2023Call4.

(1) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\sin(3x) = \boxed{a} + \boxed{b}x + \boxed{c}x^2 + \frac{\boxed{d}}{\boxed{e}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{a}$:

NUMERICAL | 2 points

0 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

-9 ✓	

$\boxed{e}$:

NUMERICAL | 1 point

2 ✓	

$$x\sqrt{1+x^2} = \boxed{h} + \boxed{i}x + \boxed{j}x^2 + \frac{\boxed{k}}{\boxed{l}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{h}$:

NUMERICAL | 2 points

0 ✓	

$\boxed{i}$:

NUMERICAL | 1 point

1 ✓	

j:

NUMERICAL 1 point

0 ✓	

k:

NUMERICAL 1 point

1 ✓	

l:

NUMERICAL 1 point

2 ✓	

$$\exp(5x^3) = \boxed{\text{n}} + \boxed{\text{o}}x + \boxed{\text{p}}x^2 + \boxed{\text{q}}x^3 + o(x^3) \text{ as } x \to 0.$$

n:

NUMERICAL 2 points

1 ✓	

o:

NUMERICAL 1 point

0 ✓	

p:

NUMERICAL 1 point

0 ✓	

q:

NUMERICAL 2 points

5 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 0} \frac{\sin(3x) + \alpha x\sqrt{1+x^2} + \beta x^2}{\exp(5x^3) - 1}.$$

This limit converges for $\alpha = \boxed{\text{r}}, \beta = \boxed{\text{s}}$.

r:

NUMERICAL 6 points

-3 ✓	

s:

NUMERICAL 6 points

0 ✓	

In that case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

v:

NUMERICAL | 3 points

-6 ✓	

w:

NUMERICAL | 3 points

5 ✓	

(2) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\sin(-3x) = \boxed{a} + \boxed{b}x + \boxed{c}x^2 + \frac{\boxed{d}}{\boxed{e}}x^3 + o(x^3) \text{ as } x \to 0.$$

a:

NUMERICAL | 2 points

0 ✓	

b:

NUMERICAL | 1 point

-3 ✓	

c:

NUMERICAL | 1 point

0 ✓	

d:

NUMERICAL | 1 point

9 ✓	

e:

NUMERICAL | 1 point

2 ✓	

$$x\sqrt{1-x^2} = \boxed{h} + \boxed{i}x + \boxed{j}x^2 + \frac{\boxed{k}}{\boxed{l}}x^3 + o(x^3) \text{ as } x \to 0.$$

h:

NUMERICAL 2 points

0 ✓	

i:

NUMERICAL 1 point

1 ✓	

j:

NUMERICAL 1 point

0 ✓	

k:

NUMERICAL 1 point

-1 ✓	

l:

NUMERICAL 1 point

2 ✓	

$\exp(4x^3) = \boxed{n} + \boxed{o}x + \boxed{p}x^2 + \boxed{q}x^3 + o(x^3)$ as $x \to 0$.

n:

NUMERICAL 2 points

1 ✓	

o:

NUMERICAL 1 point

0 ✓	

p:

NUMERICAL 1 point

0 ✓	

q:

NUMERICAL 2 points

4 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{\sin(-3x) + \alpha x\sqrt{1-x^2} + \beta x^2}{\exp(4x^3) - 1}.$$

This limit converges for $\alpha = \boxed{r}, \beta = \boxed{s}$.

r:

NUMERICAL 6 points

3 ✓	

s:

NUMERICAL | 6 points

0 ✓	

In that case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

$\boxed{\text{v}}$:

NUMERICAL | 3 points

3 ✓	

$\boxed{\text{w}}$:

NUMERICAL | 3 points

4 ✓	

(3) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let $z = 1 + i\sqrt{3}$. Fill in the blanks. $z = \boxed{\text{a}}(\cos \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}} + i \sin \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}})$, where $0 \le \frac{\boxed{\text{b}}\pi}{\boxed{\text{c}}} < 2\pi$.

