2024Call3.

(1) **Q1**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$(x^2+1)^{\frac{3}{2}} = \boxed{\text{a}} + \boxed{\text{b}}x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}x^2 + \boxed{\text{e}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{\text{a}}$:

NUMERICAL | marked out of 2

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | marked out of 1

0 ✓	

$\boxed{\text{c}}$:

NUMERICAL | marked out of 2

3 ✓	

$\boxed{\text{d}}$:

NUMERICAL | marked out of 2

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | marked out of 1

0 ✓	

$$x\log(1+3x) = \boxed{\text{h}} + \boxed{\text{i}}x + \boxed{\text{j}}x^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{\text{h}}$:

NUMERICAL | marked out of 1

0 ✓	

$\boxed{\text{i}}$:

NUMERICAL | marked out of 1

0 ✓	

j:

NUMERICAL marked out of 2

3 ✓	

k:

NUMERICAL marked out of 2

-9 ✓	

l:

NUMERICAL marked out of 2

2 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 0} \frac{(x^2+1)^{\frac{3}{2}} + x\log(1+3x) + \alpha + \beta x^2}{x^2 \sin(x)}.$$

This limit converges for $\alpha = \boxed{\text{s}}, \beta = \frac{\boxed{\text{t}}}{\boxed{\text{u}}}$.

s:

NUMERICAL marked out of 8

-1 ✓	

t:

NUMERICAL marked out of 4

-9 ✓	

u:

NUMERICAL marked out of 4

2 ✓	

In that case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

v:

NUMERICAL marked out of 4

-9 ✓	

w:

NUMERICAL marked out of 4

2 ✓	

(2) **Q1**

EMBEDDED ANSWERS penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the

answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$(x^2+1)^{\frac{4}{3}} = \boxed{\text{a}} + \boxed{\text{b}}x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}x^2 + \boxed{\text{e}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{\text{a}}$:

NUMERICAL | marked out of 2

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | marked out of 1

0 ✓	

$\boxed{\text{c}}$:

NUMERICAL | marked out of 2

4 ✓	

$\boxed{\text{d}}$:

NUMERICAL | marked out of 2

3 ✓	

$\boxed{\text{e}}$:

NUMERICAL | marked out of 1

0 ✓	

$$x\log(1+5x) = \boxed{\text{h}} + \boxed{\text{i}}x + \boxed{\text{j}}x^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{\text{h}}$:

NUMERICAL | marked out of 1

0 ✓	

$\boxed{\text{i}}$:

NUMERICAL | marked out of 1

0 ✓	

$\boxed{\text{j}}$:

NUMERICAL | marked out of 2

5 ✓	

$\boxed{\text{k}}$:

NUMERICAL | marked out of 2

-25 ✓	

k:

NUMERICAL | marked out of 2

2 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 0} \frac{(x^2+1)^{\frac{4}{3}} + x\log(1+5x) + \alpha + \beta x^2}{x^2 \sin(x)}.$$

This limit converges for $\alpha = [\text{s}], \beta = \frac{[\text{t}]}{[\text{u}]}$.

s:

NUMERICAL | marked out of 8

-1 ✓	

t:

NUMERICAL | marked out of 4

-19 ✓	

u:

NUMERICAL | marked out of 4

3 ✓	

In that case, the limit is $\frac{[\text{v}]}{[\text{w}]}$.

v:

NUMERICAL | marked out of 4

-25 ✓	

w:

NUMERICAL | marked out of 4

2 ✓	

(3) **Q2**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{[\text{a}]}{[\text{b}]}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Determine $r = \boxed{\text{a}}, \theta = \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}}$ such that $re^{i\theta} = -4\sqrt{2} + i4\sqrt{2}$ and $0 \leq \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}} < 2\pi$.

$\boxed{\text{a}}$:

NUMERICAL | marked out of 4

8 ✓	

$\boxed{\text{b}}$:

NUMERICAL | marked out of 2

3 ✓	

$\boxed{\text{c}}$:

NUMERICAL | marked out of 2

4 ✓	

Compute $(-4\sqrt{2}+i4\sqrt{2})^{\frac{1}{3}} = \sqrt{\boxed{\text{d}}}+i\sqrt{\boxed{\text{e}}}$, in the range where $\sqrt{\boxed{\text{d}}} > 0$ and $\sqrt{\boxed{\text{e}}} > 0$.

