2023Call6.

(1) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\cos x = \boxed{\text{a}} + \boxed{\text{b}}x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}x^2 + \boxed{\text{e}}x^3 + \frac{\boxed{\text{f}}}{\boxed{\text{g}}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{f}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 2 points

24 ✓	

$$x^2\sqrt{1+3x} = \boxed{\text{h}} + \boxed{\text{i}}x + \boxed{\text{j}}x^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}x^3 + \frac{\boxed{\text{m}}}{\boxed{\text{n}}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{\text{h}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{i}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{j}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{k}}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{\text{l}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{m}}$:

NUMERICAL | 2 points

-9 ✓	

$\boxed{\text{n}}$:

NUMERICAL | 1 point

8 ✓	

$$(\log(1+x^2))^2 = \boxed{\text{o}} + \boxed{\text{p}}x + \boxed{\text{q}}x^2 + \boxed{\text{r}}x^3 + \boxed{\text{s}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{\text{o}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{p}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{q}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{r}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{s}}$:

NUMERICAL | 4 points

1 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{2\cos x + x^2\sqrt{1+3x} + \alpha + \beta x^3}{(\log(1+x^2))^2}.$$

This limit converges for $\alpha = \boxed{\text{t}}, \beta = \frac{\boxed{\text{u}}}{\boxed{\text{v}}}$.

$\boxed{\text{t}}$:

NUMERICAL | 8 points

-2 ✓	

$\boxed{\text{u}}$:

NUMERICAL | 4 points

-3 ✓	

$\boxed{\text{v}}$:

NUMERICAL | 4 points

2 ✓	

In that case, the limit is $\frac{\boxed{\text{w}}}{\boxed{\text{x}}}$.

$\boxed{\text{w}}$:

NUMERICAL | 6 points

-25 ✓	

$\boxed{\text{x}}$:

NUMERICAL | 2 points

24 ✓	

(2) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\cos x = \boxed{\text{a}} + \boxed{\text{b}}x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}x^2 + \boxed{\text{e}}x^3 + \frac{\boxed{\text{f}}}{\boxed{\text{g}}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{\text{a}}$:

NUMERICAL 1 point

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL 1 point

0 ✓	

$\boxed{\text{c}}$:

NUMERICAL 1 point

-1 ✓	

$\boxed{\text{d}}$:

NUMERICAL 1 point

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL 1 point

0 ✓	

$\boxed{\text{f}}$:

NUMERICAL 1 point

1 ✓	

$\boxed{\text{g}}$:

NUMERICAL 2 points

24 ✓	

$$x^2\sqrt{1+x} = \boxed{\text{h}} + \boxed{\text{i}}x + \boxed{\text{j}}x^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}x^3 + \frac{\boxed{\text{m}}}{\boxed{\text{n}}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{\text{h}}$:

NUMERICAL 1 point

0 ✓	

$\boxed{\text{i}}$:

NUMERICAL 1 point

0 ✓	

$\boxed{\text{j}}$:

NUMERICAL 1 point

1 ✓	

$\boxed{\text{k}}$:

NUMERICAL 1 point

1 ✓	

$\boxed{\text{l}}$:

NUMERICAL 1 point

2 ✓	

$\boxed{\text{m}}$:

NUMERICAL 2 points

-1 ✓	

$\boxed{\text{n}}$:

NUMERICAL 1 point

8 ✓	

$$(\log(1+2x^2))^2 = \boxed{\text{o}} + \boxed{\text{p}}x + \boxed{\text{q}}x^2 + \boxed{\text{r}}x^3 + \boxed{\text{s}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{\text{o}}$:

NUMERICAL 1 point

0 ✓	

$\boxed{\text{p}}$:

NUMERICAL 1 point

0 ✓	

$\boxed{\text{q}}$:

NUMERICAL 1 point

0 ✓	

$\boxed{\text{r}}$:

NUMERICAL 1 point

0 ✓	

$\boxed{\text{s}}$:

NUMERICAL 4 points

4 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{2\cos x + x^2\sqrt{1+x} + \alpha + \beta x^3}{(\log(1+2x^2))^2}.$$

This limit converges for $\alpha = \boxed{\text{t}}$, $\beta = \dfrac{\boxed{\text{u}}}{\boxed{\text{v}}}$.

