2024Call1.

(1) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\log(1+3x) = \boxed{a} + \boxed{b}x + \frac{\boxed{c}}{\boxed{d}}x^2 + \boxed{e}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{a}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{c}$:

NUMERICAL | 2 points

-9 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{e}$:

NUMERICAL | 1 point

9 ✓	

$$\sin(2x) = \boxed{h} + \boxed{i}x + \boxed{j}x^2 + \frac{\boxed{k}}{\boxed{l}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{h}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{i}$:

NUMERICAL | 1 point

2 ✓	

[j]:

NUMERICAL 1 point

0 ✓	

[k]:

NUMERICAL 2 points

-4 ✓	

[l]:

NUMERICAL 1 point

3 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 0} \frac{\log(1+3x) + \sin(2x) + \alpha x + \beta x^2}{x^2 \sin(x)}.$$

This limit converges for $\alpha =$ [s], $\beta = \frac{[t]}{[u]}$.

[s]:

NUMERICAL 6 points

-5 ✓	

[t]:

NUMERICAL 3 points

9 ✓	

[u]:

NUMERICAL 3 points

2 ✓	

In that case, the limit is $\frac{[v]}{[w]}$.

[v]:

NUMERICAL 3 points

23 ✓	

[w]:

NUMERICAL 3 points

3 ✓	

(2) **Q1**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\log(1-3x) = \boxed{\text{a}} + \boxed{\text{b}}x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}x^2 + \boxed{\text{e}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

-3 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

-9 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

-9 ✓	

$$\sin(-x) = \boxed{\text{h}} + \boxed{\text{i}}x + \boxed{\text{j}}x^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{\text{h}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{i}}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{\text{j}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{k}}$:

NUMERICAL | 2 points

1 ✓	

l:

NUMERICAL 1 point

6 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{\log(1-3x)+\sin(-x)+\alpha x+\beta x^2}{x^2\sin(x)}.$$

This limit converges for $\alpha =$ s, $\beta = \frac{t}{u}$.

s:

NUMERICAL 6 points

4 ✓	

t:

NUMERICAL 3 points

9 ✓	

u:

NUMERICAL 3 points

2 ✓	

In that case, the limit is $\frac{v}{w}$.

v:

NUMERICAL 3 points

-53 ✓	

w:

NUMERICAL 3 points

6 ✓	

(3) **Q2**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Determine $r = \boxed{\text{a}}, \theta = \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}}$ such that $re^{i\theta} = -8 + i8\sqrt{3}$ and $0 \leq \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}} < 2\pi$.

$\boxed{\text{a}}$:

NUMERICAL | 4 points

16 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 2 points

2 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

3 ✓	

Compute $(-8 + i8\sqrt{3})^{\frac{1}{4}} = \sqrt{\boxed{\text{d}}} + i\boxed{\text{e}}$, in the range where ($\sqrt{\boxed{\text{d}}} > 0$ and) $\boxed{\text{e}} > 0$.

$\boxed{\text{d}}$:

NUMERICAL | 4 points

3 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 4 points

1 ✓	

Consider the series $\sum_{n=0}^{\infty} \frac{n}{n^2+1} z^n$.

Calculate the partial sum $\sum_{n=0}^{2} \frac{n}{n^2+1} z^n = \frac{\boxed{\text{j}}}{\boxed{\text{k}}} + i\frac{\boxed{\text{l}}}{\boxed{\text{m}}}$ with $z = i$.

$\boxed{\text{j}}$:

NUMERICAL | 2 points

-2 ✓	

$\boxed{\text{k}}$:

NUMERICAL | 2 points

5 ✓	

$\boxed{\text{l}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{m}}$:

NUMERICAL | 2 points

2 ✓	

Find $0 < r = \boxed{\text{p}}$ such that the series above converges for all $z \in \mathbb{C}$, $|z| < r$.

$\boxed{\text{p}}$:

NUMERICAL | 8 points

1 ✓	

For $z = -1$, the series

MULTI | 8 points | Single

- converges absolutely
- converges but not absolutely ✓
- does not converge

(4) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Determine $r = \boxed{\text{a}}$, $\theta = \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}}$ such that $re^{i\theta} = -8 + i8\sqrt{3}$ and $0 \le \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}} < 2\pi$.

