2024Call2.

(1) **Q1**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\log(x) = \boxed{a} + \boxed{b}(x-1) + \frac{\boxed{c}}{\boxed{d}}(x-1)^2 + \frac{\boxed{e}}{\boxed{f}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{a}$:

NUMERICAL | marked out of 1

0 ✓	

$\boxed{b}$:

NUMERICAL | marked out of 1

1 ✓	

$\boxed{c}$:

NUMERICAL | marked out of 1

-1 ✓	

$\boxed{d}$:

NUMERICAL | marked out of 1

2 ✓	

$\boxed{e}$:

NUMERICAL | marked out of 1

1 ✓	

$\boxed{f}$:

NUMERICAL | marked out of 1

3 ✓	

$$(x-1)x^{\frac{1}{2}} = \boxed{j} + \boxed{k}(x-1) + \frac{\boxed{l}}{\boxed{m}}(x-1)^2 + \frac{\boxed{o}}{\boxed{p}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{j}$:

NUMERICAL | marked out of 1

0 ✓	

k:

NUMERICAL | marked out of 1

1 ✓	

l:

NUMERICAL | marked out of 1

1 ✓	

m:

NUMERICAL | marked out of 1

2 ✓	

o:

NUMERICAL | marked out of 1

-1 ✓	

p:

NUMERICAL | marked out of 1

8 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 1} \frac{\log x - (x-1)x^{\frac{1}{2}} + \alpha(x-1)^2}{(x-1)^\beta}.$$

If $\alpha = 0$, there is only one $\beta = \boxed{\text{s}}$ such that this limit exists as a real number and different from 0.

s:

NUMERICAL | marked out of 6

2 ✓	

If $\beta = 3$, there is only one $\alpha = \boxed{\text{t}}$ such that this limit exists as a real number.

t:

NUMERICAL | marked out of 6

1 ✓	

In the latter case, the limit is $\frac{\boxed{\text{u}}}{\boxed{\text{v}}}$.

u:

NUMERICAL | marked out of 3

11 ✓	

v:

NUMERICAL | marked out of 3

24 ✓	

(2) **Q1**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\log(x) = \boxed{\text{a}} + \boxed{\text{b}}(x-1) + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}(x-1)^2 + \frac{\boxed{\text{e}}}{\boxed{\text{f}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{a}}$:

NUMERICAL | marked out of 1

0 ✓	

$\boxed{\text{b}}$:

NUMERICAL | marked out of 1

1 ✓	

$\boxed{\text{c}}$:

NUMERICAL | marked out of 1

-1 ✓	

$\boxed{\text{d}}$:

NUMERICAL | marked out of 1

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | marked out of 1

1 ✓	

$\boxed{\text{f}}$:

NUMERICAL | marked out of 1

3 ✓	

$$(x-1)x^{\frac{1}{3}} = \boxed{\text{j}} + \boxed{\text{k}}(x-1) + \frac{\boxed{\text{l}}}{\boxed{\text{m}}}(x-1)^2 + \frac{\boxed{\text{o}}}{\boxed{\text{p}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{j}}$:

NUMERICAL | marked out of 1

0 ✓	

$\boxed{\text{k}}$:

NUMERICAL | marked out of 1

1 ✓	

$\boxed{\text{l}}$:

NUMERICAL | marked out of 1

1 ✓	

$\boxed{\text{m}}$:

NUMERICAL | marked out of 1

3 ✓	

$\boxed{\text{o}}$:

NUMERICAL | marked out of 1

-1 ✓	

$\boxed{\text{p}}$:

NUMERICAL | marked out of 1

9 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to 1} \frac{\log x - (x-1)x^{\frac{1}{2}} + \alpha(x-1)^2}{(x-1)^\beta}.$$

If $\alpha = 0$, there is only one $\beta = \boxed{\text{s}}$ such that this limit exists as a real number and different from 0.

$\boxed{\text{s}}$:

NUMERICAL | marked out of 6

2 ✓	

If $\beta = 3$, there is only one $\alpha = \frac{\boxed{\text{t}}}{\boxed{\text{u}}}$ such that this limit exists as a real number.

$\boxed{\text{t}}$:

NUMERICAL | marked out of 3

5 ✓	

$\boxed{\text{u}}$:

NUMERICAL | marked out of 3

6 ✓	

In the latter case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

$\boxed{\text{v}}$:

NUMERICAL | marked out of 3

4 ✓	

$\boxed{\text{w}}$:

NUMERICAL | marked out of 3

9 ✓	

(3) **Q2**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Determine $r = 2^{\alpha}, \alpha = \frac{\boxed{a}}{\boxed{b}}, \theta = \frac{\boxed{c}\pi}{\boxed{d}}$ such that $re^{i\theta} = -8 + i8$, where $0 \le \theta < 2\pi$.

