2023Call3.

(1) **Q1**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

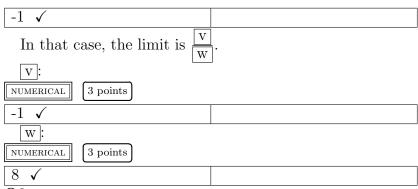
$$\cos(x-1) = \boxed{a} + \boxed{b}(x-1) + \boxed{c}(x-1)^2 + \boxed{e}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

a: NUMERICAL 1 point
1 🗸
b: NUMERICAL 1 point
0 🗸
<u>C</u> :
NUMERICAL 2 points
-1 ✓
d:
NUMERICAL 1 point
2 🗸
e:
NUMERICAL 1 point
0 /

$$(x-1)\sqrt{x} = \boxed{g} + \boxed{h}(x-1) + \frac{\boxed{i}}{\boxed{j}}(x-1)^2 + \frac{\boxed{k}}{\boxed{1}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

g: NUMERICAL 1 point	
0 🗸	
h:	
NUMERICAL 1 point	

	1 🗸	
	<u>i</u> :	
	NUMERICAL 1 point	
	1 🗸	
	j:	
	NUMERICAL 1 point	
	2 🗸	
	k:	
	NUMERICAL 1 point	
	-1 🗸	
	NUMERICAL 1 point	
	8 🗸	
$\sin((x-$	$(-1)^3$ = m+n(x-1)+o(x-1) ² +p(x-1) ³ +o((x-1) ³) as x -	$\rightarrow 1$
	<u> </u>	
	NUMERICAL 2 points	
	0 🗸	
	n:	
	NUMERICAL 1 point	
	0 🗸	
	0:	
	NUMERICAL 1 point	
	0 🗸	
	[p]:	
	NUMERICAL 2 points	
	For various $\alpha, \beta \in \mathbb{R}$, study the limit:	
	$\lim_{x \to 1} \frac{\cos(x-1) + (x-1)\sqrt{x} + \alpha + \beta(x-1)}{\sin((x-1)^3)}.$	
	This limit converges for $\alpha = \boxed{q}, \beta = \boxed{r}$.	
	q:	
	NUMERICAL 6 points	
	-1 ✓	
	NUMERICAL 6 points	

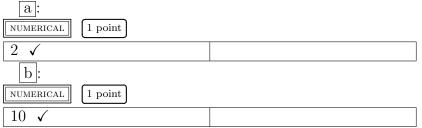


(2) **Q2**CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

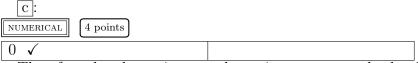
but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$. Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n-1}{n!} (x+1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For x = i, calculate the partial sum $\sum_{n=0}^{2} \frac{3^{n}-1}{n!} (x+1)^{n} = \boxed{\mathbf{a}} + \boxed{\mathbf{b}} i$.

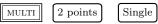


In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n - 1}{n!} |x + 1|^n$. Complete the formula.

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\boxed{\mathbf{c}}$$



Therefore, by the ratio test, the series converges absolutely for



- all *x*. ✓
- -3 < x < -1.
- -3 < x < 1.
- -3 < x < 1. $-\frac{5}{4} < x < -\frac{3}{4}$. $-\frac{3}{2} < x < -\frac{1}{2}$. $\frac{1}{2} < x < \frac{3}{2}$. $\frac{3}{4} < x < \frac{5}{4}$. -1 < x < 1.

- \bullet -1 < x < 3.
- x = 0.
- 1 < x < 3.

For the case $x = -\frac{4}{3}$, the series



- converges absolutely. \checkmark
- converges but not absolutely.
- diverges.

Calculate the sum $\sum_{n=0}^{\infty} \frac{5}{2^n} = d$.