$\boxed{\text{a}}$:

NUMERICAL | 4 points

16 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 2 points

2 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

3 ✓	

Compute $(-8 + i8\sqrt{3})^{\frac{1}{2}} = \boxed{\text{d}} + i\boxed{\text{e}}\sqrt{\boxed{\text{f}}}$, in the range where ($\boxed{\text{d}} > 0$ and) $\boxed{\text{e}}\sqrt{\boxed{\text{f}}} > 0$.

$\boxed{\text{d}}$:

NUMERICAL | 4 points

2 ✓	

[e]:

[NUMERICAL] [2 points]

2 ✓	

[f]:

[NUMERICAL] [2 points]

3 ✓	

Consider the series $\sum_{n=0}^{\infty} \frac{n}{n^2+1} z^n$.

Calculate the partial sum $\sum_{n=0}^{2} \frac{n}{n^2+1} z^n = \frac{\boxed{j}}{\boxed{k}} + i\frac{\boxed{l}}{\boxed{m}}$ with $z = 1 + i$.

[j]:

[NUMERICAL] [2 points]

1 ✓	

[k]:

[NUMERICAL] [2 points]

2 ✓	

[l]:

[NUMERICAL] [2 points]

13 ✓	

[m]:

[NUMERICAL] [2 points]

10 ✓	

Find $0 < r = \boxed{p}$ such that the series above converges for all $z \in \mathbb{C}$, $|z| < r$.

[p]:

[NUMERICAL] [8 points]

1 ✓	

For $z = 1$, the series

[MULTI] [8 points] [Single]

- converges absolutely
- converges but not absolutely
- does not converge ✓

(5) **Q3**

[CLOZE] [0.10 penalty]

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function for $x \in \mathbb{R}$

$$f(x) = \sqrt{(x^4+1)/(x^2+1)}.$$

This function has two oblique asymptotes. They are $y = \boxed{a}x + \boxed{b}, \boxed{c}x + \boxed{d}$ with $\boxed{a} < \boxed{c}$.

$\boxed{a}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

0 ✓	

One has

$$f'(1) = \frac{\boxed{e}}{\boxed{f}}.$$

$\boxed{e}$:

NUMERICAL | 6 points

1 ✓	

$\boxed{f}$:

NUMERICAL | 2 points

2 ✓	

The function $f(x)$ has $\boxed{g}$ stationary point(s) in the interval $[0,1]$.

$\boxed{g}$:

NUMERICAL | 4 points

2 ✓	

Choose the behaviour of $f(x)$ in the interval $(0, 1)$.

MULTI | 4 points | Single

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(6) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function for $x \in \mathbb{R}$

$$f(x) = \sqrt{(x^4 + 1)/(x^2 + 1)}.$$

This function has two oblique asymptotes. They are $y = \boxed{a}x + \boxed{b}, \boxed{c}x + \boxed{d}$ with $\boxed{a} < \boxed{c}$.

$\boxed{a}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

0 ✓	

One has

$$f'(-1) = \frac{\boxed{e}}{\boxed{f}}.$$

$\boxed{e}$:

NUMERICAL | 6 points

-1 ✓	

$\boxed{f}$:

NUMERICAL | 2 points

2 ✓	

The function $f(x)$ has $\boxed{\text{g}}$ stationary point(s) in the domain.

$\boxed{\text{g}}$:

NUMERICAL | 4 points

3 ✓	

Choose the behaviour of $f(x)$ in the interval $(1, 2)$.

MULTI | 4 points | Single

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(7) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^{\log 3} \frac{1}{e^x + 1} dx.$$

Let us apply the change of variables $x = \log t$. Then we have

$$\int_0^{\log 3} \frac{1}{e^x + 1} dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} \frac{\boxed{\text{c}}t + \boxed{\text{d}}}{\boxed{\text{e}}t^2 + \boxed{\text{f}}t + \boxed{\text{g}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

0 ✓	

$\boxed{\text{d}}$:

NUMERICAL 2 points

1 ✓	

e:

NUMERICAL 2 points

1 ✓	

f:

NUMERICAL 2 points

1 ✓	

g:

NUMERICAL 2 points

0 ✓	

Choose a primitive of $\frac{1}{t(t+1)}$.

