2023Call1.

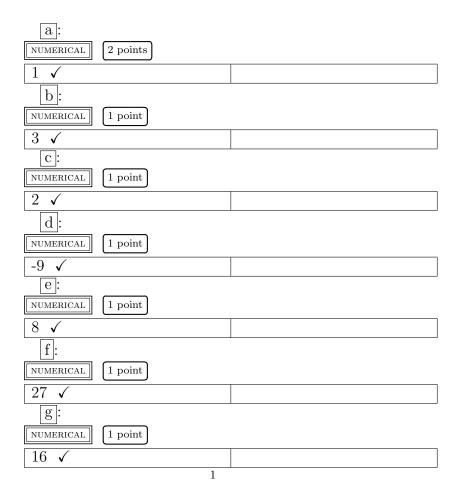
(1) **Q1**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

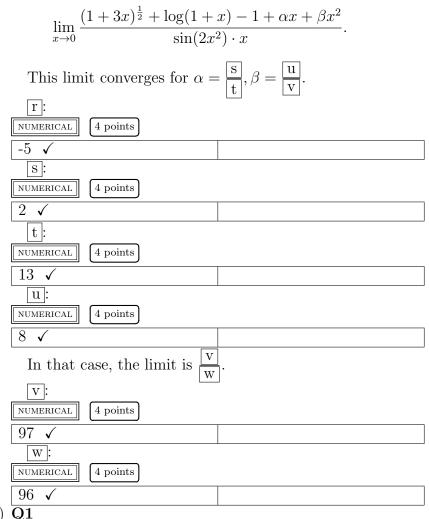
Complete the formulae.

$$(1+3x)^{\frac{1}{2}} = \boxed{a} + \frac{\boxed{b}}{\boxed{c}}x + \frac{\boxed{d}}{\boxed{e}}x^2 + \frac{\boxed{f}}{\boxed{g}}x^3 + o(x^3) \text{ as } x \to 0$$



$\log(1+x) = \boxed{\mathbf{h}} + \boxed{\mathbf{i}}x + \frac{\boxed{\mathbf{j}}}{\boxed{\mathbf{k}}}x^2 + \frac{\boxed{\mathbf{l}}}{\boxed{\mathbf{m}}}x^3 + o(x^3) \text{ as } x \to 0.$
h: NUMERICAL 2 points
NUMERICAL 2 points
j:
NUMERICAL 1 point
k:
NUMERICAL 1 point
I NUMERICAL 1 point
$1 \sqrt{m}$:
Image: Numerical 1 point
$\sin(2x^2) \cdot x = n + ox + px^2 + qx^3 + o(x^3)$ as $x \to 0$.
NUMERICAL 2 points
O: NUMERICAL 2 points
D: NUMERICAL 2 points
NUMERICAL 2 points
$2 \checkmark$

For various $\alpha, \beta \in \mathbb{R}$, study the limit:



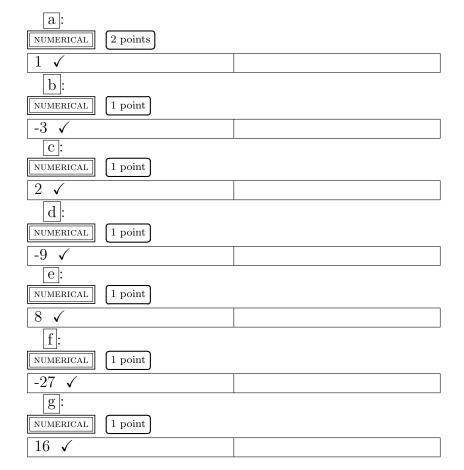
(2) Q1

0.10 penalty CLOZE

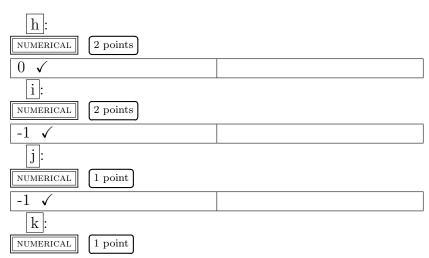
If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|\mathbf{a}|}{|\mathbf{b}|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