$\boxed{\text{a}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

3 ✓	

Fill in the blanks. $z^6 = \boxed{\text{d}} + i\boxed{\text{e}}$,

$\boxed{\text{d}}$:

NUMERICAL | 3 points

64 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

0 ✓	

Let us study the following series $\sum_{n=0}^{\infty} \frac{(3^n-2^n)(x-1)^n}{n+2}$, with various $x \in \mathbb{R}$.

Calculate the finite sum $\sum_{n=0}^{1} \frac{(3^n-2^n)(x-1)^n}{n+2} = \frac{\boxed{f}}{\boxed{g}}$ for $x = -1$.

$\boxed{f}$:

NUMERICAL 2 points

-2 ✓	

$\boxed{g}$:

NUMERICAL 2 points

3 ✓	

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \left|\frac{(3^n-2^n)(x-1)^n}{n+2}\right|$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{h}|x + \boxed{i}|^{\boxed{j}}$$

$\boxed{h}$:

NUMERICAL 2 points

3 ✓	

$\boxed{i}$:

NUMERICAL 1 point

-1 ✓	

$\boxed{j}$:

NUMERICAL 1 point

1 ✓	

Therefore, by the ratio test, the series converges absolutely for $\frac{\boxed{k}}{\boxed{l}} < x < \frac{\boxed{m}}{\boxed{n}}$.

$\boxed{k}$:

NUMERICAL 1 point

2 ✓	

$\boxed{l}$:

NUMERICAL 1 point

3 ✓	

$\boxed{m}$:

NUMERICAL 1 point

4 ✓	

n:

NUMERICAL | 1 point

3 ✓	

For the case $x = -\frac{4}{3}$, the series

MULTI | 4 points | Single | Shuffle

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(4) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let $z = \sqrt{3}+i$. Fill in the blanks. $z = \boxed{a}(\cos \frac{\boxed{b}\pi}{\boxed{c}} + i \sin \frac{\boxed{b}\pi}{\boxed{c}})$, where $0 \le \frac{\boxed{b}\pi}{\boxed{c}} < 2\pi$.

a:

NUMERICAL | 1 point

2 ✓	

b:

NUMERICAL | 1 point

1 ✓	

c:

NUMERICAL | 2 points

6 ✓	

Fill in the blanks. $z^6 = \boxed{d} + i\boxed{e}$,

d:

NUMERICAL | 3 points

-64 ✓	

e:

NUMERICAL | 1 point

0 ✓	

Let us study the following series $\sum_{n=0}^{\infty} \frac{(3^n-2^n)(x+1)^n}{n+2}$, with various $x \in \mathbb{R}$.

Calculate the finite sum $\sum_{n=0}^{1} \frac{(3^n-2^n)(x+1)^n}{n+2} = \frac{\boxed{f}}{\boxed{g}}$ for $x = 1$.

$\boxed{f}$:

NUMERICAL | 2 points

2 ✓	

$\boxed{g}$:

NUMERICAL | 2 points

3 ✓	

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \left|\frac{(3^n-2^n)(x+1)^n}{n+2}\right|$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{h}|x + \boxed{i}|^{\boxed{j}}$$

$\boxed{h}$:

NUMERICAL | 2 points

3 ✓	

$\boxed{i}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{j}$:

NUMERICAL | 1 point

1 ✓	

Therefore, by the ratio test, the series converges absolutely for $\frac{\boxed{k}}{\boxed{l}} < x < \frac{\boxed{m}}{\boxed{n}}$.

$\boxed{k}$:

NUMERICAL | 1 point

-4 ✓	

$\boxed{l}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{m}$:

NUMERICAL | 1 point

-2 ✓	

$\boxed{n}$:

NUMERICAL | 1 point

3 ✓	

For the case $x = -\frac{4}{3}$, the series

MULTI | 4 points | Single | Shuffle

- converges absolutely.
- converges but not absolutely. ✓
- diverges.