$\boxed{\text{a}}$:

NUMERICAL | marked out of 1

7 ✓	

$\boxed{\text{b}}$:

NUMERICAL | marked out of 1

2 ✓	

$\boxed{\text{c}}$:

NUMERICAL | marked out of 1

3 ✓	

$\boxed{\text{d}}$:

NUMERICAL | marked out of 1

4 ✓	

Compute $(-8 + i8)^{\frac{1}{3}} = 2^{\beta} + i2^{\gamma} = z'$, $\beta = \frac{\boxed{g}}{\boxed{h}}, \gamma = \frac{\boxed{i}}{\boxed{j}}$, in the range where $z' = r'e^{i\theta'}, 0 \le \theta' \le \frac{\pi}{2}$.

$\boxed{\text{g}}$:

NUMERICAL | marked out of 1

2 ✓	

$\boxed{\text{h}}$:

NUMERICAL | marked out of 1

3 ✓	

$\boxed{\text{i}}$:

NUMERICAL | marked out of 1

2 ✓	

$\boxed{j}$:

NUMERICAL | marked out of 1

3 ✓	

Calculate the following series.

$$\sum_{k=0}^{\infty}\left(\frac{i}{2}\right)^k = \frac{\boxed{m}}{\boxed{n}} + \frac{\boxed{o}}{\boxed{p}}i.$$

$\boxed{m}$:

NUMERICAL | marked out of 1

4 ✓	

$\boxed{n}$:

NUMERICAL | marked out of 1

5 ✓	

$\boxed{o}$:

NUMERICAL | marked out of 1

2 ✓	

$\boxed{p}$:

NUMERICAL | marked out of 1

5 ✓	

Consider the series $\sum_{n=0}^{\infty} \frac{n}{2^n+3^n} z^n$.

Find $0 < r = \boxed{r}$ such that the series above converges for all $z \in \mathbb{C}$, $|z| < r$.

$\boxed{r}$:

NUMERICAL | marked out of 4

3 ✓	

For $z = -3$, the series

MULTIPLE CHOICE | marked out of 4 | One answer only

- converges absolutely
- converges but not absolutely
- does not converge ✓

(4) **Q2**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\dfrac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Determine $r = 2^\alpha, \alpha = \dfrac{\boxed{\text{a}}}{\boxed{\text{b}}}, \theta = \dfrac{\boxed{\text{c}}\pi}{\boxed{\text{d}}}$ such that $re^{i\theta} = -4 + i4$, where $0 \le \theta < 2\pi$.

$\boxed{\text{a}}$:

NUMERICAL | marked out of 1

5 (0%)	

$\boxed{\text{b}}$:

NUMERICAL | marked out of 1

2 (0%)	

$\boxed{\text{c}}$:

NUMERICAL | marked out of 1

3 (0%)	

$\boxed{\text{d}}$:

NUMERICAL | marked out of 1

4 (0%)	

Compute $(-4 + i4)^{\frac{1}{3}} = 2^\beta + i2^\gamma = z'$, $\beta = \dfrac{\boxed{\text{g}}}{\boxed{\text{h}}}, \gamma = \dfrac{\boxed{\text{i}}}{\boxed{\text{j}}}$, in the range where $z' = r'e^{i\theta'}, 0 \le \theta' \le \frac{\pi}{2}$.

$\boxed{\text{g}}$:

NUMERICAL | marked out of 1

1 (0%)	

$\boxed{\text{h}}$:

NUMERICAL | marked out of 1

3 (0%)	

$\boxed{\text{i}}$:

NUMERICAL | marked out of 1

1 (0%)	

$\boxed{\text{j}}$:

NUMERICAL | marked out of 1

3 (0%)	

Calculate the following series.

$$\sum_{k=0}^{\infty}\left(\frac{i}{3}\right)^k = \frac{\boxed{\text{m}}}{\boxed{\text{n}}} + \frac{\boxed{\text{o}}}{\boxed{\text{p}}} i.$$

$\boxed{\text{m}}$:

NUMERICAL | marked out of 1

9 (0%)	

$\boxed{\text{n}}$:

NUMERICAL | marked out of 1

10 (0%)	

$\boxed{\text{o}}$:

NUMERICAL | marked out of 1

3 (0%)	

$\boxed{\text{p}}$:

NUMERICAL | marked out of 1

10 (0%)	

Consider the series $\sum_{n=0}^{\infty} \frac{1}{n(2^n+4^n)} z^n$.

