2024Call1.

(1) **Q1**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\log(1+3x) = \boxed{\mathbf{a}} + \boxed{\mathbf{b}}x + \boxed{\mathbf{c}}{\boxed{\mathbf{d}}}x^2 + \boxed{\mathbf{e}}x^3 + o(x^3) \text{ as } x \to 0.$$

a:	
NUMERICAL 1 point	
0 ✓	
b:	
NUMERICAL 1 point	
3 ✓	
<u>c</u> :	
NUMERICAL 2 points	
-9 ✓	
d:	
NUMERICAL 1 point	
2 🗸	
e:	
NUMERICAL 1 point	
9 🗸	

$$\sin(2x) = \boxed{\mathbf{h}} + \boxed{\mathbf{i}}x + \boxed{\mathbf{j}}x^2 + \frac{\boxed{\mathbf{k}}}{\boxed{\mathbf{l}}}x^3 + o(x^3) \text{ as } x \to 0.$$

h:	
NUMERICAL 1 point	
0 🗸	
i:	
NUMERICAL 1 point	

(2)

2 🗸
[j]:
NUMERICAL 1 point
0 ✓
k:
NUMERICAL 2 points
-4 🗸
1:
NUMERICAL 1 point
3 🗸
For various $\alpha, \beta \in \mathbb{R}$, study the limit:
$\log(1+3x)+\sin(2x)+\alpha x+\beta x^2$
$\lim_{x \to 0} \frac{\log(1+3x) + \sin(2x) + \alpha x + \beta x^2}{x^2 \sin(x)}.$
$x \to 0$ $x \to \sin(x)$
This limit converges for $\alpha = [s], \beta = [t]$.
S: NUMERICAL 6 points
-5 √
t:
NUMERICAL 3 points
9 🗸
u:
NUMERICAL 3 points
2 🗸
In that case the limit is V
In that case, the limit is w.
V:
NUMERICAL 3 points
23 ✓
W: NUMERICAL 3 points
$\frac{3}{Q1}$
CLOZE 0.10 penalty
If not specified otherwise, fill in the blanks with integers (pos-

sibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the

answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

k :

NUMERICAL

2 points

$$\log(1-3x) = \boxed{\mathbf{a}} + \boxed{\mathbf{b}} x + \boxed{\mathbf{c}} x^2 + \boxed{\mathbf{c}} x^3 + o(x^3) \text{ as } x \to 0.$$

$$\boxed{\mathbf{a}} : \\ \boxed{\mathbf{NUMERICAL}} \quad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{0}} \checkmark \\ \boxed{\mathbf{b}} : \\ \boxed{\mathbf{NUMERICAL}} \quad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{-3}} \checkmark \\ \boxed{\mathbf{c}} : \\ \boxed{\mathbf{NUMERICAL}} \quad \boxed{\mathbf{2}} \text{ points}$$

$$\boxed{\mathbf{-9}} \checkmark \\ \boxed{\mathbf{d}} : \\ \boxed{\mathbf{NUMERICAL}} \quad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{2}} \checkmark \\ \boxed{\mathbf{e}} : \\ \boxed{\mathbf{NUMERICAL}} \quad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{-9}} \checkmark \\ \boxed{\mathbf{sin}} (-x) = \boxed{\mathbf{h}} + \boxed{\mathbf{i}} x + \boxed{\mathbf{j}} x^2 + \frac{\mathbf{k}}{\mathbf{l}} x^3 + o(x^3) \text{ as } x \to 0.$$

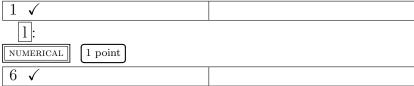
$$\boxed{\mathbf{h}} : \\ \boxed{\mathbf{NUMERICAL}} \quad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{0}} \checkmark \\ \boxed{\mathbf{j}} : \\ \boxed{\mathbf{NUMERICAL}} \quad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{0}} \checkmark \\ \boxed{\mathbf{j}} : \\ \boxed{\mathbf{NUMERICAL}} \quad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{0}} \checkmark \\ \boxed{\mathbf{j}} : \\ \boxed{\mathbf{NUMERICAL}} \quad \boxed{\mathbf{1}} \text{ point}$$

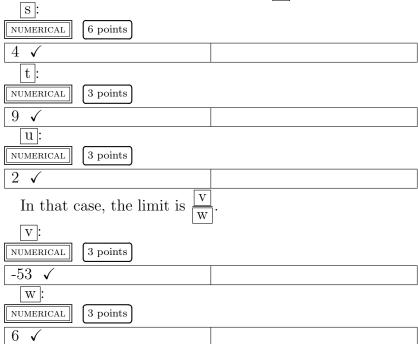
$$\boxed{\mathbf{0}} \checkmark \\ \boxed{\mathbf{j}} : \\ \boxed{\mathbf{NUMERICAL}} \quad \boxed{\mathbf{1}} \text{ point}$$



For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 0} \frac{\log(1 - 3x) + \sin(-x) + \alpha x + \beta x^2}{x^2 \sin(x)}.$$

This limit converges for $\alpha = [s], \beta = [t]$.