(5) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log \frac{x^2+2}{x^2+3x+2}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single

- -3 (-100%)
- $-\frac{5}{2}$ (-100%)
- -2 ✓
- $-\frac{3}{2}$ ✓
- -1 ✓
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 (-100%)
- $\frac{3}{2}$ (-100%)
- 2 (-100%)
- $\frac{5}{2}$ (-100%)
- 3 (-100%)

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -1$ (-100%)
- $y = 0$ ✓

- $y = 1$ (−100%)
- $x = -2$ ✓
- $x = -1$ ✓
- $x = 0$ (−100%)
- $x = 1$ (−100%)
- $x = 2$ (−100%)
- $y = x/2$ (−100%)
- $y = x/2 + 1$ (−100%)
- $y = x$ (−100%)
- $y = x + 1$ (−100%)
- $y = 2x$ (−100%)
- $y = 2x + 1$ (−100%)
- $y = -x/2$ (−100%)
- $y = -x/2 + 1$ (−100%)
- $y = -x$ (−100%)
- $y = -x + 1$ (−100%)
- $y = -2x$ (−100%)
- $y = -2x + 1$ (−100%)

One has

$$f'(1) = \frac{\boxed{a}}{\boxed{b}}.$$

a:

NUMERICAL | 4 points

-1 ✓	

b:

NUMERICAL | 4 points

6 ✓	

The function $f(x)$ has c stationary point(s) in the domain

c:

NUMERICAL | 4 points

1 ✓	

Choose the behaviour of $f(x)$ in the interval $(1, 2)$.

MULTI | 4 points | Single | Shuffle

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(6) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log \frac{2x^2+1}{2x^2+3x+1}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI 4 points Single

- -3 (-100%)
- $-\frac{5}{2}$ (-100%)
- -2 (-100%)
- $-\frac{3}{2}$ (-100%)
- -1 ✓
- $-\frac{1}{2}$ ✓
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 (-100%)
- $\frac{3}{2}$ (-100%)
- 2 (-100%)
- $\frac{5}{2}$ (-100%)
- 3 (-100%)

Choose all asymptotes of $f(x)$.

MULTI 4 points Single

- $y = -1$ (-100%)
- $y = 0$ ✓
- $y = 1$ (-100%)
- $x = -2$ (-100%)
- $x = -1$ ✓
- $x = -\frac{1}{2}$ ✓
- $x = 0$ (-100%)
- $x = \frac{1}{2}$ (-100%)
- $x = 1$ (-100%)
- $x = 2$ (-100%)
- $y = x/2$ (-100%)

- $y = x/2 + 1$ (-100%)
- $y = x$ (-100%)
- $y = x + 1$ (-100%)
- $y = 2x$ (-100%)
- $y = 2x + 1$ (-100%)
- $y = -x/2$ (-100%)
- $y = -x/2 + 1$ (-100%)
- $y = -x$ (-100%)
- $y = -x + 1$ (-100%)
- $y = -2x$ (-100%)
- $y = -2x + 1$ (-100%)

One has

$$f'(-2) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$:

NUMERICAL 4 points

7 ✓	

$\boxed{\text{b}}$:

NUMERICAL 4 points

9 ✓	

The function $f(x)$ has $\boxed{\text{c}}$ stationary point(s) in the domain

$\boxed{\text{c}}$:

NUMERICAL 4 points

1 ✓	

Choose the behaviour of $f(x)$ in the interval $(1, 2)$.

MULTI 4 points Single Shuffle

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(7) **Q4**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^{\frac{\pi}{2}} \cos^2(x)\sin\left(x+\frac{\pi}{3}\right)dx.$$

Complete the formula

$$\sin\left(x+\frac{\pi}{3}\right) = \frac{\boxed{a}}{\boxed{b}}\sin x + \frac{\sqrt{\boxed{c}}}{\boxed{d}}\cos x.$$

a:

NUMERICAL | 2 points

1 ✓	

b:

NUMERICAL | 2 points

2 ✓	

c:

NUMERICAL | 2 points

3 ✓	

d:

NUMERICAL | 2 points

2 ✓	

Choose a primitive of $\cos^2(x)\sin(x)$.

MULTI | 8 points | Single | Shuffle

- $\frac{1}{3}\cos^3(x)\sin(x)$
- $-\frac{1}{3}\cos^3(x)\sin(x)$
- $\frac{1}{3}\sin^3(x)$
- $-\frac{1}{3}\cos^3(x)$ ✓
- $\frac{1}{2}\cos^3(\sin(x))$
- $-\frac{1}{2}\cos^3(\sin(x))$
- $\frac{1}{2}\sin^3(\cos(x))$
- $-\frac{1}{2}\sin^3(\cos(x))$

Choose a primitive of $\cos^3(x)$.

