2023Call1.

(1) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$(1+3x)^{\frac{1}{2}} = \boxed{a} + \frac{\boxed{b}}{\boxed{c}}x + \frac{\boxed{d}}{\boxed{e}}x^2 + \frac{\boxed{f}}{\boxed{g}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{a}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

-9 ✓	

$\boxed{e}$:

NUMERICAL | 1 point

8 ✓	

$\boxed{f}$:

NUMERICAL | 1 point

27 ✓	

$\boxed{g}$:

NUMERICAL | 1 point

16 ✓	

$$\log(1+x) = \boxed{h} + \boxed{i}x + \frac{\boxed{j}}{\boxed{k}}x^2 + \frac{\boxed{l}}{\boxed{m}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{h}$:

NUMERICAL | 2 points

0 ✓	

$\boxed{i}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{j}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{k}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{l}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{m}$:

NUMERICAL | 1 point

3 ✓	

$$\sin(2x^2) \cdot x = \boxed{n} + \boxed{o}x + \boxed{p}x^2 + \boxed{q}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{n}$:

NUMERICAL | 2 points

0 ✓	

$\boxed{o}$:

NUMERICAL | 2 points

0 ✓	

$\boxed{p}$:

NUMERICAL | 2 points

0 ✓	

$\boxed{q}$:

NUMERICAL | 2 points

2 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{(1+3x)^{\frac{1}{2}} + \log(1+x) - 1 + \alpha x + \beta x^2}{\sin(2x^2)\cdot x}.$$

This limit converges for $\alpha = \frac{\boxed{\text{s}}}{\boxed{\text{t}}}, \beta = \frac{\boxed{\text{u}}}{\boxed{\text{v}}}$.

$\boxed{\text{r}}$:

NUMERICAL | 4 points

-5 ✓	

$\boxed{\text{s}}$:

NUMERICAL | 4 points

2 ✓	

$\boxed{\text{t}}$:

NUMERICAL | 4 points

13 ✓	

$\boxed{\text{u}}$:

NUMERICAL | 4 points

8 ✓	

In that case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

$\boxed{\text{v}}$:

NUMERICAL | 4 points

97 ✓	

$\boxed{\text{w}}$:

NUMERICAL | 4 points

96 ✓	

(2) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$(1-3x)^{\frac{1}{2}} = \boxed{\text{a}} + \frac{\boxed{\text{b}}}{\boxed{\text{c}}}x + \frac{\boxed{\text{d}}}{\boxed{\text{e}}}x^2 + \frac{\boxed{\text{f}}}{\boxed{\text{g}}}x^3 + o(x^3) \text{ as } x \to 0.$$

a:

NUMERICAL 2 points

1 ✓	

b:

NUMERICAL 1 point

-3 ✓	

c:

NUMERICAL 1 point

2 ✓	

d:

NUMERICAL 1 point

-9 ✓	

e:

NUMERICAL 1 point

8 ✓	

f:

NUMERICAL 1 point

-27 ✓	

g:

NUMERICAL 1 point

16 ✓	

$$\log(1-x) = \boxed{\text{h}} + \boxed{\text{i}}x + \frac{\boxed{\text{j}}}{\boxed{\text{k}}}x^2 + \frac{\boxed{\text{l}}}{\boxed{\text{m}}}x^3 + o(x^3) \text{ as } x \to 0.$$

h:

NUMERICAL 2 points

0 ✓	

i:

NUMERICAL 2 points

-1 ✓	

j:

NUMERICAL 1 point

-1 ✓	

k:

NUMERICAL 1 point

2 ✓	

l:

NUMERICAL 1 point

-1 ✓	

m:

NUMERICAL 1 point

3 ✓	

$$\sin(x^2) \cdot x = \boxed{\text{n}} + \boxed{\text{o}}x + \boxed{\text{p}}x^2 + \boxed{\text{q}}x^3 + o(x^3) \text{ as } x \to 0.$$

n:

NUMERICAL 2 points

0 ✓	

o:

NUMERICAL 2 points

0 ✓	

p:

NUMERICAL 2 points

0 ✓	

q:

NUMERICAL 2 points

1 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 0} \frac{(1-3x)^{\frac{1}{2}} + \log(1-x) - 1 + \alpha x + \beta x^2}{\sin(x^2) \cdot x}.$$

This limit converges for $\alpha = \frac{\boxed{\text{s}}}{\boxed{\text{t}}}, \beta = \frac{\boxed{\text{u}}}{\boxed{\text{v}}}$.

r:

NUMERICAL 4 points

5 ✓	

s:

NUMERICAL 4 points

2 ✓	

t:

NUMERICAL 4 points

13 ✓	

u:

NUMERICAL | 4 points

8 ✓	

In that case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

$\boxed{\text{v}}$:

NUMERICAL | 4 points

-97 ✓	

$\boxed{\text{w}}$:

NUMERICAL | 4 points

48 ✓	

(3) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(4^n-1)(x+1)^{2n}}{n+2}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{(4^n-1)(x+1)^{2n}}{n+2} = \boxed{\text{a}} + \boxed{\text{b}} i$.

$\boxed{\text{a}}$:

NUMERICAL | 2 points

-15 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 2 points

2 ✓	

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{(4^n-1)(x+1)^{2n}}{n+2}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{\text{c}}|x + \boxed{\text{d}}|^{\boxed{\text{e}}}$$

$\boxed{\text{c}}$:

NUMERICAL | 2 points

4 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

1 ✓	

e:

NUMERICAL 1 point

2 ✓	

Therefore, by the ratio test, the series converges absolutely for $\frac{\boxed{f}}{\boxed{g}} < x < \frac{\boxed{h}}{\boxed{i}}$.

f:

NUMERICAL 2 points

-3 ✓	

g:

NUMERICAL 2 points

2 ✓	

h:

NUMERICAL 2 points

-1 ✓	

i:

NUMERICAL 2 points

2 ✓	

For the case $x = -\frac{3}{2}$, the series

MULTI 4 points Single Shuffle

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = -1$, the series

MULTI 4 points Single Shuffle

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(4) **Q2**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(4^n-1)(x-1)^{2n}}{n+2}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{(4^n-1)(x-1)^{2n}}{n+2} = \boxed{a} + \boxed{b} i$.

$\boxed{a}$:

NUMERICAL | 2 points

-15 ✓	

$\boxed{b}$:

NUMERICAL | 2 points

-2 ✓	

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{(4^n-1)(x-1)^{2n}}{n+2}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{c} |x + \boxed{d}|^{\boxed{e}}$$

$\boxed{c}$:

NUMERICAL | 2 points

4 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{e}$:

NUMERICAL | 1 point

2 ✓	

Therefore, by the ratio test, the series converges absolutely for $\frac{\boxed{f}}{\boxed{g}} < x < \frac{\boxed{h}}{\boxed{i}}$.

$\boxed{f}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{g}$:

NUMERICAL | 2 points

2 ✓	

$\boxed{h}$:

NUMERICAL | 2 points

3 ✓	

$\boxed{i}$:

NUMERICAL | 2 points

2 ✓	

For the case $x = 1$, the series

MULTI | 4 points | Single | Shuffle

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = \frac{3}{2}$, the series

MULTI | 4 points | Single | Shuffle

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(5) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{(x+1)(e^x - 2)}{(e^x + 1)}$$

This function has two oblique asymptotes. They are $y = \boxed{\text{a}}x + \boxed{\text{b}}, \boxed{\text{c}}x + \boxed{\text{d}}$ with $\boxed{\text{a}} > \boxed{\text{c}}$.

$\boxed{\text{a}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

-2 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 2 points

-2 ✓	

One has

$$f'(0) = \frac{\boxed{\text{e}}}{\boxed{\text{f}}}.$$

$\boxed{\text{e}}$:

NUMERICAL | 4 points

1 ✓	

$\boxed{\text{f}}$:

NUMERICAL | 4 points

4 ✓	

The function $f(x)$ has $\boxed{\text{g}}$ stationary point(s) in the domain. (Hint: no need to find it (them) explicitly)

$\boxed{\text{g}}$:

NUMERICAL | 4 points

1 ✓	

Choose the behaviour of $f(x)$ in the interval $(-1, 1)$.