(3) **Q2**[CLOZE] [0.10 penalty]

If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Determine $r = [a], \theta = \frac{ b _{\pi}}{ c }$ such that $re^{i\theta} = -8 + i8\sqrt{3}$ and
$0 \le \frac{b_{\pi}}{c} < 2\pi.$
[a]: NUMERICAL 4 points
16 🗸
b:
NUMERICAL 2 points
$2 \checkmark$
C: NUMERICAL 2 points
3 🗸
Compute $(-8 + i8\sqrt{3})^{\frac{1}{4}} = \sqrt{\boxed{d}} + i\boxed{e}$, in the range where
$(\sqrt{\boxed{d}} > 0 \text{ and}) \boxed{e} > 0.$
d:
NUMERICAL 4 points
[3 ✓ [e]:
NUMERICAL 4 points
1 \(\)
Consider the series $\sum_{n=0}^{\infty} \frac{n}{n^2+1} z^n$.
Calculate the partial sum $\sum_{n=0}^{2} \frac{n}{n^2+1} z^n = \frac{\boxed{j}}{\boxed{k}} + i \frac{\boxed{l}}{\boxed{m}}$ with
z = i.
j NUMERICAL 2 points
NUMERICAL 2 points 2
k:
NUMERICAL 2 points
5 ✓
NUMERICAL 2 points
1 🗸
m:
NUMERICAL 2 points

	2 🗸
	Find $0 < r = p$ such that the series above converges for all
	$z \in \mathbb{C}, z < r.$
	p:
	NUMERICAL 8 points
	1 🗸
	For $z = -1$, the series
	MULTI 8 points Single
	• converges absolutely
	• converges but not absolutely ✓
	• does not converge
(4)	Q2
()	CLOZE 0.10 penalty
	If not specified otherwise, fill in the blanks with integers (pos-
	sibly 0 or negative). A fraction should be reduced (for ex-
	ample, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the
	answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign
	should be put on the numerator (for example $\frac{-1}{2}$ is accepted
	but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.
	Determine $r = [a], \theta = \frac{b_{\pi}}{c}$ such that $re^{i\theta} = -8 + i8\sqrt{3}$ and
	$0 \le \frac{ \mathbf{b} \pi}{ \mathbf{c} } < 2\pi.$
	a:
	NUMERICAL 4 points
	16 🗸
	b:
	NUMERICAL 2 points
	2 🗸
	<u>c</u> :
	NUMERICAL 2 points
	3 🗸
	Compute $(-8 + i8\sqrt{3})^{\frac{1}{2}} = \boxed{d} + i\boxed{e}\sqrt{\boxed{f}}$, in the range where
	$(\underline{\mathbf{d}} > 0 \text{ and}) \ \underline{\mathbf{e}} \sqrt{\underline{\mathbf{f}}} > 0.$
	<u>d</u> :
	NUMERICAL 4 points

	$2 \checkmark$
	<u>e</u> :
	NUMERICAL 2 points
	2 🗸
	f:
	NUMERICAL 2 points
	3 🗸
	Consider the series $\sum_{n=0}^{\infty} \frac{n}{n^2+1} z^n$.
	Calculate the partial sum $\sum_{n=0}^{2} \frac{n}{n^2+1} z^n = \frac{ j }{ k } + i \frac{ j }{ m }$ with
	z = 1 + i.
	[j]:
	NUMERICAL 2 points
	1 🗸
	k:
	NUMERICAL 2 points
	2 ✓
	1.
	NUMERICAL 2 points
	13 🗸
	m:
	NUMERICAL 2 points
	10 ✓
	Find $0 < r = p$ such that the series above converges for all
	$z \in \mathbb{C}, z < r.$
	[p]:
	NUMERICAL 8 points
	1 🗸
	For $z = 1$, the series
	MULTI 8 points Single
	• converges absolutely
	• converges but not absolutely
١	• does not converge ✓
)	Q3 CLOZE 0.10 penalty
	II CLOZE II. TU TU Denaity I