Find $0 < r = \boxed{r}$ such that the series above converges for all $z \in \mathbb{C}$, $|z| < r$.

$\boxed{\text{r}}$:

NUMERICAL | marked out of 4

4 (0%)	

For $z = -4$, the series

MULTIPLE CHOICE | marked out of 4 | One answer only

- converges absolutely
- converges but not absolutely ✓
- does not converge

(5) **Q3**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \arctan((x^2+1)/(x+3)).$$

This function has horizontal asymptotes. They are $y = \frac{\boxed{\text{a}}\pi}{\boxed{\text{b}}}$ for $x \to -\infty$ and $y = \frac{\boxed{\text{c}}\pi}{\boxed{\text{d}}}$ for $x \to \infty$.

$\boxed{\text{a}}$:

NUMERICAL | marked out of 1

-1 (0%)	

$\boxed{\text{b}}$:

NUMERICAL | marked out of 1

2 (0%)	

$\boxed{\text{c}}$:

NUMERICAL | marked out of 1

1 (0%)	

$\boxed{\text{d}}$:

NUMERICAL | marked out of 1

2 (0%)	

One has

$$f'(1) = \frac{\boxed{\text{e}}}{\boxed{\text{f}}}.$$

$\boxed{\text{e}}$:

NUMERICAL | marked out of 2

3 (0%)	

$\boxed{\text{f}}$:

NUMERICAL | marked out of 2

10 (0%)	

f takes a local minimum at $x = \sqrt{\boxed{\text{g}}} + \boxed{\text{h}}$.

$\boxed{\text{g}}$:

NUMERICAL | marked out of 2

10 (0%)	

$\boxed{\text{h}}$:

NUMERICAL | marked out of 2

-3 (0%)	

The function $f(x)$ has $\boxed{\text{j}}$ stationary point(s) in the domain.

$\boxed{\text{j}}$:

NUMERICAL | marked out of 4

2 (0%)	

Choose the behaviour of $f(x)$ in the interval $(-7, -5)$.

MULTIPLE CHOICE | marked out of 4 | One answer only

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(6) **Q3**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \arctan((x^2 + 1)/(x + 2)).$$

This function has horizontal asymptotes. They are $y = \frac{\boxed{a}\pi}{\boxed{b}}$ for $x \to -\infty$ and $y = \frac{\boxed{c}\pi}{\boxed{d}}$ for $x \to \infty$.

$\boxed{a}$:

NUMERICAL | marked out of 1

-1 (0%)	

$\boxed{b}$:

NUMERICAL | marked out of 1

2 (0%)	

$\boxed{c}$:

NUMERICAL | marked out of 1

1 (0%)	

$\boxed{d}$:

NUMERICAL | marked out of 1

2 (0%)	

One has

$$f'(1) = \frac{\boxed{e}}{\boxed{f}}.$$

$\boxed{e}$:

NUMERICAL | marked out of 2

4 (0%)	

$\boxed{\text{f}}$:

NUMERICAL | marked out of 2

13 (0%)	

f takes a local minimum at $x = \sqrt{\boxed{\text{g}}} + \boxed{\text{h}}$.

$\boxed{\text{g}}$:

NUMERICAL | marked out of 2

5 (0%)	

$\boxed{\text{h}}$:

NUMERICAL | marked out of 2

-2 (0%)	

The function $f(x)$ has $\boxed{\text{j}}$ stationary point(s) in the domain.

$\boxed{\text{j}}$:

NUMERICAL | marked out of 4

2 (0%)	

Choose the behaviour of $f(x)$ in the interval $(-7, -5)$.

MULTIPLE CHOICE | marked out of 4 | One answer only

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(7) **Q4**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^{\pi} x \cos x \sin\left(x + \frac{\pi}{6}\right) dx.$$

Fill in the blanks:

$$\sin\left(x + \frac{\pi}{6}\right) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}} \cos x + \frac{\sqrt{\boxed{\text{c}}}}{\boxed{\text{d}}} \sin x.$$

$\boxed{\text{a}}$:

NUMERICAL | marked out of 2

1 (0%)	

$\boxed{\text{b}}$:

NUMERICAL — marked out of 1

2 (0%)	

$\boxed{\text{c}}$:

NUMERICAL — marked out of 2

3 (0%)	

$\boxed{\text{d}}$:

NUMERICAL — marked out of 1

2 (0%)	

Fill in the blanks:

$$\int x\sin(2x)dx = \frac{\boxed{\text{g}}}{\boxed{\text{h}}}\sin(\boxed{\text{i}}x) + \frac{\boxed{\text{j}}}{\boxed{\text{k}}}x\cos(\boxed{\text{l}}x) + C.$$