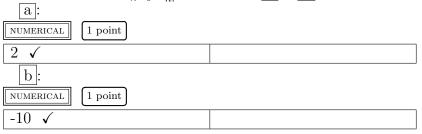
$(3) \mathbf{Q2}$

0.10 penalty CLOZE

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|a|}{|b|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted

but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$. Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n-1}{n!} (x+1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For x = -i, calculate the partial sum $\sum_{n=0}^{2} \frac{3^{n}-1}{n!} (x+1)^{n} = \boxed{\mathbf{a}} + \boxed{\mathbf{b}} i$.



In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n - 1}{n!} |x + 1|^n$. Complete the formula.

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \boxed{\mathbf{c}}$$



Therefore, by the ratio test, the series converges absolutely for

MULTI 2 points Single

- \bullet all \overline{x} .
- -3 < x < -1.

- -3 < x < 1. -3 < x < 1. $-\frac{5}{4} < x < -\frac{3}{4}$. $-\frac{3}{2} < x < -\frac{1}{2}$. $\frac{1}{2} < x < \frac{3}{2}$. $\frac{3}{4} < x < \frac{5}{4}$. -1 < x < 1.

- -1 < x < 3.
- x = 0.
- 1 < x < 3.

For the case $x = -\frac{4}{3}$, the series MULTI 2 points Single

- \bullet converges absolutely. \checkmark
- converges but not absolutely.
- diverges.

Calculate the sum $\sum_{n=0}^{\infty} \frac{3}{4^n} = d$.



 $(4) \ \mathbf{Q3}$ CLOZE 0.10 penalty

> If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as a b have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x}{1 + \exp x}.$$

Choose all the points that are **not** in the natural domain of f(x), if there is any.

MULTI 4 points Single

- \bullet -3 $\overline{(-100\%)}$
- -2 (-100%)
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0(-100%)
- $\frac{1}{2}$ (-100%)
- $\bar{1}$ (-100%)
- 2 (−100%)
- 3 (−100%)
- f is defined for all $x \in \mathbb{R}$.

Choose all asymptotes of f(x).

- $y = -e \ (-100\%)$
- $y = -1 \ (-100\%)$
- y = 0 \checkmark
- $y = 1 \ (-100\%)$
- $y = e \ (-100\%)$
- $x = -2 \ (-100\%)$
- $x = -\sqrt{3} \ (-100\%)$
- $x = -\sqrt{2} \ (-100\%)$
- $x = -1 \ (-100\%)$
- $x = 0 \ (-100\%)$
- x = 1 (-100%)
- $x = \sqrt{2} \ (-100\%)$ • $x = \sqrt{3} \ (-100\%)$
- $x = 2 \ (-100\%)$
- $y = x/2 \ (-100\%)$
- $y = x \checkmark$
- $y = 2x \ (-100\%)$
- $y = -x/2 \ (-100\%)$
- y = -x(-100%)
- y = -2x(-100%)

One has

$$f'(\log 2) = \frac{\boxed{\mathbf{a} + \boxed{\mathbf{b} \log 2}}}{\boxed{\mathbf{c}}}.$$



The function f(x) has \boxed{d} stationary point(s) in the domain. (Hint: no need to find it (them) explicitly)



Choose the behaviour of f(x) in the interval (-1,0).

MULTI 4 points Single

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(5) **Q3**[CLOZE] [0.10 penalty]

If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x}{1 + \exp(-x)}.$$

Choose all the points that are **not** in the natural domain of f(x), if there is any.

MULTI 4 points Single

- -3 (-100%)
- -2 (-100%)
- -1(-100%)
- \bullet $-\frac{1}{2}$ (-100%)
- 0 (-100%)

- $\frac{1}{2}$ (-100%) 1 (-100%)
- 2(-100%)
- 3 (−100%)
- f is defined for all $x \in \mathbb{R}$. Choose all asymptotes of f(x).