MULTI | 4 points | Single

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(6) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{(x+1)(e^x+2)}{(e^x+1)}$$

This function has two oblique asymptotes. They are $y = \boxed{\text{a}}x + \boxed{\text{b}}, \boxed{\text{c}}x + \boxed{\text{d}}$ with $\boxed{\text{a}} > \boxed{\text{c}}$.

$\boxed{\text{a}}$:

NUMERICAL | 2 points

2 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 2 points

2 ✓	

c:

NUMERICAL | 2 points

1 ✓	

d:

NUMERICAL | 2 points

1 ✓	

One has

$$f'(0) = \frac{\boxed{\text{e}}}{\boxed{\text{f}}}.$$

e:

NUMERICAL | 4 points

5 ✓	

f:

NUMERICAL | 4 points

4 ✓	

The function $f(x)$ has g stationary point(s) in the domain. (Hint: no need to find it (them) explicitly)

g:

NUMERICAL | 4 points

0 ✓	

Choose the behaviour of $f(x)$ in the interval $(-1, 1)$.

MULTI | 4 points | Single

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(7) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_1^2 \frac{\log x}{x^3} dx.$$

By noting that we can find easily a primitive of $\frac{1}{x^3}$, we can apply integration by parts. Fill in the blanks.

$$\int_1^2 \frac{\log x}{x^3} dx = [\boxed{\text{a}}]_1^2 - \int_1^2 \boxed{\text{b}} dx.$$

Choose correct functions.

$\boxed{\text{a}}$:

MULTI | 3 points | Single

- $\log x/x^2$
- $(\log x)^2/x^2$
- $-\log x/2x^2$ ✓
- $\log x/x^2$
- $-3\log x/x^3$
- $1/3x^2$
- $1/x^3$
- $-1/2x^3$
- $1/x^2 \log x$
- $-1/x^3 \log x$

$\boxed{\text{b}}$:

MULTI | 3 points | Single

- $\log x/x^2$
- $(\log x)^2/x^2$
- $-\log x/2x^2$
- $\log x/x^2$
- $-3\log x/x^3$
- $1/3x^2$
- $1/x^3$
- $-1/2x^3$ ✓
- $1/x^2 \log x$
- $-1/x^3 \log x$

By continuing the calculation, we obtain

$$\int_1^2 \frac{\log x}{x^3} dx = \frac{\boxed{\text{c}}}{\boxed{\text{d}}} - \frac{\log \boxed{\text{e}}}{\boxed{\text{f}}}.$$

$\boxed{\text{c}}$:

NUMERICAL | 3 points

3 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 3 points

16 ✓	

[e]:

NUMERICAL | 3 points

2 ✓	

[f]:

NUMERICAL | 3 points

8 ✓	

(8) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_1^3 \frac{\log x}{x^3} dx.$$

By noting that we can find easily a primitive of $\frac{1}{x^3}$, we can apply integration by parts. Fill in the blanks.

$$\int_1^3 \frac{\log x}{x^3} dx = [\boxed{a}]_1^3 - \int_1^3 \boxed{b} dx.$$

Choose correct functions.

[a]:

MULTI | 3 points | Single

- $\log x/x^2$
- $(\log x)^2/x^2$
- $-\log x/2x^2$ ✓
- $\log x/x^2$
- $-3\log x/x^3$
- $1/3x^2$
- $1/x^3$
- $-1/2x^3$
- $1/x^2 \log x$
- $-1/x^3 \log x$

[b]:

MULTI | 3 points | Single

- $\log x/x^2$

- $(\log x)^2/x^2$
- $-\log x/2x^2$
- $\log x/x^2$
- $-3\log x/x^3$
- $1/3x^2$
- $1/x^3$
- $-1/2x^3$ ✓
- $1/x^2 \log x$
- $-1/x^3 \log x$