$\boxed{\text{t}}$:

NUMERICAL | 8 points

-2 ✓	

$\boxed{\text{u}}$:

NUMERICAL | 4 points

-1 ✓	

$\boxed{\text{v}}$:

NUMERICAL | 4 points

2 ✓	

In that case, the limit is $\frac{\boxed{\text{w}}}{\boxed{\text{x}}}$.

$\boxed{\text{w}}$:

NUMERICAL | 6 points

-1 ✓	

$\boxed{\text{x}}$:

NUMERICAL | 2 points

96 ✓	

(3) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let $z = -\sqrt{3} + i$. Fill in the blanks. $z = \boxed{\text{a}}(\cos \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}} + i \sin \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}})$, where $0 \leq \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}} < 2\pi$.

$\boxed{\text{a}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 2 points

5 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 1 point

6 ✓	

Fill in the blanks. $z^6 = \boxed{\text{d}} + i\boxed{\text{e}}$,

$\boxed{\text{d}}$:

NUMERICAL | 3 points

-64 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

0 ✓	

Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n-1}{n!}(x+1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = 1$, calculate the partial sum $\sum_{n=0}^{2} \frac{3^n-1}{n!}(x+1)^n = \boxed{\text{f}}$.

$\boxed{\text{f}}$:

NUMERICAL | 4 points

20 ✓	

In order to use the ration test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n-1}{n!}|x+1|^n$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{\text{g}}$$

$\boxed{\text{g}}$:

NUMERICAL | 4 points

0 ✓	

Therefore, by the ratio test, the series converges absolutely for

MULTI | 4 points | Single

- all x. ✓
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-2 < x < 2$.
- $-\frac{3}{2} < x < \frac{1}{2}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $-1 < x < 1$.
- $-1 < x < 3$.
- $-\frac{1}{2} < x < \frac{1}{2}$.
- $x = 0$.
- $1 < x < 3$.

Calculate the following series.

$$\sum_{n=0}^{\infty} \frac{5}{4^n} = \frac{\boxed{\text{j}}}{\boxed{\text{k}}}$$

$\boxed{\text{j}}$:

NUMERICAL | 2 points

20 ✓	

$\boxed{\text{k}}$:

NUMERICAL | 2 points

3 ✓	

(4) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let $z = 1 + \sqrt{3}i$. Fill in the blanks. $z = \boxed{\text{a}}(\cos \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}} + i \sin \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}})$, where $0 \leq \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}} < 2\pi$.

$\boxed{\text{a}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 1 point

3 ✓	

Fill in the blanks. $z^6 = \boxed{\text{d}} + i\boxed{\text{e}}$,

$\boxed{\text{d}}$:

NUMERICAL | 3 points

64 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

0 ✓	

Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n-1}{n!}(x+1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = 2$, calculate the partial sum $\sum_{n=0}^{2} \frac{3^n-1}{n!}(x+1)^n = \boxed{\text{f}}$.

$\boxed{\text{f}}$:

NUMERICAL | 4 points

42 ✓	

In order to use the ration test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n-1}{n!}|x+1|^n$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{\text{g}}$$

$\boxed{\text{g}}$:

NUMERICAL | 4 points

0 ✓	

Therefore, by the ratio test, the series converges absolutely for

MULTI | 4 points | Single

- all x. ✓
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-2 < x < 2$.
- $-\frac{3}{2} < x < \frac{1}{2}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $-1 < x < 1$.
- $-1 < x < 3$.
- $-\frac{1}{2} < x < \frac{1}{2}$.
- $x = 0$.
- $1 < x < 3$.

Calculate the following series.

$$\sum_{n=0}^{\infty} \frac{2}{5^n} = \frac{\boxed{\text{j}}}{\boxed{\text{k}}}$$

$\boxed{\text{j}}$:

NUMERICAL | 2 points

5 ✓	

$\boxed{\text{k}}$:

NUMERICAL | 2 points

2 ✓	

(5) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x^3}{x^2 + 4x + 3}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single

- -6 (-100%)
- -5 (-100%)
- -4 (-100%)
- -3 ✓
- -2 (-100%)
- -1 ✓
- 0 (-100%)
- 1 (-100%)
- 2 (-100%)
- 3 (-100%)
- 4 (-100%)
- 5 (-100%)
- 6 (-100%)

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -4$ (-100%)
- $y = -1$ (-100%)
- $y = 0$ (-100%)
- $y = 1$ (-100%)
- $y = 4$ (-100%)
- $x = -4$ (-100%)
- $x = -3$ ✓
- $x = -2$ (-100%)
- $x = -1$ ✓
- $x = 0$ (-100%)
- $x = 1$ (-100%)