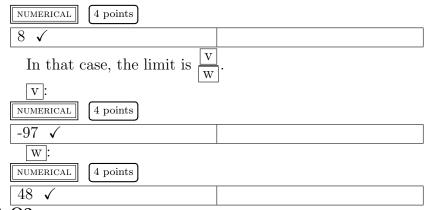
$$(1-3x)^{\frac{1}{2}} = \boxed{\mathbf{a}} + \frac{\boxed{\mathbf{b}}}{\boxed{\mathbf{c}}}x + \frac{\boxed{\mathbf{d}}}{\boxed{\mathbf{e}}}x^2 + \frac{\boxed{\mathbf{f}}}{\boxed{\mathbf{g}}}x^3 + o(x^3) \text{ as } x \to 0.$$



$$\log(1-x) = \boxed{\mathbf{h}} + \boxed{\mathbf{i}}x + \frac{\boxed{\mathbf{j}}}{\boxed{\mathbf{k}}}x^2 + \frac{\boxed{\mathbf{l}}}{\boxed{\mathbf{m}}}x^3 + o(x^3) \text{ as } x \to 0.$$



2 🗸		
NUMERICAL 1 point		
-1 🗸		
NUMERICAL 1 point		
3 🗸		
$\sin(x^2) \cdot x = \boxed{\mathbf{n}} + \boxed{\mathbf{o}}x + \boxed{\mathbf{p}}x^2 + \frac{\mathbf{n}}{\mathbf{n}}x^2 + $	$+ \boxed{\mathbf{q}} x^3 + o(x^3) \text{ as } x \to 0.$	
n: NUMERICAL 2 points		
0 🗸		
O: NUMERICAL 2 points		
0 🗸		
p :		
NUMERICAL 2 points		
q NUMERICAL 2 points		
$1 \checkmark$		
For various $\alpha, \beta \in \mathbb{R}$, study t	he limit:	
$\lim_{x \to 0} \frac{(1-3x)^{\frac{1}{2}} + \log(1-x) - 1 + \alpha x + \beta x^2}{\sin(x^2) \cdot x}.$		
$ \frac{1}{x \to 0} \qquad $	$\cdot x$	
This limit converges for $\alpha = \frac{s}{t}$, $\beta = \frac{u}{v}$.		
r NUMERICAL 4 points		
$5 \checkmark$		
s:		
NUMERICAL 4 points		
2 🗸		
t:		
NUMERICAL 4 points		
u:		



 $(3) \mathbf{Q}\overline{\mathbf{2}}$

CLOZE 0.10 penalty

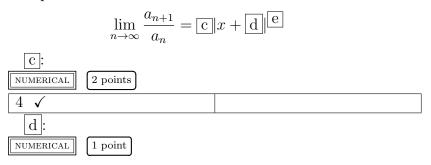
If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(4^n-1)(x+1)^{2n}}{n+2}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For x = i, calculate the partial sum $\sum_{n=0}^{2} \frac{(4^n - 1)(x+1)^{2n}}{n+2} = \boxed{a} + \boxed{b}i$.

a: NUMERICAL 2 points	
-15 🗸	
b:	
NUMERICAL 2 points	
2 🗸	

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{(4^n - 1)(x+1)^{2n}}{n+2}$. Complete the formula.

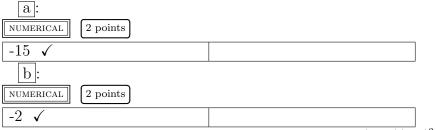


	1 🗸		
	e:		
	NUMERICAL 1 point		
	$2 \checkmark$		
	Therefore, by the ratio test, the series converges absolutely		
	f		
	for $\frac{ \mathbf{I} }{ \mathbf{\sigma} } < x < \frac{ \mathbf{I} }{ \mathbf{i} }$.		
	NUMERICAL 2 points		
	-3 🗸		
	g:		
	NUMERICAL 2 points		
	$2 \checkmark$		
	NUMERICAL 2 points		
	i:		
	NUMERICAL 2 points		
	$2 \checkmark$		
	For the case $x = -\frac{3}{2}$, the series		
	MULTI 4 points Single Shuffle		
	• converges absolutely.		
	• converges but not absolutely.		
	• diverges. \checkmark		
	For the case $x = -1$, the series		
	MULTI 4 points Single Shuffle		
	• converges absolutely. \checkmark		
	• converges but not absolutely.		
	• diverges.		
(4)	Q2		
. /	CLOZE 0.10 penalty		
	If not specified otherwise, fill in the blanks with integers (po		
	sibly 0 or negative). A fraction should be reduced (for ex-		
	$\frac{1}{2}$ is accorded but not $\frac{2}{2}$ and if it is projective and the		