By continuing the calculation, we obtain

$$\int_1^3 \frac{\log x}{x^3} dx = \frac{\boxed{c}}{\boxed{d}} - \frac{\log \boxed{e}}{\boxed{f}}.$$

$\boxed{c}$:

NUMERICAL | 3 points

2 ✓	

$\boxed{d}$:

NUMERICAL | 3 points

9 ✓	

$\boxed{e}$:

NUMERICAL | 3 points

3 ✓	

$\boxed{f}$:

NUMERICAL | 3 points

18 ✓	

(9) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_1^5 \frac{\log x}{x^3} dx.$$

By noting that we can find easily a primitive of $\frac{1}{x^3}$, we can apply integration by parts. Fill in the blanks.

$$\int_1^5 \frac{\log x}{x^3} dx = [\boxed{\text{a}}]_1^5 - \int_1^5 \boxed{\text{b}}\, dx.$$

Choose correct functions.

a:

MULTI | 3 points | Single

- $\log x/x^2$
- $(\log x)^2/x^2$
- $-\log x/2x^2$ ✓
- $\log x/x^2$
- $-3\log x/x^3$
- $1/3x^2$
- $1/x^3$
- $-1/2x^3$
- $1/x^2 \log x$
- $-1/x^3 \log x$

b:

MULTI | 3 points | Single

- $\log x/x^2$
- $(\log x)^2/x^2$
- $-\log x/2x^2$
- $\log x/x^2$
- $-3\log x/x^3$
- $1/3x^2$
- $1/x^3$
- $-1/2x^3$ ✓
- $1/x^2 \log x$
- $-1/x^3 \log x$

By continuing the calculation, we obtain

$$\int_1^5 \frac{\log x}{x^3} dx = \frac{\boxed{\text{c}}}{\boxed{\text{d}}} - \frac{\log \boxed{\text{e}}}{\boxed{\text{f}}}.$$

c:

NUMERICAL | 3 points

6 ✓	

d:

NUMERICAL | 3 points

25 ✓	

e:

NUMERICAL | 3 points

5 ✓	

f:

NUMERICAL | 3 points

50 ✓	

(10) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_1^7 \frac{\log x}{x^3} dx.$$

By noting that we can find easily a primitive of $\frac{1}{x^3}$, we can apply integration by parts. Fill in the blanks.

$$\int_1^7 \frac{\log x}{x^3} dx = [\boxed{a}]_1^7 - \int_1^7 \boxed{b} dx.$$

Choose correct functions.

a:

MULTI | 3 points | Single

- $\log x/x^2$
- $(\log x)^2/x^2$
- $-\log x/2x^2$ ✓
- $\log x/x^2$
- $-3\log x/x^3$
- $1/3x^2$
- $1/x^3$
- $-1/2x^3$
- $1/x^2 \log x$
- $-1/x^3 \log x$

b:

MULTI | 3 points | Single

- $\log x/x^2$

- $(\log x)^2/x^2$
- $-\log x/2x^2$
- $\log x/x^2$
- $-3\log x/x^3$
- $1/3x^2$
- $1/x^3$
- $-1/2x^3$ ✓
- $1/x^2 \log x$
- $-1/x^3 \log x$

By continuing the calculation, we obtain

$$\int_1^7 \frac{\log x}{x^3} dx = \frac{\boxed{\text{c}}}{\boxed{\text{d}}} - \frac{\log \boxed{\text{e}}}{\boxed{\text{f}}}.$$

c:

NUMERICAL | 3 points

12 ✓	

d:

NUMERICAL | 3 points

49 ✓	

e:

NUMERICAL | 3 points

7 ✓	

f:

NUMERICAL | 3 points

98 ✓	

(11) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = y(x)^2 \cos(\sqrt{x})/\sqrt{x}.$$

MULTI | 4 points | Single

- $y(x) = (\sin(\sqrt{x}) + C)^2$

- $y(x) = -2\cos(\sqrt{x} + C)^2$
- $y(x) = (\sin(\sqrt{x}) + C)^2$
- $y(x) = -\cos(\sqrt{x} + C)^2$
- $y(x) = -1/(2\sin(\sqrt{x}) + C)$ ✓
- $y(x) = -1/\sin(2\sqrt{x} + C)$
- $y(x) = 1/(\cos(\sqrt{x}) + C)$
- $y(x) = 1/(2\cos(\sqrt{x}) + C)$