$\boxed{\text{g}}$:

NUMERICAL — marked out of 1

1 (0%)	

$\boxed{\text{h}}$:

NUMERICAL — marked out of 1

4 (0%)	

$\boxed{\text{i}}$:

NUMERICAL — marked out of 1

2 (0%)	

$\boxed{\text{j}}$:

NUMERICAL — marked out of 1

-1 (0%)	

$\boxed{\text{k}}$:

NUMERICAL — marked out of 1

2 (0%)	

$\boxed{\text{l}}$:

NUMERICAL — marked out of 1

2 (0%)	

It holds that

$$\int_0^\pi x\cos x\sin\left(x+\frac{\pi}{6}\right)dx = \frac{\boxed{\text{o}}}{\boxed{\text{p}}}\pi^2 - \frac{\sqrt{\boxed{\text{q}}}}{\boxed{\text{r}}}\pi.$$

$\boxed{\text{o}}$:

NUMERICAL | marked out of 5

1 (0%)	

$\boxed{\text{p}}$:

NUMERICAL | marked out of 5

8 (0%)	

$\boxed{\text{q}}$:

NUMERICAL | marked out of 4

3 (0%)	

$\boxed{\text{r}}$:

NUMERICAL | marked out of 4

8 (0%)	

(8) **Q4**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^\pi x\cos x\sin\left(x+\frac{\pi}{3}\right)dx.$$

Fill in the blanks:

$$\sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{\boxed{\text{a}}}}{\boxed{\text{b}}}\cos x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}\sin x.$$

$\boxed{\text{a}}$:

NUMERICAL | marked out of 2

3 (0%)	

$\boxed{\text{b}}$:

NUMERICAL | marked out of 1

2 (0%)	

$\boxed{\text{c}}$:

NUMERICAL marked out of 2

1 (0%)	

$\boxed{\text{d}}$:

NUMERICAL marked out of 1

2 (0%)	

Fill in the blanks:

$$\int x\cos(2x)dx = \frac{\boxed{\text{g}}}{\boxed{\text{h}}} x\sin(\boxed{\text{i}}x) + \frac{\boxed{\text{j}}}{\boxed{\text{k}}}\cos(\boxed{\text{l}}x) + C.$$

$\boxed{\text{g}}$:

NUMERICAL marked out of 1

1 (0%)	

$\boxed{\text{h}}$:

NUMERICAL marked out of 1

2 (0%)	

$\boxed{\text{i}}$:

NUMERICAL marked out of 1

2 (0%)	

$\boxed{\text{j}}$:

NUMERICAL marked out of 1

1 (0%)	

$\boxed{\text{k}}$:

NUMERICAL marked out of 1

4 (0%)	

$\boxed{\text{l}}$:

NUMERICAL marked out of 1

2 (0%)	

It holds that

$$\int_0^{\pi} x\cos x\sin\left(x + \frac{\pi}{3}\right)dx = \frac{\sqrt{\boxed{\text{o}}}}{\boxed{\text{p}}}\pi^2 - \frac{\sqrt{\boxed{\text{q}}}}{\boxed{\text{r}}}\pi.$$

$\boxed{\text{o}}$:

NUMERICAL marked out of 5

3 (0%)	

$\boxed{\text{p}}$:

NUMERICAL | marked out of 5

8 (0%)	

$\boxed{\text{q}}$:

NUMERICAL | marked out of 4

-1 (0%)	

$\boxed{\text{r}}$:

NUMERICAL | marked out of 4

8 (0%)	

(9) **Q5**

EMBEDDED ANSWERS | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral based on definition.

$$\int_0^\infty \frac{1}{\sqrt{x}} \exp(\alpha\sqrt{x})dx$$

.

We split the integral into two parts: $\int_0^1 x\exp(\alpha x^2)dx + \int_1^\infty x\exp(\alpha x^2)dx$
$\int_1^\infty \frac{1}{\sqrt{x}} \exp(\alpha\sqrt{x})dx$ converges for the following α.

MULTIPLE CHOICE | marked out of 2 | One answer only

- no such α
- $\alpha > -\frac{1}{4}$
- $\alpha < -\frac{1}{4}$
- $\alpha > -1$
- $\alpha < -1$
- $\alpha > 0$
- $\alpha < 0$ ✓
- $\alpha > 1$
- $\alpha < 1$
- $\alpha > \frac{1}{4}$
- $\alpha < \frac{1}{4}$
- all $\alpha \in \mathbb{R}$

$\int_0^1 \frac{1}{\sqrt{x}} \exp(\alpha\sqrt{x})dx$ converges for the following α.

