2023Call5.

(1) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$x^{\frac{1}{3}} = \boxed{a} + \frac{\boxed{b}}{\boxed{c}}(x-1) + \frac{\boxed{d}}{\boxed{e}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

$\boxed{a}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{e}$:

NUMERICAL | 2 points

9 ✓	

$$(x-2)\log x = \boxed{g} + \boxed{h}(x-1) + \frac{\boxed{i}}{\boxed{j}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

$\boxed{g}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{h}$:

NUMERICAL | 2 points

-1 ✓	

i:

NUMERICAL 2 points

3 ✓	

j:

NUMERICAL 1 point

2 ✓	

$(x-1)\exp(x-1) = \boxed{\text{m}} + \boxed{\text{n}}(x-1) + \boxed{\text{o}}(x-1)^2 + o((x-1)^2)$ as $x \to 1$.

m:

NUMERICAL 2 points

0 ✓	

n:

NUMERICAL 2 points

1 ✓	

o:

NUMERICAL 2 points

1 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to 1} \frac{x^{\frac{1}{3}} + (x-2)\log x + \alpha + \beta(x-1)}{(x-1)\exp(x-1) - (x-1)}.$$

This limit converges for $\alpha = \boxed{\text{q}}, \beta = \frac{\boxed{\text{r}}}{\boxed{\text{s}}}$.

q:

NUMERICAL 4 points

-1 ✓	

r:

NUMERICAL 4 points

2 ✓	

s:

NUMERICAL 4 points

3 ✓	

In that case, the limit is $\frac{\boxed{\text{t}}}{\boxed{\text{u}}}$.

t:

NUMERICAL 3 points

25 ✓	

u:

NUMERICAL | 3 points

18 ✓	

(2) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$x^{\frac{1}{2}} = \boxed{\text{a}} + \frac{\boxed{\text{b}}}{\boxed{\text{c}}}(x-1) + \frac{\boxed{\text{d}}}{\boxed{\text{e}}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

a:

NUMERICAL | 1 point

1 ✓	

b:

NUMERICAL | 1 point

1 ✓	

c:

NUMERICAL | 1 point

2 ✓	

d:

NUMERICAL | 1 point

-1 ✓	

e:

NUMERICAL | 2 points

8 ✓	

$$(x-2)\log x = \boxed{\text{g}} + \boxed{\text{h}}(x-1) + \frac{\boxed{\text{i}}}{\boxed{\text{j}}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

g:

NUMERICAL | 1 point

0 ✓	

h:

NUMERICAL 2 points

-1 ✓	

i:

NUMERICAL 2 points

3 ✓	

j:

NUMERICAL 1 point

2 ✓	

$(x-1)\exp(x-1) = \boxed{\text{m}} + \boxed{\text{n}}(x-1) + \boxed{\text{o}}(x-1)^2 + o((x-1)^2)$ as $x \to 1$.

m:

NUMERICAL 2 points

0 ✓	

n:

NUMERICAL 2 points

1 ✓	

o:

NUMERICAL 2 points

1 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to 1} \frac{x^{\frac{1}{2}} + (x-2)\log x + \alpha + \beta(x-1)}{(x-1)\exp(x-1) - (x-1)}.$$

This limit converges for $\alpha = \boxed{\text{q}}$, $\beta = \frac{\boxed{\text{r}}}{\boxed{\text{s}}}$.

q:

NUMERICAL 4 points

-1 ✓	

r:

NUMERICAL 4 points

1 ✓	

s:

NUMERICAL 4 points

2 ✓	

In that case, the limit is $\frac{\boxed{t}}{\boxed{u}}$.

$\boxed{t}$:

NUMERICAL | 3 points

11 ✓	

$\boxed{u}$:

NUMERICAL | 3 points

8 ✓	

(3) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let $z = 1 + i$. Fill in the blanks. $z = \sqrt{\boxed{a}}(\cos \frac{\boxed{b}\pi}{\boxed{c}} + i \sin \frac{\boxed{b}\pi}{\boxed{c}})$, where $0 \le \frac{\boxed{b}\pi}{\boxed{c}} < 2\pi$.

$\boxed{a}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{b}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

4 ✓	

Fill in the blanks. $z^{10} = \boxed{d} + i\boxed{e}$,

$\boxed{d}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{e}$:

NUMERICAL | 3 points

32 ✓	

Let us study the following series $\sum_{n=0}^{\infty} \frac{(4^n-1)(x-1)^{2n}}{n+1}$, with various $x \in \mathbb{R}$.

