2023Call3.

(1) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\cos(x-1) = \boxed{a} + \boxed{b}(x-1) + \frac{\boxed{c}}{\boxed{d}}(x-1)^2 + \boxed{e}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{a}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{c}$:

NUMERICAL | 2 points

-1 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{e}$:

NUMERICAL | 1 point

0 ✓	

$$(x-1)\sqrt{x} = \boxed{g} + \boxed{h}(x-1) + \frac{\boxed{i}}{\boxed{j}}(x-1)^2 + \frac{\boxed{k}}{\boxed{l}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{g}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{h}$:

NUMERICAL | 1 point

1 ✓	

i:

NUMERICAL 1 point

1 ✓	

j:

NUMERICAL 1 point

2 ✓	

k:

NUMERICAL 1 point

-1 ✓	

l:

NUMERICAL 1 point

8 ✓	

$\sin((x-1)^3) = \boxed{\text{m}} + \boxed{\text{n}}(x-1) + \boxed{\text{o}}(x-1)^2 + \boxed{\text{p}}(x-1)^3 + o((x-1)^3)$ as $x \to 1$.

m:

NUMERICAL 2 points

0 ✓	

n:

NUMERICAL 1 point

0 ✓	

o:

NUMERICAL 1 point

0 ✓	

p:

NUMERICAL 2 points

1 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 1} \frac{\cos(x-1) + (x-1)\sqrt{x} + \alpha + \beta(x-1)}{\sin((x-1)^3)}.$$

This limit converges for $\alpha = \boxed{\text{q}}$, $\beta = \boxed{\text{r}}$.

q:

NUMERICAL 6 points

-1 ✓	

r:

NUMERICAL 6 points

-1 ✓	

In that case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

$\boxed{\text{v}}$:

NUMERICAL 3 points

-1 ✓	

$\boxed{\text{w}}$:

NUMERICAL 3 points

8 ✓	

(2) **Q2**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n-1}{n!}(x+1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{3^n-1}{n!}(x+1)^n = \boxed{\text{a}} + \boxed{\text{b}}i$.

$\boxed{\text{a}}$:

NUMERICAL 1 point

2 ✓	

$\boxed{\text{b}}$:

NUMERICAL 1 point

10 ✓	

In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n-1}{n!}|x+1|^n$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{\text{c}}$$

$\boxed{\text{c}}$:

NUMERICAL 4 points

0 ✓	

Therefore, by the ratio test, the series converges absolutely for

MULTI 2 points Single

- all x. ✓
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-\frac{5}{4} < x < -\frac{3}{4}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $\frac{1}{2} < x < \frac{3}{2}$.
- $\frac{3}{4} < x < \frac{5}{4}$.
- $-1 < x < 1$.
- $-1 < x < 3$.
- $x = 0$.
- $1 < x < 3$.

For the case $x = -\frac{4}{3}$, the series

MULTI | 2 points | Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

Calculate the sum $\sum_{n=0}^{\infty} \frac{5}{2^n} =$ [d].

[d]:

NUMERICAL | 2 points

10 ✓	

(3) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n - 1}{n!}(x+1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = -i$, calculate the partial sum $\sum_{n=0}^{2} \frac{3^n - 1}{n!}(x+1)^n =$ [a] + [b]i.

[a]:

NUMERICAL | 1 point

2 ✓	

[b]:

NUMERICAL | 1 point

-10 ✓	

In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n - 1}{n!}|x+1|^n$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{\text{c}}$$

$\boxed{\text{c}}$:

NUMERICAL 4 points

0 ✓	

Therefore, by the ratio test, the series converges absolutely for

MULTI 2 points Single

- all x. ✓
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-\frac{5}{4} < x < -\frac{3}{4}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $\frac{1}{2} < x < \frac{3}{2}$.
- $\frac{3}{4} < x < \frac{5}{4}$.
- $-1 < x < 1$.
- $-1 < x < 3$.
- $x = 0$.
- $1 < x < 3$.

For the case $x = -\frac{4}{3}$, the series

MULTI 2 points Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

Calculate the sum $\sum_{n=0}^{\infty} \frac{3}{4^n} = \boxed{\text{d}}$.

$\boxed{\text{d}}$:

NUMERICAL 2 points

4 ✓	

(4) **Q3**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x}{1 + \exp x}.$$

Choose all the points that are **not** in the natural domain of $f(x)$, if there is any.

