2023Call2.

(1) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\log x = \boxed{\text{a}} + \boxed{\text{b}}(x-1) + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}(x-1)^2 + \frac{\boxed{\text{e}}}{\boxed{\text{f}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{f}}$:

NUMERICAL | 1 point

3 ✓	

$$(x-1)\sqrt{x+3} = \boxed{\text{g}} + \boxed{\text{h}}(x-1) + \frac{\boxed{\text{i}}}{\boxed{\text{j}}}(x-1)^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{g}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{h}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{i}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{j}}$:

NUMERICAL | 1 point

4 ✓	

$\boxed{\text{k}}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{\text{l}}$:

NUMERICAL | 1 point

64 ✓	

$\exp((x-1)^3) = \boxed{\text{m}} + \boxed{\text{n}}(x-1) + \boxed{\text{o}}(x-1)^2 + \boxed{\text{p}}(x-1)^3 + o((x-1)^3)$ as $x \to 1$.

$\boxed{\text{m}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{m}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{o}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{p}}$:

NUMERICAL | 2 points

1 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 1} \frac{\log x + \alpha(x-1)\sqrt{x+3} + \beta(x-1)}{\exp((x-1)^3) - 1}.$$

This limit converges for $\alpha = \boxed{\text{q}}$, $\beta = \boxed{\text{r}}$.

$\boxed{\text{q}}$:

NUMERICAL | 6 points

2 ✓	

$\boxed{\text{r}}$:

NUMERICAL | 6 points

-5 ✓	

In that case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

$\boxed{\text{v}}$:

NUMERICAL | 3 points

29 ✓	

$\boxed{\text{w}}$:

NUMERICAL | 3 points

96 ✓	

(2) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\log x = \boxed{\text{a}} + \boxed{\text{b}}(x-1) + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}(x-1)^2 + \frac{\boxed{\text{e}}}{\boxed{\text{f}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL 1 point

1 ✓	

$\boxed{\text{f}}$:

NUMERICAL 1 point

3 ✓	

$$(x-1)\sqrt{x} = \boxed{\text{g}} + \boxed{\text{h}}(x-1) + \frac{\boxed{\text{i}}}{\boxed{\text{j}}}(x-1)^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{g}}$:

NUMERICAL 1 point

0 ✓	

$\boxed{\text{h}}$:

NUMERICAL 1 point

1 ✓	

$\boxed{\text{i}}$:

NUMERICAL 1 point

1 ✓	

$\boxed{\text{j}}$:

NUMERICAL 1 point

2 ✓	

$\boxed{\text{k}}$:

NUMERICAL 1 point

-1 ✓	

$\boxed{\text{l}}$:

NUMERICAL 1 point

8 ✓	

$$\exp(2(x-1)^3) = \boxed{\text{m}} + \boxed{\text{n}}(x-1) + \boxed{\text{o}}(x-1)^2 + \boxed{\text{p}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{m}}$:

NUMERICAL 2 points

1 ✓	

$\boxed{\text{m}}$:

NUMERICAL 1 point

0 ✓	

$\boxed{\text{o}}$:

NUMERICAL 1 point

0 ✓	

$\boxed{\text{p}}$:

NUMERICAL 2 points

2 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 1} \frac{\log x + \alpha(x-1)\sqrt{x} + \beta(x-1)}{\exp(2(x-1)^3) - 1}.$$

This limit converges for $\alpha = \boxed{\text{q}}$, $\beta = \boxed{\text{r}}$.

$\boxed{\text{q}}$:

NUMERICAL 6 points

1 ✓	

$\boxed{\text{r}}$:

NUMERICAL 6 points

-2 ✓	

In that case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

$\boxed{\text{v}}$:

NUMERICAL 3 points

5 ✓	

$\boxed{\text{w}}$:

NUMERICAL 3 points

48 ✓	

(3) **Q2**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{n}{(1+x)^{2n}}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{n}{(1+x)^{2n}} = \frac{\boxed{\text{a}}}{\boxed{\text{b}}} + \frac{\boxed{\text{c}}}{\boxed{\text{d}}} i$.

$\boxed{\text{a}}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

2 ✓	

In order to use the ratio test for $x \in \mathbb{R}, x \neq -1$, we put $a_n = \frac{n}{(1+x)^{2n}}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{\text{e}}}{|\boxed{\text{f}} + x|^{\boxed{\text{g}}}}$$

$\boxed{\text{e}}$:

NUMERICAL | 4 points

1 ✓	

$\boxed{\text{f}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 2 points

2 ✓	

Therefore, by the ratio test, the series converges absolutely for $x < \boxed{\text{h}}, \boxed{\text{i}} < x$.

$\boxed{\text{h}}$:

NUMERICAL | 2 points

-2 ✓	

$\boxed{\text{i}}$:

NUMERICAL | 2 points

0 ✓	

For the case $x = -2$, the series

MULTI | 4 points | Single

- converges absolutely.