Multiple choice | marked out of 2 | One answer only

- no such α
- $\alpha > -\frac{1}{4}$
- $\alpha < -\frac{1}{4}$
- $\alpha > -1$
- $\alpha < -1$
- $\alpha > 0$
- $\alpha < 0$
- $\alpha > 1$
- $\alpha < 1$
- $\alpha > \frac{1}{4}$
- $\alpha < \frac{1}{4}$
- all $\alpha \in \mathbb{R}$ ✓

Take $\alpha = -\frac{3}{4}$. In this case, $\int_0^\infty \frac{1}{\sqrt{x}} \exp(\alpha\sqrt{x})dx = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}$ (if the integral is divergent, write $\frac{1}{0}$).

$\boxed{\text{a}}$:

Numerical | marked out of 1

8 ✓	

$\boxed{\text{b}}$:

Numerical | marked out of 1

3 ✓	

Choose all improper integrals that are convergent.

Multiple choice | marked out of 4 | One answer only

- $\int_0^\infty \frac{1}{\sqrt{x}} \exp(\sqrt{x})dx$ (−100%)
- $\int_0^\infty \sqrt{x} \exp(\sqrt{x})dx$ (−100%)
- $\int_0^\infty \frac{1}{\sqrt{x}} \exp(-\sqrt{x})dx$ ✓
- $\int_0^\infty \sqrt{x} \exp(-\sqrt{x})dx$ ✓
- $\int_0^\infty \frac{1}{x} \exp(-x)dx$ (−100%)
- $\int_0^\infty x \exp(-x + x^2)dx$ (−100%)
- $\int_0^\infty x \exp(-x + \sqrt{x})dx$ ✓

(10) **Q5**

Embedded answers | penalty 0.10

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral based on definition.

$$\int_0^\infty \frac{1}{\sqrt{x}} \exp(\alpha\sqrt{x})dx$$

.

We split the integral into two parts: $\int_0^1 x\exp(\alpha x^2)dx + \int_1^\infty x\exp(\alpha x^2)dx$

$\int_0^1 \frac{1}{\sqrt{x}} \exp(\alpha\sqrt{x})dx$ converges for the following α.

Multiple choice | marked out of 2 | One answer only

- no such α
- $\alpha > -\frac{1}{4}$
- $\alpha < -\frac{1}{4}$
- $\alpha > -1$
- $\alpha < -1$
- $\alpha > 0$
- $\alpha < 0$
- $\alpha > 1$
- $\alpha < 1$
- $\alpha > \frac{1}{4}$
- $\alpha < \frac{1}{4}$
- all $\alpha \in \mathbb{R}$ ✓

$\int_1^\infty \frac{1}{\sqrt{x}} \exp(\alpha\sqrt{x})dx$ converges for the following α.

Multiple choice | marked out of 2 | One answer only

- no such α
- $\alpha > -\frac{1}{4}$
- $\alpha < -\frac{1}{4}$
- $\alpha > -1$
- $\alpha < -1$
- $\alpha > 0$
- $\alpha < 0$ ✓
- $\alpha > 1$
- $\alpha < 1$
- $\alpha > \frac{1}{4}$
- $\alpha < \frac{1}{4}$
- all $\alpha \in \mathbb{R}$

Take $\alpha = -\frac{4}{3}$. In this case, $\int_0^\infty \frac{1}{\sqrt{x}} \exp(\alpha\sqrt{x})dx = \frac{\boxed{a}}{\boxed{b}}$ (if the integral is divergent, write $\frac{1}{0}$).

$\boxed{a}$:

Numerical | marked out of 1

3 (0%)	

$\boxed{\text{b}}$:

NUMERICAL | marked out of 1

2 (0%)	

Choose all improper integrals that are convergent.

MULTIPLE CHOICE | marked out of 4 | One answer only

- $\int_0^\infty \frac{1}{\sqrt{x}} \exp(-\sqrt{x})dx$ ✓
- $\int_0^\infty \sqrt{x} \exp(-\sqrt{x})dx$ ✓
- $\int_0^\infty \frac{1}{\sqrt{x}} \exp(\sqrt{x})dx$ (−100%)
- $\int_0^\infty \sqrt{x} \exp(\sqrt{x})dx$ (−100%)
- $\int_0^\infty \frac{1}{x} \exp(-x)dx$ (−100%)
- $\int_0^\infty x \exp(x - x^2)dx$ ✓
- $\int_0^\infty x \exp(-x + \sqrt{x})dx$ ✓

Total of marks: 220