MULTI | 4 points | Single

- -3 (-100%)
- -2 (-100%)
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 (-100%)
- 2 (-100%)
- 3 (-100%)
- f is defined for all $x \in \mathbb{R}$. ✓

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -e$ (-100%)
- $y = -1$ (-100%)
- $y = 0$ ✓
- $y = 1$ (-100%)
- $y = e$ (-100%)
- $x = -2$ (-100%)
- $x = -\sqrt{3}$ (-100%)
- $x = -\sqrt{2}$ (-100%)
- $x = -1$ (-100%)
- $x = 0$ (-100%)
- $x = 1$ (-100%)
- $x = \sqrt{2}$ (-100%)
- $x = \sqrt{3}$ (-100%)
- $x = 2$ (-100%)
- $y = x/2$ (-100%)
- $y = x$ ✓
- $y = 2x$ (-100%)
- $y = -x/2$ (-100%)
- $y = -x$ (-100%)
- $y = -2x$ (-100%)

One has

$$f'(\log 2) = \frac{\boxed{\text{a}} + \boxed{\text{b}} \log 2}{\boxed{\text{c}}}.$$

a:

NUMERICAL 2 points

3 ✓	

b:

NUMERICAL 2 points

-2 ✓	

c:

NUMERICAL 4 points

9 ✓	

The function $f(x)$ has d stationary point(s) in the domain. (Hint: no need to find it (them) explicitly)

d:

NUMERICAL 4 points

1 ✓	

Choose the behaviour of $f(x)$ in the interval $(-1, 0)$.

MULTI 4 points Single

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(5) **Q3**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x}{1 + \exp(-x)}.$$

Choose all the points that are **not** in the natural domain of $f(x)$, if there is any.

MULTI 4 points Single

- -3 (-100%)
- -2 (-100%)
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)

- $\frac{1}{2}$ (-100%)
- 1 (-100%)
- 2 (-100%)
- 3 (-100%)
- f is defined for all $x \in \mathbb{R}$. ✓

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -e$ (-100%)
- $y = -1$ (-100%)
- $y = 0$ ✓
- $y = 1$ (-100%)
- $y = e$ (-100%)
- $x = -2$ (-100%)
- $x = -\sqrt{3}$ (-100%)
- $x = -\sqrt{2}$ (-100%)
- $x = -1$ (-100%)
- $x = 0$ (-100%)
- $x = 1$ (-100%)
- $x = \sqrt{2}$ (-100%)
- $x = \sqrt{3}$ (-100%)
- $x = 2$ (-100%)
- $y = x/2$ (-100%)
- $y = x$ ✓
- $y = 2x$ (-100%)
- $y = -x/2$ (-100%)
- $y = -x$ (-100%)
- $y = -2x$ (-100%)

One has

$$f'(\log 3) = \frac{\boxed{\text{a}} + \boxed{\text{b}} \log 3}{\boxed{\text{c}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 2 points

12 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 2 points

3 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 4 points

16 ✓	

The function $f(x)$ has [d] stationary point(s) in the domain. (Hint: no need to find it (them) explicitly)

[d]:

NUMERICAL | 4 points

1 ✓	

Choose the behaviour of $f(x)$ in the interval $(-2, 0)$.

MULTI | 4 points | Single

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(6) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 x^2 e^{3x} dx.$$

By applying the integration by parts, we have

$$\int_0^1 x^2 e^{3x} dx = [\frac{\boxed{a}}{\boxed{b}} x^2 e^{3x}]_0^1 - \int_0^1 \frac{\boxed{c}}{\boxed{d}} x^{\boxed{e}} e^{3x} dx.$$

[a]:

NUMERICAL | 1 point

1 ✓	

[b]:

NUMERICAL | 1 point

3 ✓	

[c]:

NUMERICAL | 2 points

2 ✓	

[d]:

NUMERICAL | 1 point

3 ✓	

e:

NUMERICAL | 1 point

1 ✓	

By continuing the calculation, we obtain

$$\int_0^1 x^2 e^{3x} dx = \frac{\boxed{f}}{\boxed{g}} + \frac{\boxed{h}}{\boxed{i}} e^3.$$

f:

NUMERICAL | 2 points

-2 ✓	

g:

NUMERICAL | 1 point

27 ✓	

h:

NUMERICAL | 2 points

5 ✓	

i:

NUMERICAL | 1 point

27 ✓	

(7) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 x^2 e^{4x} dx.$$

By applying the integration by parts, we have

$$\int_0^1 x^2 e^{4x} dx = [\frac{\boxed{a}}{\boxed{b}} x^2 e^{4x}]_0^1 - \int_0^1 \frac{\boxed{c}}{\boxed{d}} x^{\boxed{e}} e^{4x} dx.$$

a:

NUMERICAL | 1 point

1 ✓	

b:

NUMERICAL | 1 point

4 ✓	

c:

NUMERICAL | 2 points

1 ✓	

d:

NUMERICAL | 1 point

2 ✓	

e:

NUMERICAL | 1 point

1 ✓	

By continuing the calculation, we obtain

$$\int_0^1 x^2 e^{4x} dx = \frac{\boxed{\text{f}}}{\boxed{\text{g}}} + \frac{\boxed{\text{h}}}{\boxed{\text{i}}} e^4.$$

f:

NUMERICAL | 2 points

-1 ✓	

g:

NUMERICAL | 1 point

32 ✓	

h:

NUMERICAL | 2 points

5 ✓	

i:

NUMERICAL | 1 point

32 ✓	

(8) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral based on definition.

$$\int_0^\infty x \exp(-x^2) dx$$

.

Choose a primitive of the integrated function.

MULTI | 4 points | Single

- $-x^2 \exp(-x^2)$
- $3x^3 \exp(-x^2)$
- $-\exp(-x^2)/2$ ✓
- $2x \exp(-x)$
- $-\exp(-x^3)/3$
- $\exp(-x^2)/2$

Calculate the integral $\int_0^\infty x \exp(-x^2) dx = \frac{\boxed{a}}{\boxed{b}}$

a:

NUMERICAL | 1 point

1 ✓	

b:

NUMERICAL | 1 point

2 ✓	

Choose all the value of s for which the integral $\int_0^\infty x^s \exp(-x^2) dx$ converges.

MULTI | 4 points | Single

- 0.1 ✓
- 0.2 ✓
- 0.5 ✓
- 1 ✓
- 1.5 ✓
- 2 ✓
- 3 ✓

Choose the value of s for which the series $\sum_{n=1}^\infty n^s \exp(-n^2)$ converges.

MULTI | 4 points | Single

- 0.1 ✓
- 0.2 ✓
- 0.5 ✓
- 1 ✓
- 1.5 ✓

- 2 ✓
- 3 ✓

Consider the following three improper integrals.

$$(1)\int_1^\infty e^{-100x}dx.(2)\int_0^\infty \log(x{+}e)/(x^2{+}1)^2dx, \quad (3)\int_1^\infty \log(x)/(x^2{+}1)^2dx, \quad (4)\int_0^\infty (x{+}1)/(x^2$$

Give the correct order. ([c]) < ([d]) < ([e]) < ([f]).

[c]:

NUMERICAL | 1 point

1 ✓	

[d]:

NUMERICAL | 1 point

3 ✓	

[e]:

NUMERICAL | 1 point

2 ✓	

[f]:

NUMERICAL | 1 point

4 ✓	

(9) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral based on definition.

$$\int_0^\infty x^2 \exp(-x^3)dx$$

.

Choose a primitive of the integrated function.

MULTI | 4 points | Single

- $-x^2\exp(-x^2)$
- $3x^3\exp(-x^2)$
- $-\exp(-x^2)/2$
- $2x\exp(-x)$

- $-\exp(-x^3)/3$ ✓
- $\exp(-x^2)/2$

Calculate the integral $\int_0^\infty x^2 \exp(-x^3)dx = \frac{\boxed{a}}{\boxed{b}}$

a:

NUMERICAL | 1 point

1 ✓	

b:

NUMERICAL | 1 point

3 ✓	

Choose all the value of s for which the integral $\int_0^\infty x^s \exp(-x^3)dx$ converges.

MULTI | 4 points | Single

- 0.1 ✓
- 0.2 ✓
- 0.5 ✓
- 1 ✓
- 1.5 ✓
- 2 ✓
- 3 ✓

Choose the value of s for which the series $\sum_{n=1}^\infty n^s \exp(-n^3)$ converges.

MULTI | 4 points | Single

- 0.1 ✓
- 0.2 ✓
- 0.5 ✓
- 1 ✓
- 1.5 ✓
- 2 ✓
- 3 ✓

Consider the following three improper integrals.

$$(1) \int_0^\infty \log(x+e)/(x^2+1)^2 dx, \quad (2) \int_1^\infty \log(x)/(x^2+1)^2 dx, \quad (3) \int_0^\infty (x+1)/(x^2+1)^2 dx, \quad (4) \int_1^\infty$$

Give the correct order. ($\boxed{c}$) < ($\boxed{d}$) < ($\boxed{e}$) < ($\boxed{f}$).

c:

NUMERICAL | 1 point

4 ✓	

d:

NUMERICAL 1 point

2 ✓	

e:

NUMERICAL 1 point

1 ✓	

f:

NUMERICAL 1 point

3 ✓	

Total of marks: 168