Calculate the finite sum $\sum_{n=0}^{2} \frac{(4^n-1)(x-1)^{2n}}{n+1} = \frac{\boxed{f}}{\boxed{g}}$ for $x = 0$.

f:

NUMERICAL | 2 points

13 ✓	

g:

NUMERICAL | 2 points

2 ✓	

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \left| \frac{(4^n-1)(x-1)^{2n}}{n+1} \right|$. Complete the formula.

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \boxed{h} |x + \boxed{i}|^{\boxed{j}}$$

h:

NUMERICAL | 1 point

4 ✓	

i:

NUMERICAL | 2 points

-1 ✓	

j:

NUMERICAL | 1 point

2 ✓	

Therefore, by the root test, the series converges absolutely for $\frac{\boxed{k}}{\boxed{l}} < x < \frac{\boxed{m}}{\boxed{n}}$.

k:

NUMERICAL | 1 point

1 ✓	

l:

NUMERICAL | 1 point

2 ✓	

m:

NUMERICAL | 1 point

3 ✓	

n:

NUMERICAL | 1 point

2 ✓	

For the case $x = \frac{3}{2}$, the series

MULTI | 4 points | Single

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(4) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let $z = -1 + i$. Fill in the blanks. $z = \sqrt{\boxed{a}}(\cos \frac{\boxed{b}\pi}{\boxed{c}} + i \sin \frac{\boxed{b}\pi}{\boxed{c}})$, where $0 \leq \frac{\boxed{b}\pi}{\boxed{c}} < 2\pi$.

$\boxed{a}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{b}$:

NUMERICAL | 2 points

3 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

4 ✓	

Fill in the blanks. $z^{10} = \boxed{d} + i\boxed{e}$,

$\boxed{d}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{e}$:

NUMERICAL | 3 points

-32 ✓	

Let us study the following series $\sum_{n=0}^{\infty} \frac{(4^n-1)(x+1)^{2n}}{n+1}$, with various $x \in \mathbb{R}$.

Calculate the finite sum $\sum_{n=0}^{2} \frac{(4^n-1)(x+1)^{2n}}{n+1} = \frac{\boxed{\text{f}}}{\boxed{\text{g}}}$ for $x = 0$.

$\boxed{\text{f}}$:

NUMERICAL | 2 points

13 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 2 points

2 ✓	

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \left|\frac{(4^n-1)(x+1)^{2n}}{n+1}\right|$. Complete the formula.

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \boxed{\text{h}}|x + \boxed{\text{i}}|^{\boxed{\text{j}}}$$

$\boxed{\text{h}}$:

NUMERICAL | 1 point

4 ✓	

$\boxed{\text{i}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{j}}$:

NUMERICAL | 1 point

2 ✓	

Therefore, by the root test, the series converges absolutely for $\frac{\boxed{\text{k}}}{\boxed{\text{l}}} < x < \frac{\boxed{\text{m}}}{\boxed{\text{n}}}$.

$\boxed{\text{k}}$:

NUMERICAL | 1 point

-3 ✓	

$\boxed{\text{l}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{m}}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{\text{n}}$:

NUMERICAL | 1 point

2 ✓	

For the case $x = -\frac{2}{3}$, the series

MULTI | 4 points | Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(5) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \sqrt{\frac{x^4}{x^2 + 3x - 4}}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single

- -6 (-100%)
- -5 (-100%)
- -4 ✓
- -3 ✓
- -2 ✓
- -1 ✓
- 1 ✓
- 2 (-100%)
- 3 (-100%)
- 4 (-100%)
- 5 (-100%)
- 6 (-100%)

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -4$ (-100%)
- $y = -1$ (-100%)
- $y = 0$ (-100%)
- $y = 1$ (-100%)
- $y = 4$ (-100%)

- $x = -4$ ✓
- $x = -2$ (−100%)
- $x = -\sqrt{2}$ (−100%)
- $x = -1$ (−100%)
- $x = 0$ (−100%)
- $x = 1$ ✓
- $x = \sqrt{2}$ (−100%)
- $x = 2$ (−100%)
- $x = 4$ (−100%)
- $y = x/2$ (−100%)
- $y = x$ ✓
- $y = 2x$ (−100%)
- $y = -x/2$ (−100%)
- $y = -x$ ✓
- $y = -2x$ (−100%)

One has

$$f'(2) = \frac{\boxed{\text{a}}\sqrt{\boxed{\text{b}}}}{\boxed{\text{c}}}.$$

[a]:

NUMERICAL | 4 points

5 ✓	

[b]:

NUMERICAL | 2 points

6 ✓	

[c]:

NUMERICAL | 2 points

18 ✓	

The function $f(x)$ has [d] stationary point(s) in the domain

[d]:

NUMERICAL | 4 points

2 ✓	

Choose the behaviour of $f(x)$ in the interval $(1, 3)$.