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|a|}{|b|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted

but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$. Let us consider the following function for $x \in \mathbb{R}$

$$f(x) = \sqrt{(x^4 + 1)/(x^2 + 1)}.$$

This function has two oblique asymptotes. They are y =[a]x + [b], [c]x + [d] with [a] < [c]

NUMERICAL 1 point
-1 ✓
b:
NUMERICAL 1 point
0 🗸
<u>c</u> :
NUMERICAL 1 point
1 🗸
d:
NUMERICAL 1 point
0 🗸
One has
$f'(1) = \frac{\boxed{\mathrm{e}}}{\boxed{\mathrm{f}}}.$
e:
NUMERICAL 6 points
1 🗸
f:
NUMERICAL 2 points
$2 \checkmark$
The function $f(x)$ has g stationary point(s) in the interval
[0,1].
NUMERICAL 4 points
2 \(\)

Choose the behaviour of f(x) in the interval (0,1).

MULTI 4 points Single

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing \checkmark

(6) **Q3**

CLOZE 0.10 penalty

One has

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function for $x \in \mathbb{R}$

$$f(x) = \sqrt{(x^4 + 1)/(x^2 + 1)}$$
.

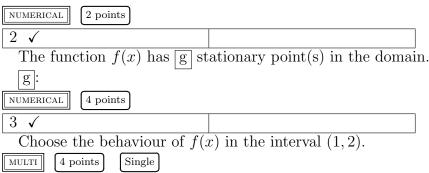
This function has two oblique asymptotes. They are y = [a]x + [b], [c]x + [d] with [a] < [c].

a : 1 point NUMERICAL **-1** ✓ b : NUMERICAL 1 point 0 🗸 c : NUMERICAL 1 point 1 dNUMERICAL 1 point 0 🗸

 $f'(-1) = \frac{\boxed{e}}{\boxed{f}}.$

e:
NUMERICAL 6 points

-1
f:



- monotonically decreasing
- ullet monotonically increasing \checkmark
- neither decreasing nor increasing

(7) **Q4**CLOZE 0.10 penalty

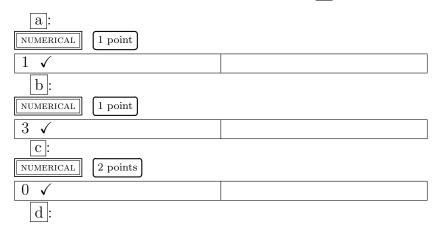
If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

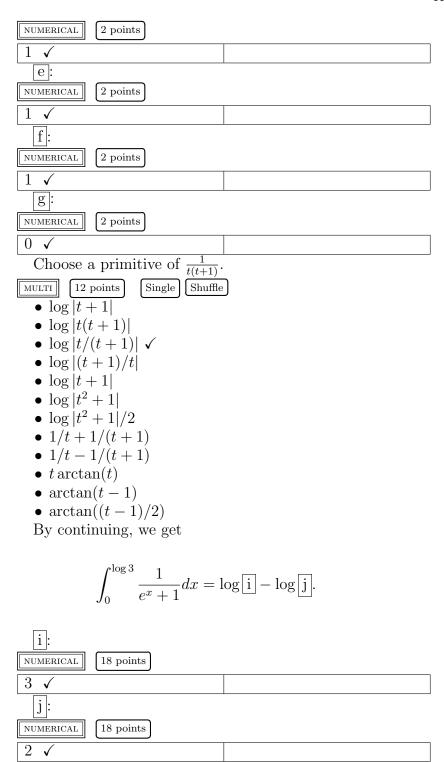
Let us calculate the following integral.

$$\int_0^{\log 3} \frac{1}{e^x + 1} dx.$$

Let us apply the change of variables $x = \log t$. Then we have

$$\int_0^{\log 3} \frac{1}{e^x + 1} dx = \int_{\boxed{\textbf{a}}}^{\boxed{\textbf{b}}} \frac{\boxed{\textbf{c}} t + \boxed{\textbf{d}}}{\boxed{\textbf{e}} t^2 + \boxed{\textbf{f}} t + \boxed{\textbf{g}}}.$$





(8) **Q4**[CLOZE] [0.10 penalty]

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^{\log 4} \frac{1}{e^x + 1} dx.$$