- converges but not absolutely.
- diverges. ✓

Calculate the following series.

$$\sum_{n=0}^{\infty} \frac{5}{3^n} = \frac{\boxed{j}}{\boxed{k}}$$

$\boxed{j}$:

NUMERICAL | 2 points

15 ✓	

$\boxed{k}$:

NUMERICAL | 2 points

2 ✓	

(4) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{n}{(x-1)^{2n}}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{n}{(x-1)^{2n}} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} i$.

$\boxed{a}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

2 ✓	

In order to use the ratio test for $x \in \mathbb{R}, x \neq 1$, we put $a_n = \frac{n}{(x-1)^{2n}}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{\mathrm{e}}}{|x + \boxed{\mathrm{f}}|^{\boxed{\mathrm{g}}}}$$

$\boxed{\mathrm{e}}$:

NUMERICAL | 4 points

1 ✓	

$\boxed{\mathrm{f}}$:

NUMERICAL | 2 points

-1 ✓	

$\boxed{\mathrm{g}}$:

NUMERICAL | 2 points

2 ✓	

Therefore, by the ratio test, the series converges absolutely for $x < \boxed{\mathrm{h}}, \boxed{\mathrm{i}} < x$.

$\boxed{\mathrm{h}}$:

NUMERICAL | 2 points

0 ✓	

$\boxed{\mathrm{i}}$:

NUMERICAL | 2 points

2 ✓	

For the case $x = \frac{1}{2}$, the series

MULTI | 4 points | Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

Calculate the following series.

$$\sum_{n=0}^{\infty} \frac{5}{4^n} = \frac{\boxed{\mathrm{j}}}{\boxed{\mathrm{k}}}$$

$\boxed{\mathrm{j}}$:

NUMERICAL | 2 points

20 ✓	

$\boxed{\mathrm{k}}$:

NUMERICAL | 2 points

3 ✓	

(5) **Q3**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \exp\left(\frac{1 - x^3}{(x-3)^2}\right).$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$ (with this expression). Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI 4 points Single

- -3 (-100%)
- -2 (-100%)
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 (-100%)
- 2 (-100%)
- 3 ✓

Choose all asymptotes of $f(x)$.

MULTI 4 points Single

- $y = -e$ (-100%)
- $y = -1$ (-100%)
- $y = 0$ ✓
- $y = 1$ (-100%)
- $y = e$ (-100%)
- $x = -2$ (-100%)
- $x = -\sqrt{3}$ (-100%)
- $x = -\sqrt{2}$ (-100%)
- $x = -1$ (-100%)
- $x = 0$ (-100%)
- $x = 1$ (-100%)
- $x = \sqrt{2}$ (-100%)
- $x = \sqrt{3}$ (-100%)

- $x = 2$ (-100%)
- $y = x/2$ (-100%)
- $y = x$ (-100%)
- $y = 2x$ (-100%)
- $y = -x/2$ (-100%)
- $y = -x$ (-100%)
- $y = -2x$ (-100%)

One has

$$f'(1) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 4 points

-3 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 4 points

4 ✓	

The function $f(x)$ has $\boxed{\text{c}}$ stationary point(s) in the domain. (Hint: no need to find it (them) explicitly)

$\boxed{\text{c}}$:

NUMERICAL | 4 points

3 ✓	

Choose the behaviour of $f(x)$ in the interval $(-1, 0)$.

MULTI | 4 points | Single

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(6) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \exp\left(\frac{x^3 - 1}{(x+3)^2}\right).$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$ (with this expression). Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single

- -3 ✓
- -2 (-100%)
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 (-100%)
- 2 (-100%)
- 3 (-100%)

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -e$ (-100%)
- $y = -1$ (-100%)
- $y = 0$ ✓
- $y = 1$ (-100%)
- $y = e$ (-100%)
- $x = -2$ (-100%)
- $x = -\sqrt{3}$ (-100%)
- $x = -\sqrt{2}$ (-100%)
- $x = -1$ (-100%)
- $x = 0$ (-100%)
- $x = 1$ (-100%)
- $x = \sqrt{2}$ (-100%)
- $x = \sqrt{3}$ (-100%)
- $x = 2$ (-100%)
- $y = x/2$ (-100%)
- $y = x$ (-100%)
- $y = 2x$ (-100%)
- $y = -x/2$ (-100%)
- $y = -x$ (-100%)
- $y = -2x$ (-100%)

One has

$$f'(1) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 4 points

3 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 4 points

16 ✓	

The function $f(x)$ has $\boxed{\text{c}}$ stationary point(s) in the domain. (Hint: no need to find it (them) explicitly)

$\boxed{\text{c}}$:

NUMERICAL | 4 points

1 ✓	

Choose the behaviour of $f(x)$ in the interval $(-1, 0)$.