MULTI | 4 points | Single | Shuffle

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(6) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \sqrt{\frac{x^4}{x^2 - 3x - 4}}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single

- -6 (-100%)
- -5 (-100%)
- -4 (-100%)
- -3 (-100%)
- -2 (-100%)
- -1 ✓
- 1 ✓
- 2 ✓
- 3 ✓
- 4 ✓
- 5 (-100%)
- 6 (-100%)

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -4$ (-100%)
- $y = -1$ (-100%)
- $y = 0$ (-100%)
- $y = 1$ (-100%)
- $y = 4$ (-100%)
- $x = -4$ (-100%)
- $x = -2$ (-100%)
- $x = -\sqrt{2}$ (-100%)
- $x = -1$ ✓
- $x = 0$ (-100%)
- $x = 1$ (-100%)
- $x = \sqrt{2}$ (-100%)

- $x = 2$ (-100%)
- $x = 4$ ✓
- $y = x/2$ (-100%)
- $y = x$ ✓
- $y = 2x$ (-100%)
- $y = -x/2$ (-100%)
- $y = -x$ ✓
- $y = -2x$ (-100%)

One has

$$f'(5) = \frac{\boxed{\text{a}}\sqrt{\boxed{\text{b}}}}{\boxed{\text{c}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 4 points

-55 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 2 points

6 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

72 ✓	

The function $f(x)$ has $\boxed{\text{d}}$ stationary point(s) in the domain

$\boxed{\text{d}}$:

NUMERICAL | 4 points

2 ✓	

Choose the behaviour of $f(x)$ in the interval $(-3, -2)$.

MULTI | 4 points | Single | Shuffle

- monotonically decreasing ✓
- monotonically increasing
- neither decreasing nor increasing

(7) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 \frac{2x^2-8}{x^3-3x^2+x-3}dx.$$

Complete the formula

$$\frac{2x^2-8}{x^3-3x^2+x-3} = \frac{\boxed{\text{a}}x+\boxed{\text{b}}}{x^2+\boxed{\text{c}}} + \frac{\boxed{\text{d}}}{x+\boxed{\text{e}}}.$$

a:

NUMERICAL | 1 point

1 ✓	

b:

NUMERICAL | 1 point

3 ✓	

c:

NUMERICAL | 2 points

1 ✓	

d:

NUMERICAL | 1 point

1 ✓	

e:

NUMERICAL | 5 points

-3 ✓	

Choose a primitive of $\frac{x}{x^2+1}$.

MULTI | 5 points | Single

- $\arctan x$
- $\arctan(x+1)$
- $\frac{x}{2}\arctan(x)$
- $x\arctan(x^2+1)$
- $\frac{1}{4}\log(x^2+1)$
- $\frac{1}{2}\log(x^2+1)$ ✓
- $\log(x(x^2+1))$
- $\frac{1}{4}\arcsin(x^2+1)$
- $\frac{1}{2}\arcsin(x^2+1)$
- $\arcsin(x(x^2+1))$

By continuing, we get

$$\int_0^1 \frac{2x^2-8}{x^3-3x^2+x-3}dx = \frac{\boxed{\text{f}}}{\boxed{\text{g}}}\pi + \frac{\boxed{\text{h}}}{\boxed{\text{i}}}\log 2 + \boxed{\text{j}}\log 3.$$

f:

NUMERICAL | 1 point

3 ✓	

g:

NUMERICAL | 2 points

4 ✓	

h:

NUMERICAL | 4 points

3 ✓	

i:

NUMERICAL | 4 points

2 ✓	

j:

NUMERICAL | 4 points

-1 ✓	

(8) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{-1}^{0} \frac{2x^2 - 8}{x^3 - 3x^2 + x - 3} dx.$$

Complete the formula

$$\frac{2x^2 - 8}{x^3 - 3x^2 + x - 3} = \frac{\boxed{a}x + \boxed{b}}{x^2 + \boxed{c}} + \frac{\boxed{d}}{x + \boxed{e}}.$$

a:

NUMERICAL | 1 point

1 ✓	

b:

NUMERICAL | 1 point

3 ✓	

c:

NUMERICAL | 2 points

1 ✓	

d:

NUMERICAL | 1 point

1 ✓	

e:

NUMERICAL | 5 points

-3 ✓	

Choose a primitive of $\frac{x}{x^2+1}$.