MULTI 4 points Single

- $y = -e \ (-100\%)$
- $y = -1 \ (-100\%)$
- y = 0
- y = 1 (-100%)
- $y = e \ (-100\%)$
- $x = -2 \ (-100\%)$
- $x = -\sqrt{3} \ (-100\%)$
- $x = -\sqrt{2} \ (-100\%)$
- $x = -1 \ (-100\%)$
- $x = 0 \ (-100\%)$
- $x = 1 \ (-100\%)$
- $x = \sqrt{2} \ (-100\%)$
- $x = \sqrt{3} \ (-100\%)$
- x = 2 (-100%)
- $y = x/2 \ (-100\%)$
- $y = x \checkmark$
- y = 2x (-100%)
- $y = -x/2 \ (-100\%)$
- y = -x (-100%)
- y = -2x (-100%)

One has

$$f'(\log 3) = \frac{\boxed{a} + \boxed{b} \log 3}{\boxed{c}}.$$

a :	
NUMERICAL 2 points	
12 🗸	
b:	
NUMERICAL 2 points	
3 ✓	
<u>c</u> :	
NUMERICAL 4 points	
16 ✓	

The function f(x) has \boxed{d} stationary point(s) in the domain. (Hint: no need to find it (them) explicitly)



Choose the behaviour of f(x) in the interval (-2,0).

MULTI 4 points Single

- monotonically decreasing
- monotonically increasing
- ullet neither decreasing nor increasing \checkmark

(6) **Q4**CLOZE 0.10 penalty

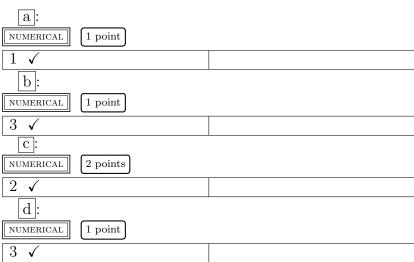
If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

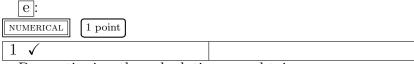
Let us calculate the following integral.

$$\int_0^1 x^2 e^{3x} dx.$$

By applying the integration by parts, we have

$$\int_0^1 x^2 e^{3x} dx = \left[\frac{\boxed{\mathbf{a}}}{\boxed{\mathbf{b}}} x^2 e^{3x}\right]_0^1 - \int_0^1 \frac{\boxed{\mathbf{c}}}{\boxed{\mathbf{d}}} x^{\boxed{\mathbf{e}}} e^{3x} dx.$$





By continuing the calculation, we obtain

$$\int_0^1 x^2 e^{3x} dx = \frac{\boxed{\mathrm{f}}}{\boxed{\mathrm{g}}} + \frac{\boxed{\mathrm{h}}}{\boxed{\mathrm{i}}} e^3.$$

f:	
NUMERICAL 2 points	
-2 √	
g:	
NUMERICAL 1 point	
27 ✓	
h:	
NUMERICAL 2 points	
5 ✓	
i:	
NUMERICAL 1 point	
27 ✓	

(7) **Q4**CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 x^2 e^{4x} dx.$$

By applying the integration by parts, we have

$$\int_0^1 x^2 e^{4x} dx = \left[\frac{\boxed{\mathbf{a}}}{\boxed{\mathbf{b}}} x^2 e^{4x} \right]_0^1 - \int_0^1 \frac{\boxed{\mathbf{c}}}{\boxed{\mathbf{d}}} x^{\boxed{\mathbf{e}}} e^{4x} dx.$$



1 🗸		
b:		
NUMERICAL 1 point		
4 🗸		
<u>c</u> :		
NUMERICAL 2 points		
1 🗸		
d:		
NUMERICAL 1 point		
2 🗸		
e:		
NUMERICAL 1 point		
1 🗸		

By continuing the calculation, we obtain

$$\int_0^1 x^2 e^{4x} dx = \frac{\boxed{f}}{\boxed{g}} + \frac{\boxed{h}}{\boxed{i}} e^4.$$

f:	
NUMERICAL 2 points	
-1 ✓	
g:	
NUMERICAL 1 point	
32 ✓	
h:	
NUMERICAL 2 points	
5 🗸	
i:	
NUMERICAL 1 point	
32 ✓	

(8) $\overline{\mathbf{Q5}}$ CLOZE 0.10 penalty

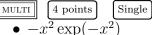
If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as \boxed{a}) have ambiguity, the negative sign

should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral based on definition.

$$\int_0^\infty x \exp(-x^2) dx$$

Choose a primitive of the integrated function.