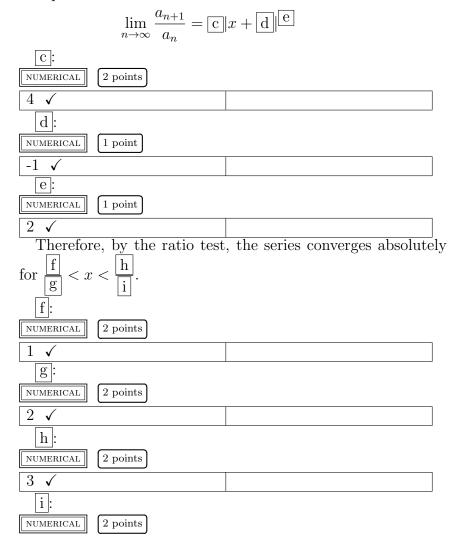
ample, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\boxed{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(4^n-1)(x-1)^{2n}}{n+2}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For x = i, calculate the partial sum $\sum_{n=0}^{2} \frac{(4^n - 1)(x-1)^{2n}}{n+2} = \boxed{a} + \boxed{b}i$.



In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{(4^n - 1)(x - 1)^{2n}}{n+2}$. Complete the formula.



$2 \checkmark$
For the case $x = 1$, the series
MULTI 4 points Single Shuffle
• converges absolutely. \checkmark
• converges but not absolutely.
• diverges.
For the case $x = \frac{3}{2}$, the series
MULTI 4 points Single Shuffle • converges absolutely.
• converges but not absolutely.

- diverges. \checkmark
- (5) **Q3**

CLOZE 0.10 penalty

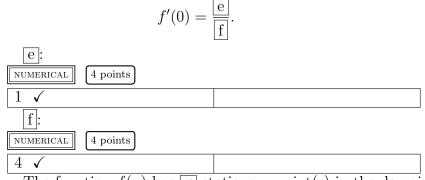
If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\begin{bmatrix} a \\ b \end{bmatrix}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$. Let us consider the following function

$$f(x) = \frac{(x+1)(e^x - 2)}{(e^x + 1)}$$

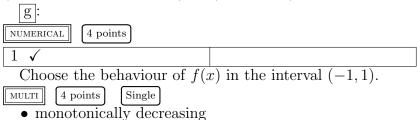
This function has two oblique asymptotes. They are y =ax + b, cx + d with a > c.

a :	
NUMERICAL 2 points	
1 🗸	
b:	
NUMERICAL 2 points	
1 🗸	
C:	
NUMERICAL 2 points	
-2 🗸	
d:	
NUMERICAL 2 points	
-2 🗸	

One has



The function f(x) has g stationary point(s) in the domain. (Hint: no need to find it (them) explicitly)



- monotonically increasing
- neither decreasing nor increasing \checkmark

(6) **Q3**

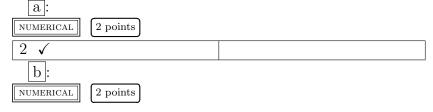
CLOZE 0.10 penalty

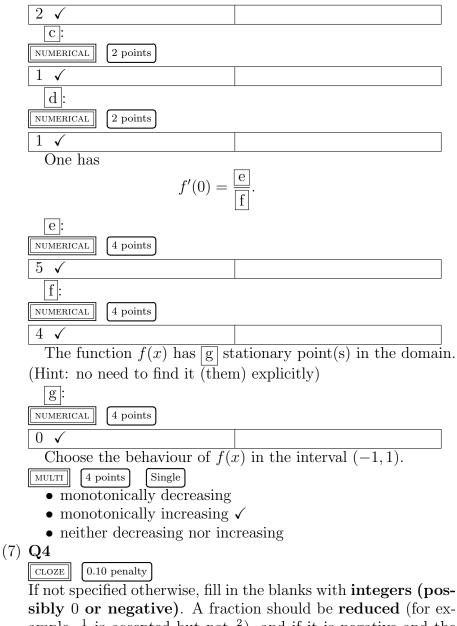
If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). log $x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{(x+1)(e^x+2)}{(e^x+1)}$$

This function has two oblique asymptotes. They are $y = [\underline{a}x + [\underline{b}], \underline{c}x + [\underline{d}]$ with $[\underline{a} > \underline{c}]$.





sibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{1}^{2} \frac{\log x}{x^3} dx.$$

By noting that we can find easily a primitive of $\frac{1}{x^3}$, we can apply integration by parts. Fill in the blanks.

$$\int_{1}^{2} \frac{\log x}{x^{3}} dx = [a]_{1}^{2} - \int_{1}^{2} b dx.$$

Choose correct functions.

a:
 3 points
 Single

 •
$$\log x/x^2$$
 • $(\log x)^2/x^2$

 • $-\log x/2x^2$
 •

 • $\log x/x^2$
 •

 • $\log x/x^2$
 •

 • $\log x/x^2$
 •

 • $\log x/x^2$
 •

 • $-3 \log x/x^3$
 •

 • $1/x^3$
 •

 • $-1/2x^3$
 •

 • $-1/x^3 \log x$
 •

 • $-1/x^3 \log x$
 •

 • $1/x^2 \log x$
 •

 • $\log x/x^2$
 •

 • $1/3x^2$
 •

 • $1/x^3$
 •

 • $-1/2x^3 \checkmark$
 •

• $-1/x^3 \log x$ By continuing the calculation, we obtain

$$\int_{1}^{2} \frac{\log x}{x^{3}} dx = \frac{\boxed{c}}{\boxed{d}} - \frac{\log \boxed{e}}{\boxed{f}}.$$

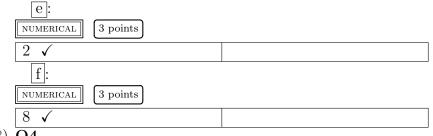
$$\boxed{c:}$$

$$\boxed{\text{NUMERICAL}} \quad \boxed{3 \text{ points}}$$

$$\boxed{d:}$$

$$\boxed{\text{NUMERICAL}} \quad \boxed{3 \text{ points}}$$

$$\boxed{16 \checkmark}$$



(8) **Q4**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\begin{bmatrix} a \\ b \end{bmatrix}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$. Let us calculate the following integral.

$$\int_{1}^{3} \frac{\log x}{x^3} dx.$$

By noting that we can find easily a primitive of $\frac{1}{x^3}$, we can apply integration by parts. Fill in the blanks.

$$\int_{1}^{3} \frac{\log x}{x^{3}} dx = [a]_{1}^{3} - \int_{1}^{3} b dx.$$

Choose correct functions.

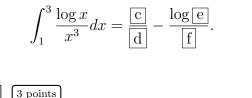
a:MULTI3 points•
$$\log x/x^2$$
• $(\log x)^2/x^2$ • $-\log x/2x^2 \checkmark$ • $\log x/x^2$ • $-3 \log x/x^3$ • $1/3x^2$ • $1/3x^2$ • $1/x^3$ • $-1/2x^3$ • $1/x^2 \log x$ • $-1/x^3 \log x$ b:MULTI3 points• $\log x/x^2$

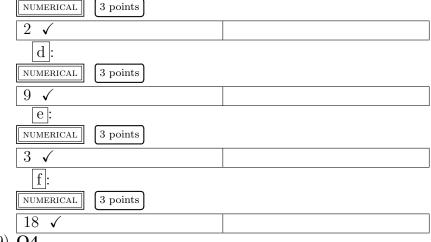
- $(\log x)^2/x^2$ • $-\log x/2x^2$ • $\log x/x^2$ • $-3\log x/x^3$ • $1/3x^2$ • $1/x^3$ • $-1/2x^3 \checkmark$
- 1/2x $1/x^2 \log x$
- $1/x \log x$

[C]:

• $-1/x^3 \log x$

By continuing the calculation, we obtain





 $(9) \overline{\mathbf{Q4}}$

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{1}^{5} \frac{\log x}{x^3} dx$$

By noting that we can find easily a primitive of $\frac{1}{x^3}$, we can apply integration by parts. Fill in the blanks.

$$\int_{1}^{5} \frac{\log x}{x^{3}} dx = [a]_{1}^{5} - \int_{1}^{5} b dx.$$

Choose correct functions.

a:

 MULTI
 3 points
 Single

 •
$$\log x/x^2$$
 • $\log x/2x^2 \checkmark$

 • $\log x/x^2$
 • $\log x/x^2$

 • $\log x/x^2$
 • $\log x/x^3$

 • $1/3x^2$
 • $1/x^3$

 • $-1/2x^3$
 • $-1/2x^3$

 • $-1/x^3 \log x$
 • $-1/x^3 \log x$

 b:
 Single

 • $\log x/x^2$
 • $(\log x)^2/x^2$

 • $\log x/x^2$
 • $\log x/2x^2$

 • $\log x/x^2$
 • $-\log x/2x^2$

 • $\log x/x^2$
 • $-3\log x/x^3$

 • $1/3x^2$
 • $1/3x^2$

 • $1/3x^2$
 • $1/x^3$

 • $-1/2x^3 \checkmark$
 • $-1/2x^3 \checkmark$

By continuing the calculation, we obtain

$$\int_{1}^{5} \frac{\log x}{x^{3}} dx = \frac{\boxed{c}}{\boxed{d}} - \frac{\log \boxed{e}}{\boxed{f}}.$$

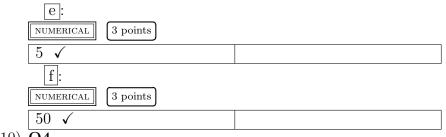
$$\boxed{c:}$$

$$\boxed{\text{NUMERICAL}} \quad 3 \text{ points}$$

$$\boxed{d:}$$

$$\boxed{\text{NUMERICAL}} \quad 3 \text{ points}$$

$$\boxed{25 \checkmark}$$



(10) **Q4**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\begin{bmatrix} a \\ b \end{bmatrix}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$. Let us calculate the following integral.

$$\int_{1}^{7} \frac{\log x}{x^3} dx.$$

By noting that we can find easily a primitive of $\frac{1}{x^3}$, we can apply integration by parts. Fill in the blanks.

$$\int_{1}^{7} \frac{\log x}{x^{3}} dx = [a]_{1}^{7} - \int_{1}^{7} b dx.$$

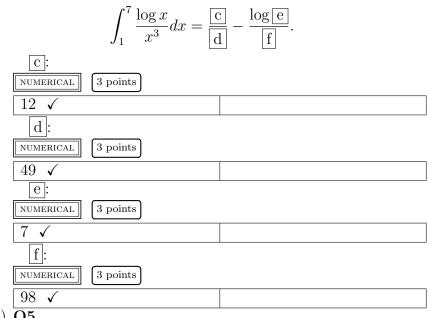
Choose correct functions.

a]:
MULTI 3 points Single
•
$$\log x/x^2$$

• $(\log x)^2/x^2$
• $-\log x/2x^2 \checkmark$
• $\log x/x^2$
• $-3\log x/x^3$
• $1/3x^2$
• $1/x^3$
• $-1/2x^3$
• $1/x^2\log x$
• $-1/x^3\log x$
b]:
MULTI 3 points Single

- $(\log x)^2 / x^2$
- $-\log x/2x^2$
- $\log x/x^2$
- $-3 \log x/x^3$ • $1/3x^2$
- 1/3x• $1/x^3$
- 1/x• $-1/2x^3 \checkmark$
- -1/2x • $1/x^2 \log x$
- $1/x \log x$
- $-1/x^3 \log x$

By continuing the calculation, we obtain



(11) $\overline{Q5}$

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = y(x)^2 \cos(\sqrt{x})/\sqrt{x}$$

$$\boxed{\text{MULTI}} \begin{array}{c} 4 \text{ points} \\ \bullet y(x) = (\sin(\sqrt{x}) + C)^2 \end{array}$$

•
$$y(x) = -2\cos(\sqrt{x} + C)^2$$

• $y(x) = (\sin(\sqrt{x}) + C)^2$
• $y(x) = -\cos(\sqrt{x} + C)^2$
• $y(x) = -1/(2\sin(\sqrt{x}) + C) \checkmark$
• $y(x) = -1/\sin(2\sqrt{x} + C)$
• $y(x) = 1/(\cos(\sqrt{x}) + C)$
• $y(x) = 1/(2\cos(\sqrt{x}) + C)$
Determine $C = a$ with the initial condition $y(0) = -1$
a:
NUMERICAL 2 points

Choose the general solution of the following differential equation.