- $x = 2$ (-100%)
- $x = 3$ (-100%)
- $x = 4$ (-100%)
- $y = x$ (-100%)
- $y = x + 4$ (-100%)
- $y = x - 4$ ✓
- $y = -x$ (-100%)
- $y = -x + 4$ (-100%)
- $y = -x - 4$ (-100%)

One has

$$f'(1) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 4 points

9 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 4 points

32 ✓	

The function $f(x)$ has $\boxed{\text{d}}$ stationary point(s) in the domain

$\boxed{\text{d}}$:

NUMERICAL | 4 points

3 ✓	

Choose the behaviour of $f(x)$ in the interval $(1, 3)$.

MULTI | 4 points | Single | Shuffle

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(6) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x^3}{x^2 - 4x + 3}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single

- -6 (-100%)
- -5 (-100%)
- -4 (-100%)
- -3 (-100%)
- -2 (-100%)
- -1 (-100%)
- 0 (-100%)
- 1 ✓
- 2 (-100%)
- 3 ✓
- 4 (-100%)
- 5 (-100%)
- 6 (-100%)

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -4$ (-100%)
- $y = -1$ (-100%)
- $y = 0$ (-100%)
- $y = 1$ (-100%)
- $y = 4$ (-100%)
- $x = -4$ (-100%)
- $x = -3$ (-100%)
- $x = -2$ (-100%)
- $x = -1$ (-100%)
- $x = 0$ (-100%)
- $x = 1$ ✓
- $x = 2$ (-100%)
- $x = 3$ ✓
- $x = 4$ (-100%)
- $y = x$ (-100%)
- $y = x + 4$ ✓
- $y = x - 4$ (-100%)
- $y = -x$ (-100%)
- $y = -x + 4$ (-100%)
- $y = -x - 4$ (-100%)

One has

$$f'(4) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$:

NUMERICAL 4 points

-112 ✓	

$\boxed{\text{b}}$:

NUMERICAL 4 points

9 ✓	

The function $f(x)$ has $\boxed{\text{d}}$ stationary point(s) in the domain

$\boxed{\text{d}}$:

NUMERICAL 4 points

3 ✓	

Choose the behaviour of $f(x)$ in the interval $(1, 3)$.

MULTI 4 points Single Shuffle

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(7) **Q4**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 \arctan x dx.$$

We integrate this by parts. Consider $\arctan x = 1 \cdot \arctan x$, we can write

$$\int_0^1 \arctan x dx = [x^{\boxed{\text{a}}} \arctan x]_0^1 - \int_0^1 \frac{x^{\boxed{\text{b}}}}{\boxed{\text{c}}x^{\boxed{\text{d}}} + \boxed{\text{e}}} dx.$$

$\boxed{\text{a}}$:

NUMERICAL 1 point

1 ✓	

$\boxed{b}$:

NUMERICAL 1 point

1 ✓	

$\boxed{c}$:

NUMERICAL 2 points

1 ✓	

$\boxed{d}$:

NUMERICAL 1 point

2 ✓	

$\boxed{e}$:

NUMERICAL 1 point

1 ✓	

By continuing, we get

$$\int_0^1 \arctan x dx = \frac{\boxed{f}}{\boxed{g}}\pi + \frac{\boxed{h}}{\boxed{i}}\log 2.$$

$\boxed{f}$:

NUMERICAL 3 points

1 ✓	

$\boxed{g}$:

NUMERICAL 3 points

4 ✓	

$\boxed{h}$:

NUMERICAL 3 points

-1 ✓	

$\boxed{i}$:

NUMERICAL 3 points

2 ✓	

(8) **Q4**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{-1}^{0} \arctan x dx.$$

We integrate this by parts. Consider $\arctan x = 1 \cdot \arctan x$, we can write

$$\int_{-1}^{0} \arctan x dx = [x^{\boxed{\text{a}}} \arctan x]_0^1 - \int_0^1 \frac{x^{\boxed{\text{b}}}}{\boxed{\text{c}}x^{\boxed{\text{d}}} + \boxed{\text{e}}} dx.$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

1 ✓	

By continuing, we get

$$\int_{-1}^{0} \arctan x dx = \frac{\boxed{\text{f}}}{\boxed{\text{g}}}\pi + \frac{\boxed{\text{h}}}{\boxed{\text{i}}} \log 2.$$

$\boxed{\text{f}}$:

NUMERICAL | 3 points

-1 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 3 points

4 ✓	

$\boxed{\text{h}}$:

NUMERICAL | 3 points

1 ✓	

$\boxed{\text{i}}$:

NUMERICAL | 3 points

2 ✓	

(9) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = 2y(x)^2 \cos(x^2)x.$$

MULTI | 2 points | Single

- $y(x) = x^2 \cos(x^2) + C$
- $y(x) = (x^2 \cos(x^2) + C)^{\frac{1}{2}}$
- $y(x) = \sin(x^2) + C$
- $y(x) = \sin(x^2 + C)$
- $y(x) = 1/(x^2 \cos(x^2) + C)$
- $y(x) = 1/x^2 \cos(x^2) + C$
- $y(x) = 1/(-\sin(x^2) + C)$ ✓
- $y(x) = 1/\sin(x^2 + C)$

Determine $C = \boxed{\text{a}}$ with the initial condition $y(0) = \frac{1}{3}$

$\boxed{\text{a}}$:

NUMERICAL | 2 points

3 ✓	

Choose the general solution of the following differential equation.

$$y''(x) - 3y'(x) - 4y(x) = 0.$$

MULTI | 2 points | Single

- $y(x) = C_1 \exp(2x) + C_2 \exp(-2x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(4x)$ ✓
- $y(x) = C_1 \exp(x) + C_2 \exp(4x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(-4x)$
- $y(x) = C_1 \sin(-4x) + C_2 \cos(x)$

- $y(x) = C_1 \sin(-2x) + C_2 \cos(2x)$
- $y(x) = C_1 \sin(2x) + C_2 \cos(-2x)$
- $y(x) = C_1 \sin(x) + C_2 \cos(4x)$

Find a solution $y(x)$ such that $y(0) = 4$ and $y'(0) = 6$. $C_1 = \boxed{\text{b}}$, $C_2 = \boxed{\text{c}}$.

$\boxed{\text{b}}$:

NUMERICAL 2 points

2 ✓	

$\boxed{\text{c}}$:

NUMERICAL 2 points

2 ✓	

For these values of C_1, C_2, find a value $A = \boxed{\text{d}}$ such that $\lim_{x\to\infty} y(x)/\exp(Ax) = 2$.

$\boxed{\text{d}}$:

NUMERICAL 2 points

4 ✓	

(10) **Q5**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = 2y(x)^2 \cos(x^2)x.$$

MULTI 2 points Single

- $y(x) = x^2 \cos(x^2) + C$
- $y(x) = (x^2 \cos(x^2) + C)^{\frac{1}{2}}$
- $y(x) = \sin(x^2) + C$
- $y(x) = \sin(x^2 + C)$
- $y(x) = 1/(x^2 \cos(x^2) + C)$
- $y(x) = 1/x^2 \cos(x^2) + C$
- $y(x) = 1/(-\sin(x^2) + C)$ ✓
- $y(x) = 1/\sin(x^2 + C)$

Determine $C = \boxed{\text{a}}$ with the initial condition $y(0) = \frac{1}{2}$

$\boxed{\text{a}}$:

NUMERICAL 2 points

2 ✓	

Choose the general solution of the following differential equation.

$$y''(x) - 3y'(x) - 4y(x) = 0.$$

MULTI 2 points Single

- $y(x) = C_1 \exp(2x) + C_2 \exp(-2x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(4x)$ ✓
- $y(x) = C_1 \exp(x) + C_2 \exp(4x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(-4x)$
- $y(x) = C_1 \sin(-4x) + C_2 \cos(x)$
- $y(x) = C_1 \sin(-2x) + C_2 \cos(2x)$
- $y(x) = C_1 \sin(2x) + C_2 \cos(-2x)$
- $y(x) = C_1 \sin(x) + C_2 \cos(4x)$

Find a solution $y(x)$ such that $y(0) = 4$ and $y'(0) = 1$. $C_1 = \boxed{\text{b}}$, $C_2 = \boxed{\text{c}}$.

$\boxed{\text{b}}$:

NUMERICAL 2 points

3 ✓	

$\boxed{\text{c}}$:

NUMERICAL 2 points

1 ✓	

For these values of C_1, C_2, find a value $A = \boxed{\text{d}}$ such that $\lim_{x\to-\infty} y(x)/\exp(Ax) = 3$.

$\boxed{\text{d}}$:

NUMERICAL 2 points

-1 ✓	

Total of marks: 252