MULTI | 5 points | Single

- $\arctan x$
- $\arctan(x+1)$
- $\frac{x}{2}\arctan(x)$
- $x\arctan(x^2+1)$
- $\frac{1}{4}\log(x^2+1)$
- $\frac{1}{2}\log(x^2+1)$ ✓
- $\log(x(x^2+1))$
- $\frac{1}{4}\arcsin(x^2+1)$
- $\frac{1}{2}\arcsin(x^2+1)$
- $\arcsin(x(x^2+1))$

By continuing, we get

$$\int_{-1}^{0}\frac{2x^2-8}{x^3-3x^2+x-3}dx = \frac{\boxed{f}}{\boxed{g}}\pi + \frac{\boxed{h}}{\boxed{i}}\log 2 + \boxed{j}\log 3.$$

f:

NUMERICAL | 1 point

3 ✓	

g:

NUMERICAL | 2 points

4 ✓	

h:

NUMERICAL | 4 points

-5 ✓	

i:

NUMERICAL | 4 points

2 ✓	

j:

NUMERICAL | 4 points

1 ✓	

(9) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Consider the following improper integral for various $\alpha \in \mathbb{R}, \alpha \neq 0$.

$$\int_0^\infty x e^{\alpha x^2} dx.$$

By definition of improper integral, this is $\lim_{\beta \to \infty} \int_0^\beta x \exp(\alpha x^2) dx$. Choose the primitive of $x \exp(\alpha x^2)$ (for $\alpha \neq 0$).

MULTI | 1 point | Single

- $\frac{1}{2}x^2 + \exp(\alpha x^2)$
- $\frac{1}{2}x^2 \exp(\alpha x^2)$
- $\frac{1}{\alpha}x^\alpha \exp(\alpha x^2)$
- $\frac{1}{2}x^\alpha \exp(\alpha x^2)$
- $\exp(\alpha x^2)/2\alpha$ ✓
- $\exp(\alpha x^2)/2$
- $\exp(\alpha x^3/3)$
- $x \exp(\alpha x^3/3)$

Choose all values of α such that the improper integral is convergent.

MULTI | 1 point | Single

- none of them (−100%)
- -6 ✓
- $-\pi$ ✓
- $-e$ ✓
- -2 ✓
- -1 ✓
- $-\frac{1}{2}$ ✓
- 0 (−100%)
- $\frac{1}{2}$ (−100%)
- 1 (−100%)
- 2 (−100%)

- π (−100%)
- e (−100%)
- 6 (−100%)

Among the correct options above, take the value of α such that the improper integral is the smallest, and compute the value: $\int_0^\infty xe^{\alpha x^2}dx = \frac{\boxed{a}}{\boxed{b}}$.

a:

NUMERICAL | 1 point

1 ✓	

b:

NUMERICAL | 1 point

12 ✓	

Choose the values of β such that the following integral converges.

$$\int_0^\infty xe^{-x^2+\beta x}.$$

MULTI | 1 point | Single

- none of them (−100%)
- -5 ✓
- $-\pi$ ✓
- $-e$ ✓
- -2 ✓
- -1 ✓
- $-\frac{1}{2}$ ✓
- 0 ✓
- $\frac{1}{2}$ ✓
- 1 ✓
- 2 ✓
- e ✓
- π ✓
- 5 ✓

Choose the values of γ such that the following integral converges.

$$\int_0^\infty x^\gamma e^{-x^2}.$$

MULTI | 1 point | Single

- none of them (−100%)
- -5 (−100%)
- $-\pi$ (−100%)

- $-e$ (-100%)
- -2 (-100%)
- -1 (-100%)
- $-\frac{1}{2}$ ✓
- 0 ✓
- $\frac{1}{2}$ ✓
- 1 ✓
- 2 ✓
- e ✓
- π ✓
- 5 ✓

Total of marks: 234