$$y''(x) - 2y'(x) - 3y(x) = 0.$$

$$\begin{array}{c} \hline \text{MULTI} & 2 \text{ points} & \text{Single} \\ \bullet & y(x) = C_1 \exp(x) + C_2 \exp(-2x) \\ \bullet & y(x) = C_1 \exp(-x) + C_2 \exp(2x) \\ \bullet & y(x) = C_1 \exp(-x) + C_2 \exp(3x) \checkmark \\ \bullet & y(x) = C_1 \exp(-x) + C_2 \exp(3x) \checkmark \\ \bullet & y(x) = C_1 \sin(-3x) + C_2 \cos(x) \\ \bullet & y(x) = C_1 \sin(x) + C_2 \cos(-3x) \\ \bullet & y(x) = C_1 \sin(x) + C_2 \cos(-2x) \\ \hline \text{Find a solution } y(x) \text{ such that } y(0) = 4 \text{ and } y'(0) = 0. \ C_1 = \hline b, C_2 = c. \\ \hline b: \\ \hline \text{NUMERICAL} & 1 \text{ point} \\ \hline 1 \checkmark \\ \hline \text{Find a solution } y(x) \text{ such that } y(0) = 4 \text{ and } \lim_{x \to \infty} y(0) = 0. \\ C_1 = \hline d, C_2 = c. \\ \hline d: \\ \hline \text{NUMERICAL} & 1 \text{ point} \\ \hline 4 \checkmark \\ \hline e: \\ \hline \text{NUMERICAL} & 1 \text{ point} \\ \hline 0 \checkmark \\ \end{array}$$

(12) **Q5**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (pos-sibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

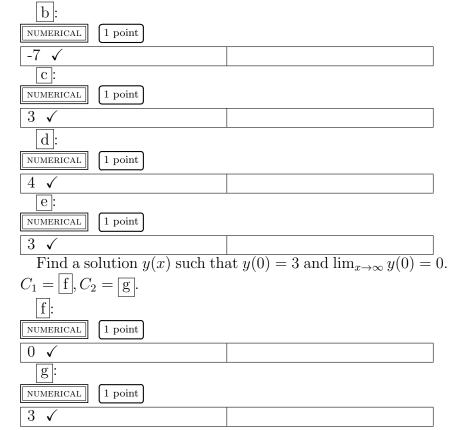
$$y'(x) = y(x)^{2} \sin(\sqrt{x})/\sqrt{x}.$$

$$\boxed{\text{MULTI}} \quad \underbrace{4 \text{ points}}_{\text{single}} \quad \underbrace{\text{Single}}_{\text{single}} \quad \underbrace{y(x) = (\sin(\sqrt{x}) + C)^{2}}_{\text{single}} \quad \underbrace{y(x) = -2\cos(\sqrt{x} + C)^{2}}_{\text{single}} \quad \underbrace{y(x) = (\sin(\sqrt{x}) + C)^{2}}_{\text{single}} \quad \underbrace{y(x) = -\cos(\sqrt{x} + C)^{2}}_{\text{single}} \quad \underbrace{y(x) = -1/(2\sin(\sqrt{x}) + C)}_{\text{single}} \quad \underbrace{y(x) = -1/(\sin(\sqrt{x}) + C)}_{\text{single}} \quad \underbrace{y(x) = 1/(\cos(\sqrt{x}) + C)}_{\text{single}} \quad \underbrace{y(x) = 1/(2\cos(\sqrt{x}) + C)}_{\text{single}} \quad \underbrace{y(x) = 1$$

Choose the general solution of the following differential equation.

$$y''(x) + y'(x) - 2y(x) = 0.$$

$$\boxed{\text{MULTI}} \begin{array}{c} 2 \text{ points} & \text{Single} \\ \bullet \ y(x) = C_1 \exp(x) + C_2 \exp(-2x) \checkmark \\ \bullet \ y(x) = C_1 \exp(-x) + C_2 \exp(2x) \\ \bullet \ y(x) = C_1 \exp(x) + C_2 \exp(-3x) \\ \bullet \ y(x) = C_1 \exp(-x) + C_2 \exp(3x) \\ \bullet \ y(x) = C_1 \sin(-3x) + C_2 \cos(x) \\ \bullet \ y(x) = C_1 \sin(x) + C_2 \cos(-3x) \\ \bullet \ y(x) = C_1 \sin(x) + C_2 \cos(-2x) \\ \bullet \ y(x) = C_1 \sin(x) + C_2 \cos(-2x) \\ \text{Find a solution } y(x) \text{ such that } y(0) = -1 \text{ and } y'(0) = -5 \\ C_1 = \boxed{\frac{b}{c}}, C_2 = \boxed{\frac{d}{c}}.$$



Total of marks: 290