$\boxed{\text{d}}$:

NUMERICAL | marked out of 4

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | marked out of 4

2 ✓	

Consider the series $\sum_{n=0}^{\infty} \frac{n}{4^n+1} z^n$.

Calculate the partial sum $\sum_{n=0}^{2} \frac{n}{4^n+1} z^n = \frac{\boxed{\text{j}}}{\boxed{\text{k}}} + i\frac{\boxed{\text{l}}}{\boxed{\text{m}}}$ with $z = i$.

$\boxed{\text{j}}$:

NUMERICAL | marked out of 2

-2 ✓	

$\boxed{\text{k}}$:

NUMERICAL | marked out of 2

17 ✓	

$\boxed{\text{l}}$:

NUMERICAL | marked out of 2

1 ✓	

$\boxed{\text{m}}$:

NUMERICAL | marked out of 2

5 ✓	

Find the largest $r = \boxed{\text{p}} > 0$ such that the series above converges for all $z \in \mathbb{C}$ with $|z| < r$.

$\boxed{\text{p}}$:

NUMERICAL marked out of 8

4 ✓	

For $z = -4$, the series

MULTIPLE CHOICE marked out of 8 One answer only

- converges absolutely
- converges but not absolutely
- does not converge ✓

(4) **Q2**

EMBEDDED ANSWERS penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Determine $r = \boxed{\text{a}}\sqrt{2}, \theta = \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}}$ such that $re^{i\theta} = -4 - 4i$ and $0 \leq \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}} < 2\pi$.

$\boxed{\text{a}}$:

NUMERICAL marked out of 4

4 (0%)	

$\boxed{\text{b}}$:

NUMERICAL marked out of 2

5 (0%)	

$\boxed{\text{c}}$:

NUMERICAL marked out of 2

4 (0%)	

Compute $(-4 - 4i)^{\frac{1}{5}} = \boxed{\text{d}} + i\boxed{\text{e}}$, in the range where $\boxed{\text{d}} > 0$ and $\boxed{\text{e}} > 0$.

$\boxed{\text{d}}$:

NUMERICAL marked out of 4

1 (0%)	

e:

NUMERICAL | marked out of 4

1 (0%)	

Consider the series $\sum_{n=0}^{\infty} \frac{n}{3^n+1} z^n$.

Calculate the partial sum $\sum_{n=0}^{2} \frac{n}{3^n+1} z^n = \frac{\boxed{\text{j}}}{\boxed{\text{k}}} + i\frac{\boxed{\text{l}}}{\boxed{\text{m}}}$ with $z = i$.

j:

NUMERICAL | marked out of 2

-1 (0%)	

k:

NUMERICAL | marked out of 2

5 (0%)	

l:

NUMERICAL | marked out of 2

1 (0%)	

m:

NUMERICAL | marked out of 2

4 (0%)	

Find the largest $r = \boxed{\text{p}} > 0$ such that the series above converges for all $z \in \mathbb{C}$ with $|z| < r$.

p:

NUMERICAL | marked out of 8

3 (0%)	

For $z = -3$, the series

MULTIPLE CHOICE | marked out of 8 | One answer only

- converges absolutely
- converges but not absolutely
- does not converge ✓

(5) **Q3**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function for $x \in \mathbb{R}$

$$f(x) = \log \frac{2x^2+1}{2x^2+x-1}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that **are** in the natural domain of $f(x)$.

MULTIPLE CHOICE | marked out of 4 | Multiple answers allowed

- -3 (19.9997%)
- -2 (19.9997%)
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 (19.9997%)
- 2 (19.9997%)
- 3 (19.9997%)

The graph of this function crosses the x-axis at $x =$ [a].

[a]:

NUMERICAL | marked out of 4

2 (20%)	

One has

$$f'(2) = \frac{[e]}{[f]}.$$

[e]:

NUMERICAL | marked out of 2

-1 (20%)	

[f]:

NUMERICAL | marked out of 2

9 (20%)	

The function $f(x)$ has [g] stationary point(s) in the domain.

[g]:

NUMERICAL | marked out of 4

1 (20%)	

Choose the behaviour of $f(x)$ in the interval $(-4, -3)$.

MULTIPLE CHOICE | marked out of 4 | One answer only

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(6) **Q3**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function for $x \in \mathbb{R}$

$$f(x) = \log \frac{2x^2+1}{2x^2-x-1}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that **are** in the natural domain of $f(x)$.