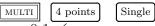


- $3x^3 \exp(-x^2)$
- $\bullet \exp(-x^2)/2 \checkmark$
- $2x \exp(-x)$
- $-\exp(-x^3)/3$
- $\bullet \exp(-x^2)/2$

Calculate the integral $\int_0^\infty x \exp(-x^2) dx = \boxed{a}$



Choose all the value of s for which the integral $\int_0^\infty x^s \exp(-x^2) dx$ converges.



- 0.1 √
- 0.2 ✓
- 0.5 ✓
- 1 ✓
- 1.5 ✓
- 2 √
- 3 √

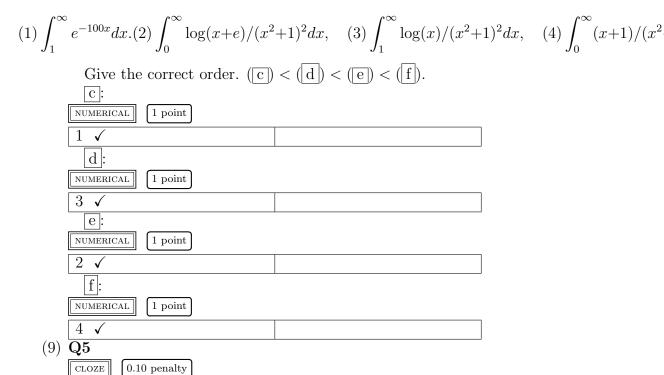
Choose the value of s for which the series $\sum_{n=1}^{\infty} n^s \exp(-n^2)$ converges.

MULTI 4 points Single

- $\overline{\bullet} \ 0.1 \overline{\checkmark}$
- 0.2 ✓
- 0.5 ✓
- 1 ✓
- 1.5 ✓

- 2 √
- 3 √

Consider the following three improper integrals.



If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral based on definition.

$$\int_0^\infty x^2 \exp(-x^3) dx$$

Choose a primitive of the integrated function.

MULTI 4 points Single

- $-x^{2} \exp(-x^{2})$ $3x^{3} \exp(-x^{2})$
- $\exp(-x^2)/2$
- $2x \exp(-x)$

•
$$-\exp(-x^3)/3$$
 \checkmark

• $\exp(-x^2)/2$

Calculate the integral $\int_0^\infty x^2 \exp(-x^3) dx = \boxed{\text{a}}$

a:

NUMERICAL 1 point

1

b:

NUMERICAL 1 point

3

Choose all the value of s for which the integral $\int_0^\infty x^s \exp(-x^3) dx$ converges.

MULTI 4 points Single

• 0.1 ✓

- 0.1 V
- 0.2 ✓
- 0.5 ✓
- 1 ✓
- 1.5 ✓
- 2 √
- 3 √

Choose the value of s for which the series $\sum_{n=1}^{\infty} n^s \exp(-n^3)$ converges.

MULTI 4 points Single

- 0.1 ✓
- 0.2 ✓
- 0.5 ✓
- 1 ✓
- 1.5 ✓
- 2 √
- 3 ✓

Consider the following three improper integrals.

$$(1) \int_0^\infty \log(x+e)/(x^2+1)^2 dx, \quad (2) \int_1^\infty \log(x)/(x^2+1)^2 dx, \quad (3) \int_0^\infty (x+1)/(x^2+1)^2 dx, \quad (4) \int_1^\infty \log(x+e)/(x^2+1)^2 dx$$

Give the correct order. (c) < (d) < (e) < (f).

C:
NUMERICAL 1 point

4

d:

NUMERICAL 1 point	
2 🗸	
e:	
NUMERICAL 1 point	
1 🗸	
f:	
NUMERICAL 1 point	
3 ✓	

Total of marks: 168