MULTI 12 points Single Shuffle

- $\log|t+1|$
- $\log|t(t+1)|$
- $\log|t/(t+1)|$ ✓
- $\log|(t+1)/t|$
- $\log|t+1|$
- $\log|t^2+1|$
- $\log|t^2+1|/2$
- $1/t+1/(t+1)$
- $1/t-1/(t+1)$
- $t\arctan(t)$
- $\arctan(t-1)$
- $\arctan((t-1)/2)$

By continuing, we get

$$\int_0^{\log 3} \frac{1}{e^x+1}dx = \log \boxed{\text{i}} - \log \boxed{\text{j}}.$$

i:

NUMERICAL 18 points

3 ✓	

j:

NUMERICAL 18 points

2 ✓	

(8) **Q4**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^{\log 4} \frac{1}{e^x + 1} dx.$$

Let us apply the change of variables $x = \log t$. Then we have

$$\int_0^{\log 4} \frac{1}{e^x + 1} dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} \frac{\boxed{\text{c}}\, t + \boxed{\text{d}}}{\boxed{\text{e}}\, t^2 + \boxed{\text{f}}\, t + \boxed{\text{g}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

0 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{f}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 2 points

0 ✓	

Choose a primitive of $\frac{1}{t(t+1)}$.

MULTI | 12 points | Single | Shuffle

- $\log|t+1|$
- $\log|t(t+1)|$
- $\log|t/(t+1)|$ ✓
- $\log|(t+1)/t|$
- $\log|t+1|$
- $\log|t^2+1|$
- $\log|t^2+1|/2$
- $1/t+1/(t+1)$
- $1/t-1/(t+1)$
- $t\arctan(t)$
- $\arctan(t-1)$
- $\arctan((t-1)/2)$

By continuing, we get

$$\int_0^{\log 4}\frac{1}{e^x+1}dx = \log\boxed{\text{i}} - \log\boxed{\text{j}}.$$

$\boxed{\text{i}}$:

NUMERICAL | 18 points

8 ✓	

$\boxed{\text{j}}$:

NUMERICAL | 18 points

5 ✓	

(9) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = x\exp(x^2)y(x)^2$$

.

MULTI | 3 points | Single | Shuffle

- $y(x) = C/(\exp x^2)$
- $y(x) = 1/(\exp x^2) + C$
- $y(x) = 1/(\exp(-x^2) + C)$

- $y(x) = -2/(\exp x^2 + C)$ ✓
- $y(x) = 2/(\exp x^2 + C)$
- $y(x) = \exp x^2 + C$
- $y(x) = \exp(x^2 + C)$

Determine $C =$ [a] with the initial condition $y(0) = \frac{1}{2}$

[a]:

NUMERICAL | 3 points

-5 ✓	

Consider the following differential equation.

$$y''(x) + 2y'(x) + 5y(x) = 0$$

.

A solution satisfying $y(0) = 3$ and $y'(0) = -1$ can be written as $y(x) = \exp($[f]$x)($[g]$\cos($[h]$x) +$ [i]$\sin($[j]$x))$.

[f]:

NUMERICAL | 2 points

-1 ✓	

[g]:

NUMERICAL | 1 point

3 ✓	

[h]:

NUMERICAL | 1 point

2 ✓	

[i]:

NUMERICAL | 1 point

1 ✓	

[j]:

NUMERICAL | 1 point

2 ✓	

Determine the limit $\lim_{x\to\infty} y(x) =$ [k].

[k]:

NUMERICAL | 3 points

0 ✓	

(10) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = x\exp(x^2)y(x)^2$$

.

MULTI | 3 points | Single | Shuffle

- $y(x) = C/(\exp x^2)$
- $y(x) = 1/(\exp x^2) + C$
- $y(x) = 1/(\exp(-x^2) + C)$
- $y(x) = -2/(\exp x^2 + C)$ ✓
- $y(x) = 2/(\exp x^2 + C)$
- $y(x) = \exp x^2 + C$
- $y(x) = \exp(x^2 + C)$

Determine $C = \boxed{\text{a}}$ with the initial condition $y(0) = \frac{1}{3}$

$\boxed{\text{a}}$:

NUMERICAL | 3 points

-7 ✓	

Consider the following differential equation.

$$y''(x) + 4y'(x) + 5y(x) = 0$$

.

A solution satisfying $y(0) = 2$ and $y'(0) = -2$ can be written as $y(x) = \exp(\boxed{\text{f}}x)(\boxed{\text{g}}\cos(\boxed{\text{h}}x) + \boxed{\text{i}}\sin(\boxed{\text{j}}x))$.

$\boxed{\text{f}}$:

NUMERICAL | 2 points

-2 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{h}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{i}}$:

NUMERICAL | 1 point

2 ✓	

j:

NUMERICAL | 1 point

1 ✓	

Determine the limit $\lim_{x\to\infty} y(x) =$ k.

k:

NUMERICAL | 3 points

0 ✓	

Total of marks: 330