MULTI | 8 points | Single | Shuffle

- $-\frac{1}{4}\cos^4(x)$
- $\frac{1}{4}\sin^4(x)$
- $-\cos(x)+\frac{1}{3}\cos^3(x)$
- $\cos(x)-\frac{1}{3}\cos^3(x)$
- $-\cos(x)+\frac{1}{3}\cos^3(x)$
- $\sin(x)-\frac{1}{3}\sin^3(x)$ ✓
- $x-\frac{1}{3}\sin^3(x)$

- $x - \frac{1}{3}\cos^3(x)$
- $x + \frac{1}{4}\sin^4(x)$
- $x - \frac{1}{4}\cos^4(x)$

By continuing, we get

$$\int_0^{\frac{\pi}{2}} \cos^2(x)\sin\left(x+\frac{\pi}{3}\right)dx = \frac{\boxed{e}}{\boxed{f}} + \frac{\sqrt{\boxed{g}}}{\boxed{h}}$$

.

$\boxed{e}$:

NUMERICAL | 6 points

1 ✓	

$\boxed{f}$:

NUMERICAL | 6 points

6 ✓	

$\boxed{g}$:

NUMERICAL | 6 points

3 ✓	

$\boxed{h}$:

NUMERICAL | 6 points

3 ✓	

(8) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^{\frac{\pi}{2}} \cos^2(x)\sin\left(x+\frac{\pi}{3}\right)dx.$$

Complete the formula

$$\sin\left(x+\frac{\pi}{6}\right) = \frac{\sqrt{\boxed{a}}}{\boxed{b}}\sin x + \frac{\boxed{c}}{\boxed{d}}\cos x.$$

$\boxed{a}$:

NUMERICAL | 2 points

3 ✓	

b:

NUMERICAL | 2 points

2 ✓	

c:

NUMERICAL | 2 points

1 ✓	

d:

NUMERICAL | 2 points

2 ✓	

Choose a primitive of $\cos^2(x)\sin(x)$.

MULTI | 8 points | Single | Shuffle

- $\frac{1}{3}\cos^3(x)\sin(x)$
- $-\frac{1}{3}\cos^3(x)\sin(x)$
- $\frac{1}{3}\sin^3(x)$
- $-\frac{1}{3}\cos^3(x)$ ✓
- $\frac{1}{2}\cos^3(\sin(x))$
- $-\frac{1}{2}\cos^3(\sin(x))$
- $\frac{1}{2}\sin^3(\cos(x))$
- $-\frac{1}{2}\sin^3(\cos(x))$

Choose a primitive of $\cos^3(x)$.

MULTI | 8 points | Single | Shuffle

- $-\frac{1}{4}\cos^4(x)$
- $\frac{1}{4}\sin^4(x)$
- $-\cos(x)+\frac{1}{3}\cos^3(x)$
- $\cos(x)-\frac{1}{3}\cos^3(x)$
- $-\cos(x)+\frac{1}{3}\cos^3(x)$
- $\sin(x)-\frac{1}{3}\sin^3(x)$ ✓
- $x-\frac{1}{3}\sin^3(x)$
- $x-\frac{1}{3}\cos^3(x)$
- $x+\frac{1}{4}\sin^4(x)$
- $x-\frac{1}{4}\cos^4(x)$

By continuing, we get

$$\int_0^{\frac{\pi}{2}} \cos^2(x)\sin\left(x+\frac{\pi}{6}\right)dx = \frac{\boxed{e}}{\boxed{f}} + \frac{\sqrt{\boxed{g}}}{\boxed{h}}$$

.

e:

NUMERICAL | 6 points

1 ✓	

f:

NUMERICAL | 6 points

3 ✓	

g:

NUMERICAL | 6 points

3 ✓	

h:

NUMERICAL | 6 points

6 ✓	

(9) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = y(x)x\exp(x^2).$$

MULTI | 2 points | Single

- $y(x) = (\exp(\sin x^2) + C)^2$
- $y(x) = -2/(\sin(x^2) + C)^2$
- $y(x) = (\sin(\exp x) + C)^2$
- $y(x) = -2/(\cos(x^2) + C)^2$
- $y(x) = \exp((\exp x^2)/2 + C)$ ✓
- $y(x) = -\log((\exp x^2)/2 + C)$
- $y(x) = \exp((\exp x)/2 + C)^2$
- $y(x) = -\exp((\log x^2)/2 + C)$

Determine $C = \frac{\boxed{a}}{\boxed{b}}$ with the initial condition $y(0) = e$

a:

NUMERICAL | 1 point

1 ✓	

a:

NUMERICAL | 1 point

2 ✓	

Choose the general solution of the following differential equation.

$$y''(x) + 4y'(x) + 5y(x) = 0.$$

MULTI | 2 points | Single

- $y(x) = C_1 \exp(x) + C_2 \exp(-4x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(4x)$
- $y(x) = C_1 \exp(x) + C_2 \exp(5x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(-5x)$
- $y(x) = C_1 \exp(-2x) \sin(2x) + C_2 \exp(2x) \cos(2x)$
- $y(x) = C_1 \exp(-x) \sin(2x) + C_2 \exp(-2x) \cos(x)$
- $y(x) = C_1 \exp(-2x) \sin(x) + C_2 \exp(-2x) \cos(x)$ ✓
- $y(x) = C_1 \exp(-x) \sin(5x) + C_2 \exp(-x) \cos(5x)$

Find a solution $y(x)$ such that $y(0) = 0$ and $y'(0) = 4$. $C_1 =$ [c], $C_2 =$ [d].

[c]:

NUMERICAL | 2 points

4 ✓	

[d]:

NUMERICAL | 2 points

0 ✓	

For the solution above, calculate $\lim_{x\to\infty} y(x) =$ [e].

[e]:

NUMERICAL | 2 points

0 ✓	

(10) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{[a]}{[b]}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = y(x)x\exp(x^2).$$

MULTI | 2 points | Single

- $y(x) = (\exp(\sin x^2) + C)^2$

- $y(x) = -2/(\sin(x^2) + C)^2$
- $y(x) = (\sin(\exp x) + C)^2$
- $y(x) = -2/(\cos(x^2) + C)^2$
- $y(x) = \exp((\exp x^2)/2 + C)$ ✓
- $y(x) = -\log((\exp x^2)/2 + C)$
- $y(x) = \exp((\exp x)/2 + C)^2$
- $y(x) = -\exp((\log x^2)/2 + C)$

Determine $C = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}$ with the initial condition $y(0) = e$

a:

NUMERICAL | 1 point

1 ✓	

a:

NUMERICAL | 1 point

2 ✓	

Choose the general solution of the following differential equation.

$$y''(x) + 4y'(x) - 5y(x) = 0.$$

MULTI | 2 points | Single

- $y(x) = C_1 \exp(x) + C_2 \exp(-4x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(4x)$
- $y(x) = C_1 \exp(x) + C_2 \exp(5x)$
- $y(x) = C_1 \exp(x) + C_2 \exp(-5x)$ ✓
- $y(x) = C_1 \exp(-2x) \sin(2x) + C_2 \exp(2x) \cos(2x)$
- $y(x) = C_1 \exp(-x) \sin(2x) + C_2 \exp(-2x) \cos(x)$
- $y(x) = C_1 \exp(-2x) \sin(x) + C_2 \exp(-2x) \cos(x)$
- $y(x) = C_1 \exp(-x) \sin(5x) + C_2 \exp(-x) \cos(5x)$

Find a solution $y(x)$ such that $y(0) = 2$ and $y'(0) = -10$. $C_1 = \boxed{\text{c}}, C_2 = \boxed{\text{d}}$.

c:

NUMERICAL | 2 points

0 ✓	

d:

NUMERICAL | 2 points

2 ✓	

For the solution above, calculate $\lim_{x\to\infty} y(x) = \boxed{\text{e}}$.

e:

NUMERICAL | 2 points

0 ✓	

Total of marks: 288