MULTIPLE CHOICE | marked out of 4 | Multiple answers allowed

- -3 (19.9997%)
- -2 (19.9997%)
- -1 (19.9997%)
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 (-100%)
- 2 (19.9997%)
- 3 (19.9997%)

The graph of this function crosses the x-axis at $x = \boxed{\text{a}}$.

$\boxed{\text{a}}$:

NUMERICAL | marked out of 4

-2 (20%)	

One has

$$f'(2) = \frac{\boxed{\text{e}}}{\boxed{\text{f}}}.$$

$\boxed{\text{e}}$:

NUMERICAL | marked out of 2

-23 (20%)	

$\boxed{\text{f}}$:

NUMERICAL | marked out of 2

45 (20%)	

The function $f(x)$ has $\boxed{\text{g}}$ stationary point(s) in the domain.

$\boxed{\text{g}}$:

NUMERICAL marked out of 4

1 (20%)	

Choose the behaviour of $f(x)$ in the interval $(3, 4)$.

MULTIPLE CHOICE marked out of 4 One answer only

- monotonically decreasing ✓
- monotonically increasing
- neither decreasing nor increasing

(7) **Q4**

EMBEDDED ANSWERS penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 \frac{4x}{x^3 + x^2 + 3x + 3} dx.$$

Complete the formula

$$\frac{4x}{x^3 + x^2 + 3x + 3} = \frac{\boxed{\text{a}}x + \boxed{\text{b}}}{x^2 + \boxed{\text{c}}} + \frac{\boxed{\text{d}}}{x + \boxed{\text{e}}}.$$

[a]:

NUMERICAL marked out of 2

1 (0%)	

[b]:

NUMERICAL marked out of 2

3 (0%)	

[c]:

NUMERICAL marked out of 2

3 (0%)	

[d]:

NUMERICAL marked out of 2

-1 (0%)	

[e]:

NUMERICAL marked out of 2

1 (0%)	

Choose a primitive of $\frac{1}{x^2+3}$.

MULTIPLE CHOICE | marked out of 5 | One answer only

- $\arctan 3x$
- $\arctan(x+3)$
- $3\arctan(x)$
- $\arctan(x/\sqrt{3})/\sqrt{3}$ ✓
- $\frac{1}{3}\log(x^2+3)$
- $\frac{1}{2}\log(x+3)$
- $(\log(x+3)^2$
- $-2x/(x^2+3)^2$

By continuing, we get

$$\int_0^1 \frac{4x}{x^3+x^2+3x+3}dx = \frac{\sqrt{\boxed{\text{f}}}}{\boxed{\text{g}}}\pi + \frac{\boxed{\text{h}}}{\boxed{\text{i}}}\log\boxed{\text{j}}.$$

$\boxed{\text{f}}$:

NUMERICAL | marked out of 2

3 (0%)	

$\boxed{\text{g}}$:

NUMERICAL | marked out of 2

6 (0%)	

$\boxed{\text{h}}$:

NUMERICAL | marked out of 2

-1 (0%)	

$\boxed{\text{i}}$:

NUMERICAL | marked out of 2

2 (0%)	

$\boxed{\text{j}}$:

NUMERICAL | marked out of 2

3 (0%)	

(8) **Q4**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign

should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^3 \frac{4x}{x^3+x^2+3x+3}dx.$$

Complete the formula

$$\frac{4x}{x^3+x^2+3x+3} = \frac{\boxed{\text{a}}x+\boxed{\text{b}}}{x^2+\boxed{\text{c}}} + \frac{\boxed{\text{d}}}{x+\boxed{\text{e}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | marked out of 2

1 (0%)	

$\boxed{\text{b}}$:

NUMERICAL | marked out of 2

3 (0%)	

$\boxed{\text{c}}$:

NUMERICAL | marked out of 2

3 (0%)	

$\boxed{\text{d}}$:

NUMERICAL | marked out of 2

-1 (0%)	

$\boxed{\text{e}}$:

NUMERICAL | marked out of 2

1 (0%)	

Choose a primitive of $\frac{1}{x^2+3}$.