MULTI | 4 points | Single

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(7) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_1^2 \frac{1}{3^x + 2} dx.$$

Let us change the variables $3^x = t$. Complete the formula

$$\int_1^2 \frac{1}{3^x + 2} dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} \frac{1}{\log \boxed{\text{c}} (t^2 + \boxed{\text{d}} t + \boxed{\text{e}})} dt$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

9 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

3 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

0 ✓	

By continuing, we get

$$\int_1^2 \frac{1}{3^x+2}dx = \frac{\log \frac{\boxed{\text{f}}}{\boxed{\text{g}}}}{\log \boxed{\text{h}}}.$$

$\boxed{\text{f}}$:

NUMERICAL | 4 points

15 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 4 points

11 ✓	

$\boxed{\text{h}}$:

NUMERICAL | 4 points

9 ✓	

(8) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 \frac{1}{3^x+2}dx.$$

Let us change the variables $3^x = t$. Complete the formula

$$\int_0^1 \frac{1}{3^x+2}dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} \frac{1}{\log \boxed{\text{c}}(t^2+\boxed{\text{d}}t+\boxed{\text{e}})}dt$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL 1 point

3 ✓	

$\boxed{\text{c}}$:

NUMERICAL 2 points

3 ✓	

$\boxed{\text{d}}$:

NUMERICAL 1 point

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL 1 point

0 ✓	

By continuing, we get

$$\int_0^1 \frac{1}{3^x+2}dx = \frac{\log \frac{\boxed{\text{f}}}{\boxed{\text{g}}}}{\log \boxed{\text{i}}}.$$

$\boxed{\text{f}}$:

NUMERICAL 4 points

9 ✓	

$\boxed{\text{g}}$:

NUMERICAL 4 points

5 ✓	

$\boxed{\text{h}}$:

NUMERICAL 4 points

9 ✓	

(9) **Q4**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 \frac{1}{5^x+2}dx.$$

Let us change the variables $5^x = t$. Complete the formula

$$\int_0^1 \frac{1}{5^x+2}dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} \frac{1}{\log \boxed{\text{c}}(t^2+\boxed{\text{d}}t+\boxed{\text{e}})}dt$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

5 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

5 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

0 ✓	

By continuing, we get

$$\int_0^1 \frac{1}{5^x+2}dx = \frac{\log \frac{\boxed{\text{f}}}{\boxed{\text{g}}}}{\log \boxed{\text{i}}}.$$

$\boxed{\text{f}}$:

NUMERICAL | 4 points

15 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 4 points

7 ✓	

$\boxed{\text{h}}$:

NUMERICAL | 4 points

25 ✓	

(10) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_1^2 \frac{1}{5^x+2}dx.$$

Let us change the variables $5^x = t$. Complete the formula

$$\int_1^2 \frac{1}{5^x+2}dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} \frac{1}{\log\boxed{\text{c}}(t^2+\boxed{\text{d}}t+\boxed{\text{e}})}dt$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

5 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

25 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

5 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

0 ✓	

By continuing, we get

$$\int_1^2 \frac{1}{5^x+2}dx = \frac{\log\frac{\boxed{\text{f}}}{\boxed{\text{g}}}}{\log\boxed{\text{h}}}.$$

$\boxed{\text{f}}$:

NUMERICAL | 4 points

35 ✓	

g:

NUMERICAL 4 points

27 ✓	

h:

NUMERICAL 4 points

25 ✓	

(11) **Q5**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral based on definition.

$$\int_{-\infty}^{\infty} \frac{x^2}{x^6+1} dx$$

.

We split the integral into two parts: $\int_{-\infty}^{0} \frac{x^2}{x^6+1} dx + \int_{0}^{\infty} \frac{x^2}{x^6+1} dx$ Choose a primitive of the integrated function.

MULTI 4 points Single

- $3\log(x^6+1)$
- $3x^2\log(x^6)$
- $1/(x^6+1)^2$
- $x^2/(x^6+1)^2$
- $\arctan(x^3)/3$ ✓
- $3\arctan(x^3+1)$
- $(x^6+1)^{\frac{1}{2}}$
- $3x(x^6+1)^{\frac{1}{2}}$

Calculate $\int_{-\infty}^{0} \frac{x^2}{x^6+1} dx = \frac{\boxed{a}\pi}{\boxed{b}}$

a:

NUMERICAL 1 point

1 ✓	

b:

NUMERICAL 3 points

6 ✓	

Altogether, the integral $\int_{-\infty}^{\infty} \frac{x^2}{x^6+1} dx$

MULTI | 4 points | Single

- converges, also absolutely. ✓
- converges but not absolutely.
- does not converge.