Determine $C =$ [a] with the initial condition $y(0) = -1$

[a]:

NUMERICAL | 2 points

1 ✓	

Choose the general solution of the following differential equation.

$$y''(x) - 2y'(x) - 3y(x) = 0.$$

MULTI | 2 points | Single

- $y(x) = C_1 \exp(x) + C_2 \exp(-2x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(2x)$
- $y(x) = C_1 \exp(x) + C_2 \exp(-3x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(3x)$ ✓
- $y(x) = C_1 \sin(-3x) + C_2 \cos(x)$
- $y(x) = C_1 \sin(x) + C_2 \cos(-3x)$
- $y(x) = C_1 \sin(2x) + C_2 \cos(-x)$
- $y(x) = C_1 \sin(x) + C_2 \cos(-2x)$

Find a solution $y(x)$ such that $y(0) = 4$ and $y'(0) = 0$. $C_1 =$ [b], $C_2 =$ [c].

[b]:

NUMERICAL | 1 point

3 ✓	

[c]:

NUMERICAL | 1 point

1 ✓	

Find a solution $y(x)$ such that $y(0) = 4$ and $\lim_{x\to\infty} y(0) = 0$. $C_1 =$ [d], $C_2 =$ [e].

[d]:

NUMERICAL | 1 point

4 ✓	

[e]:

NUMERICAL | 1 point

0 ✓	

(12) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = y(x)^2 \sin(\sqrt{x})/\sqrt{x}.$$

MULTI | 4 points | Single

- $y(x) = (\sin(\sqrt{x}) + C)^2$
- $y(x) = -2\cos(\sqrt{x} + C)^2$
- $y(x) = (\sin(\sqrt{x}) + C)^2$
- $y(x) = -\cos(\sqrt{x} + C)^2$
- $y(x) = -1/(2\sin(\sqrt{x}) + C)$
- $y(x) = -1/\sin(2\sqrt{x} + C)$
- $y(x) = 1/(\cos(\sqrt{x}) + C)$
- $y(x) = 1/(2\cos(\sqrt{x}) + C)$ ✓

Determine $C = \boxed{a}$ with the initial condition $y(0) = 1$

$\boxed{a}$:

NUMERICAL | 2 points

-1 ✓	

Choose the general solution of the following differential equation.

$$y''(x) + y'(x) - 2y(x) = 0.$$

MULTI | 2 points | Single

- $y(x) = C_1 \exp(x) + C_2 \exp(-2x)$ ✓
- $y(x) = C_1 \exp(-x) + C_2 \exp(2x)$
- $y(x) = C_1 \exp(x) + C_2 \exp(-3x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(3x)$
- $y(x) = C_1 \sin(-3x) + C_2 \cos(x)$
- $y(x) = C_1 \sin(x) + C_2 \cos(-3x)$
- $y(x) = C_1 \sin(2x) + C_2 \cos(-x)$
- $y(x) = C_1 \sin(x) + C_2 \cos(-2x)$

Find a solution $y(x)$ such that $y(0) = -1$ and $y'(0) = -5$. $C_1 = \frac{\boxed{b}}{\boxed{c}}, C_2 = \frac{\boxed{d}}{\boxed{e}}$.

b:

NUMERICAL 1 point

-7 ✓	

c:

NUMERICAL 1 point

3 ✓	

d:

NUMERICAL 1 point

4 ✓	

e:

NUMERICAL 1 point

3 ✓	

Find a solution $y(x)$ such that $y(0) = 3$ and $\lim_{x\to\infty} y(0) = 0$.
$C_1 = \boxed{\text{f}}, C_2 = \boxed{\text{g}}$.

f:

NUMERICAL 1 point

0 ✓	

g:

NUMERICAL 1 point

3 ✓	

Total of marks: 290