MULTIPLE CHOICE | marked out of 5 | One answer only

- $\arctan 3x$
- $\arctan(x+3)$
- $3\arctan(x)$
- $\arctan(x/\sqrt{3})/\sqrt{3}$ ✓
- $\frac{1}{3}\log(x^2+3)$
- $\frac{1}{2}\log(x+3)$
- $(\log(x+3)^2$
- $-2x/(x^2+3)^2$

By continuing, we get

$$\int_0^3 \frac{4x}{x^3+x^2+3x+3}dx = \boxed{\text{f}}\log\boxed{\text{g}} + \frac{\sqrt{\boxed{\text{h}}}}{\boxed{\text{i}}}\pi.$$

f:

NUMERICAL marked out of 2

-1 (0%)	

g:

NUMERICAL marked out of 2

2 (0%)	

h:

NUMERICAL marked out of 3

3 (0%)	

i:

NUMERICAL marked out of 3

3 (0%)	

(9) **Q5**

EMBEDDED ANSWERS penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = \cos(x)\exp(\sin(x))y(x)^2$$

.

MULTIPLE CHOICE marked out of 3 One answer only Shuffle

- $y(x) = C/(\exp(\sin(x)))$
- $y(x) = 1/(\exp(\sin(x + C)))$
- $y(x) = -1/(\exp(\sin(x)) + C)$ ✓
- $y(x) = C/(\exp(\cos(x)))$
- $y(x) = 1/(\exp(\cos(x + C)))$
- $y(x) = 1/(C - (\cos(x)))$
- $y(x) = \exp(\exp(\sin(x))) + C$
- $y(x) = \exp(C\exp(\cos(x)))$

Determine $C = \boxed{a}$ with the initial condition $y(0) = \frac{1}{2}$

a:

NUMERICAL marked out of 3

-3 (0%)	

Consider the following differential equation.

$$y''(x) - 2y'(x) + 10y(x) = 0$$

.

A solution satisfying $y(0) = 4$ and $y'(0) = 7$ can be written as $y(x) = \exp(\boxed{\text{f}}x)(\boxed{\text{g}}\cos(\boxed{\text{h}}x) + \boxed{\text{i}}\sin(\boxed{\text{j}}x))$.

$\boxed{\text{f}}$:

NUMERICAL marked out of 2

1 (0%)	

$\boxed{\text{g}}$:

NUMERICAL marked out of 1

4 (0%)	

$\boxed{\text{h}}$:

NUMERICAL marked out of 1

3 (0%)	

$\boxed{\text{i}}$:

NUMERICAL marked out of 1

1 (0%)	

$\boxed{\text{j}}$:

NUMERICAL marked out of 1

3 (0%)	

Find $r = \boxed{\text{k}}$ such that $\frac{y(x)}{e^{rx}}$ remains bounded but does not converge.

$\boxed{\text{k}}$:

NUMERICAL marked out of 3

1 (0%)	

(10) **Q5**

EMBEDDED ANSWERS penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = \cos(x)\exp(\sin(x))y(x)^2$$

.

MULTIPLE CHOICE | marked out of 3 | One answer only | Shuffle

- $y(x) = C/(\exp(\sin(x)))$
- $y(x) = 1/(\exp(\sin(x + C)))$
- $y(x) = -1/(\exp(\sin(x)) + C)$ ✓
- $y(x) = C/(\exp(\cos(x)))$
- $y(x) = 1/(\exp(\cos(x + C)))$
- $y(x) = 1/(C - (\cos(x)))$
- $y(x) = \exp(\exp(\sin(x))) + C$
- $y(x) = \exp(C\exp(\cos(x)))$

Determine $C =$ [a] with the initial condition $y(0) = 1$

[a]:

NUMERICAL | marked out of 3

-2 (0%)	

Consider the following differential equation.

$$y''(x) + 2y'(x) + 5y(x) = 0$$

.

A solution satisfying $y(0) = 3$ and $y'(0) = -5$ can be written as $y(x) = \exp([\text{f}]x)([\text{g}]\cos([\text{h}]x) + [\text{i}]\sin([\text{j}]x))$.

[f]:

NUMERICAL | marked out of 2

-1 (0%)	

[g]:

NUMERICAL | marked out of 1

3 (0%)	

[h]:

NUMERICAL | marked out of 1

2 (0%)	

[i]:

NUMERICAL | marked out of 1

-1 (0%)	

[j]:

NUMERICAL | marked out of 1

2 (0%)	

Find $r = \boxed{\text{k}}$ such that $\frac{y(x)}{e^{rx}}$ remains bounded but does not converge.

$\boxed{\text{k}}$:

NUMERICAL marked out of 3

-1 (0%)	

Total of marks: 280