Let $s > 0$. Choose the value of s for which the series $\int_0^{\infty} \frac{1}{(x^2+1)^s} dx$ converges.

MULTI | 4 points | Single

- 0.1 (−100%)
- 0.2 (−100%)
- 0.3 (−100%)
- 0.4 (−100%)
- 0.5 (−100%)
- 0.6 ✓
- 0.7 ✓
- 0.8 ✓
- 0.9 ✓
- 1 ✓
- 1.5 ✓
- 2 ✓
- 3 ✓

Choose the value of s for which the series $\sum_{n=1}^{\infty} \frac{1}{(n^2+1)^s}$ converges.

MULTI | 4 points | Single

- 0.1 (−100%)
- 0.2 (−100%)
- 0.3 (−100%)
- 0.4 (−100%)
- 0.5 (−100%)
- 0.6 ✓
- 0.7 ✓
- 0.8 ✓
- 0.9 ✓
- 1 ✓
- 1.5 ✓
- 2 ✓
- 3 ✓

Consider the following three improper integrals.

(1) $\int_1^\infty x^{100}e^{-x}dx$, (2) $\int_0^\infty x^{101}e^{-x}dx$, (3) $\int_0^\infty e^{-100x}dx$, (4) $\int_0^\infty e^{-x}dx$.

Give the correct order. ($\boxed{\text{c}}$) < ($\boxed{\text{d}}$) < ($\boxed{\text{e}}$) < ($\boxed{\text{f}}$).

$\boxed{\text{c}}$:

NUMERICAL 1 point

3 ✓	

$\boxed{\text{d}}$:

NUMERICAL 1 point

4 ✓	

$\boxed{\text{e}}$:

NUMERICAL 1 point

1 ✓	

$\boxed{\text{f}}$:

NUMERICAL 1 point

2 ✓	

(12) **Q5**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral based on definition.

$$\int_{-\infty}^{\infty} \frac{x^2}{x^6+1}dx$$

.

We split the integral into two parts: $\int_{-\infty}^{1} \frac{x^2}{x^6+1}dx + \int_{1}^{\infty} \frac{x^2}{x^6+1}dx$ Choose a primitive of the integrated function.

MULTI 4 points Single

- $\arctan(x^3)/3$ ✓
- $3\arctan(x^3+1)$
- $(x^6+1)^{\frac{1}{2}}$
- $3x(x^6+1)^{\frac{1}{2}}$
- $3\log(x^6+1)$
- $3x^2\log(x^6)$

- $1/(x^6+1)^2$
- $x^2/(x^6+1)^2$

Calculate $\int_{-\infty}^{1} \frac{x^2}{x^6+1} dx = \frac{\boxed{a}\pi}{\boxed{b}}$

$\boxed{a}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{b}$:

NUMERICAL | 3 points

4 ✓	

Altogether, the integral $\int_{-\infty}^{\infty} \frac{x^2}{x^6+1} dx$

MULTI | 4 points | Single

- converges, also absolutely. ✓
- converges but not absolutely.
- does not converge.

Let $s > 0$. Choose the value of s for which the series $\int_0^\infty \frac{1}{(x^3+1)^s} dx$ converges.

MULTI | 4 points | Single

- 0.1 (-100%)
- 0.2 (-100%)
- 0.3 (-100%)
- 0.4 ✓
- 0.5 ✓
- 0.6 ✓
- 0.7 ✓
- 0.8 ✓
- 0.9 ✓
- 1 ✓
- 1.5 ✓
- 2 ✓
- 3 ✓

Choose the value of s for which the series $\sum_{n=1}^{\infty} \frac{1}{(n^3+1)^s}$ converges.

MULTI | 4 points | Single

- 0.1 (-100%)
- 0.2 (-100%)
- 0.3 (-100%)
- 0.4 ✓
- 0.5 ✓
- 0.6 ✓

- 0.7 ✓
- 0.8 ✓
- 0.9 ✓
- 1 ✓
- 1.5 ✓
- 2 ✓
- 3 ✓

Consider the following three improper integrals.

$$(1)\int_0^\infty x^{101}e^{-x}dx, \quad (2)\int_1^\infty x^{100}e^{-x}dx, \quad (3)\int_0^\infty e^{-x}dx, \quad (4)\int_0^\infty e^{-100x}dx.$$

Give the correct order. ($\boxed{\text{c}}$) < ($\boxed{\text{d}}$) < ($\boxed{\text{e}}$) < ($\boxed{\text{f}}$).

$\boxed{\text{c}}$:

NUMERICAL 1 point

4 ✓	

$\boxed{\text{d}}$:

NUMERICAL 1 point

3 ✓	

$\boxed{\text{e}}$:

NUMERICAL 1 point

2 ✓	

$\boxed{\text{f}}$:

NUMERICAL 1 point

1 ✓	

Total of marks: 288