{2x^2 - x - 1}$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that **are** in the natural domain of f(x).





If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). log $x = \log_e x$, not $\log_{10} x$.

ar + b

d

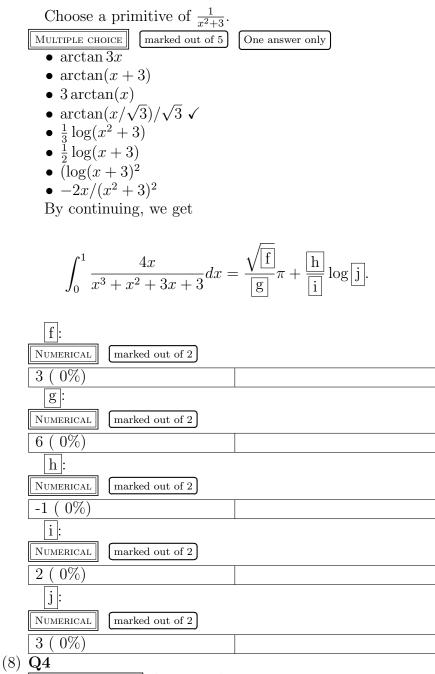
Let us calculate the following integral.

$$\int_0^1 \frac{4x}{x^3 + x^2 + 3x + 3} dx$$

Complete the formula

 Δr

$\frac{4x}{x^3 + x^2 + 3x + 3} = \frac{12x}{x^2 + 10} + \frac{14}{x + 10}.$	
a	
NUMERICAL marked out of 2	
b:	
NUMERICAL marked out of 2	
C:	
NUMERICAL marked out of 2	
d:	
NUMERICAL marked out of 2	
e:	
NUMERICAL marked out of 2	



EMBEDDED ANSWERS penalty 0.10

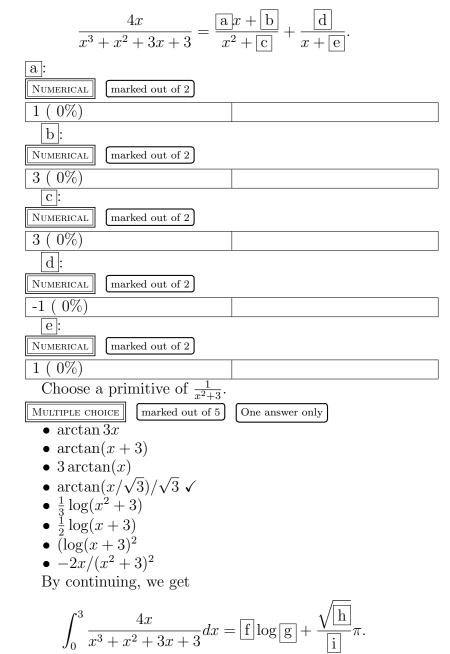
If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign

should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

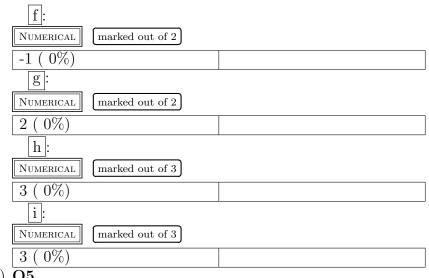
Let us calculate the following integral.

$$\int_0^3 \frac{4x}{x^3 + x^2 + 3x + 3} dx.$$

Complete the formula



12



(9) **Q5**

EMBEDDED ANSWERS [penalty 0.10]

If not specified otherwise, fill in the blanks with **integers (pos-sibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\boxed{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = \cos(x)\exp(\sin(x))y(x)^2$$

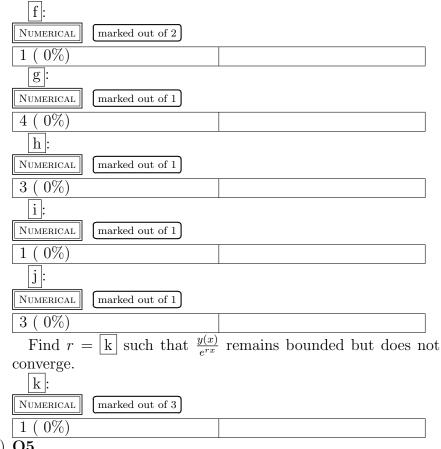
Multiple choice marked out of 3 One answer only Shuffle • $y(x) = C/(\exp(\sin(x)))$ • $y(x) = 1/(\exp(\sin(x+C)))$ • $y(x) = -1/(\exp(\sin(x)) + C) \checkmark$ • $y(x) = C/(\exp(\cos(x)))$ • $y(x) = 1/(\exp(\cos(x+C)))$ • $y(x) = 1/(C - (\cos(x)))$ • $y(x) = \exp(\exp(\sin(x))) + C$ • $y(x) = \exp(C\exp(\cos(x)))$ Determine C = [a] with the initial condition $y(0) = \frac{1}{2}$ a : NUMERICAL marked out of 3

 -3 (0%)

 Consider the following differential equation.

$$y''(x) - 2y'(x) + 10y(x) = 0$$

A solution satisfying y(0) = 4 and y'(0) = 7 can be written as $y(x) = \exp([fx)([g]\cos([hx]) + [i]\sin([jx])).$



(10) **Q5**

Embedded answers penalty 0.10

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = \cos(x)\exp(\sin(x))y(x)^2$$

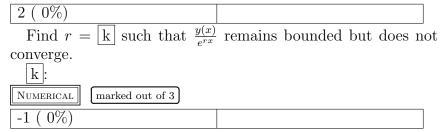
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MULTIPLE CHOICE marked out of 3 One answer only Shuffle
• $y(x) = C/(\exp(\sin(x)))$
• $y(x) = 1/(\exp(\sin(x+C)))$
• $y(x) = -1/(\exp(\sin(x)) + C) \checkmark$
• $y(x) = C/(\exp(\cos(x)))$
• $y(x) = 1/(\exp(\cos(x+C)))$
• $y(x) = 1/(C - (\cos(x)))$
• $y(x) = \exp(\exp(\sin(x))) + C$
• $y(x) = \exp(C \exp(\cos(x)))$
Determine $C = [a]$ with the initial condition $y(0) = 1$
a :
NUMERICAL marked out of 3
-2 (0%)

Consider the following differential equation.

$$y''(x) + 2y'(x) + 5y(x) = 0$$

A solution satisfying y(0) = 3 and y'(0) = -5 can be written as $y(x) = \exp(fx)(g\cos(hx) + i\sin(jx))$.

1:		
NUMERICAL	marked out of 2	
-1 (0%)		
g:		
NUMERICAL	marked out of 1	
3 (0%)		
h:		
NUMERICAL	marked out of 1	
2 (0%)		
i:		
NUMERICAL	marked out of 1	
-1 (0%)		
j:		
NUMERICAL	marked out of 1	



Total of marks: 280

16