Let us apply the change of variables $x = \log t$. Then we have

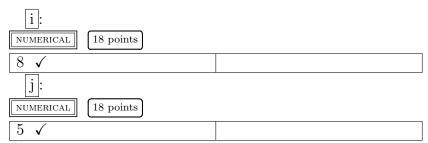
$$\int_0^{\log 4} \frac{1}{e^x + 1} dx = \int_{\boxed{\textbf{a}}}^{\boxed{\textbf{b}}} \frac{\boxed{\textbf{c}} t + \boxed{\textbf{d}}}{\boxed{\textbf{e}} t^2 + \boxed{\textbf{f}} t + \boxed{\textbf{g}}}.$$

a:
NUMERICAL 1 point
1 🗸
b :
NUMERICAL 1 point
3 🗸
<u>c</u> :
NUMERICAL 2 points
0 ✓
d:
NUMERICAL 2 points
1 🗸
<u>e</u> :
NUMERICAL 2 points
1
<u>f</u> :
NUMERICAL 2 points
1 🗸
g:
NUMERICAL 2 points
0 ✓
Choose a primitive of $\frac{1}{t(t+1)}$.
MULTI 12 points Single Shuffle

- $\log |t+1|$
- $\log |t(t+1)|$
- $\log |t/(t+1)| \checkmark$
- $\log |(t+1)/t|$
- $\log |t+1|$
- $\log |t^2 + 1|$
- $\log |t^2 + 1|/2$
- 1/t + 1/(t+1)
- 1/t 1/(t+1)
- $t \arctan(t)$
- $\arctan(t-1)$
- $\arctan((t-1)/2)$

By continuing, we get

$$\int_0^{\log 4} \frac{1}{e^x + 1} dx = \log[i] - \log[j].$$

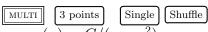


(9) **Q5**[CLOZE] [0.10 penalty]

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = x \exp(x^2) y(x)^2$$



- $\bullet \ y(x) = C/(\exp x^2)$
- $y(x) = 1/(\exp x^2) + C$
- $y(x) = 1/(\exp(-x^2) + C)$

•
$$y(x) = -2/(\exp x^2 + C)$$

• $y(x) = 2/(\exp x^2 + C)$

$$\bullet \ y(x) = \exp x^2 + C$$

$$\bullet \ y(x) = \exp(x^2 + C)$$

Determine $C = \boxed{\mathbf{a}}$ with the initial condition $y(0) = \frac{1}{2}$ $\boxed{\mathbf{a}}$:

NUMERICAL 3 points

<u>-5 √</u>

Consider the following differential equation.

$$y''(x) + 2y'(x) + 5y(x) = 0$$

A solution satisfying y(0) = 3 and y'(0) = -1 can be written as $y(x) = \exp(fx)(g\cos(hx) + i\sin(jx))$.

f: NUMERICAL 2 points <u>-1 √</u> g: NUMERICAL 1 point 3 ✓ h : NUMERICAL 1 point 2 **√** i: NUMERICAL 1 point 1 🗸 j : 1 point NUMERICAL

Determine the limit $\lim_{x\to\infty} y(x) = \boxed{\mathbf{k}}$.

k:

NUMERICAL

3 points

0 🗸

 $(10) \overline{\mathbf{Q5}}$

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the

answer boxes (such as $\frac{|a|}{|b|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = x \exp(x^2) y(x)^2$$

Single Shuffle 3 points

- $\bullet \ y(x) = C/(\exp x^2)$
- $y(x) = 1/(\exp x^2) + C$
- $y(x) = 1/(\exp(-x^2) + C)$
- $y(x) = -2/(\exp x^2 + C)$ • $y(x) = 2/(\exp x^2 + C)$
- $\bullet \ y(x) = \exp x^2 + C$
- $\bullet \ y(x) = \exp(x^2 + C)$

Determine C = [a] with the initial condition $y(0) = \frac{1}{3}$

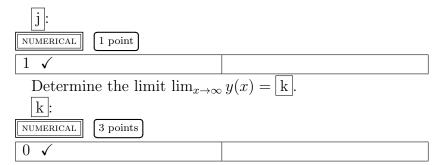
a : NUMERICAL 3 points

Consider the following differential equation.

y''(x) + 4y'(x) + 5y(x) = 0

A solution satisfying y(0) = 2 and y'(0) = -2 can be written as $y(x) = \exp(fx)(g\cos(hx) + i\sin(jx)).$

f: NUMERICAL 2 points **-2** ✓ g: NUMERICAL 1 point 2 h : 1 point NUMERICAL 1 i : 1 point NUMERICAL 2 **√**



Total of marks: 330