## 2022Call6.

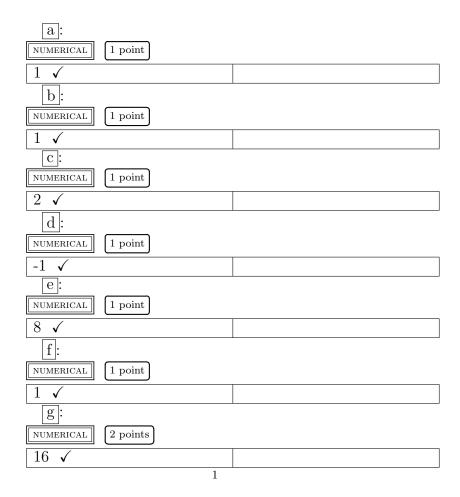
(1) **Q1** 

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Complete the formulae.

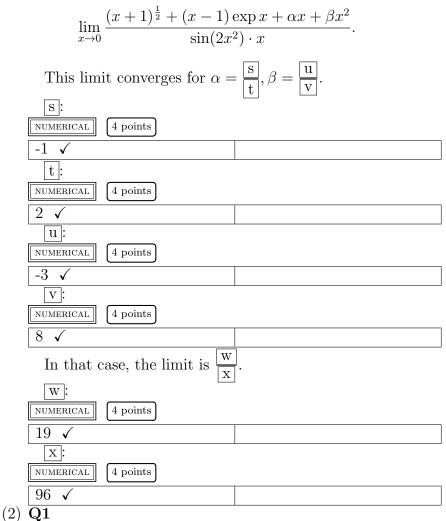
$$(x+1)^{\frac{1}{2}} = \boxed{\mathbf{a}} + \frac{\boxed{\mathbf{b}}}{\boxed{\mathbf{c}}}x + \frac{\boxed{\mathbf{d}}}{\boxed{\mathbf{e}}}x^2 + \frac{\boxed{\mathbf{f}}}{\boxed{\mathbf{g}}}x^3 \text{ as } x \to 0.$$



$(x-1) \exp x = \boxed{\mathbf{h}} + \boxed{\mathbf{i}}x + \frac{\boxed{\mathbf{j}}}{\boxed{\mathbf{k}}}x^2 + \frac{\boxed{\mathbf{l}}}{\boxed{\mathbf{m}}}x^3 + o(x^3) \text{ as } x \to 0.$
$ \begin{array}{c c}     h: \\ \hline \text{NUMERICAL} & 1 \text{ point} \\ \hline -1 & \checkmark \end{array} $
i: NUMERICAL 1 point
0 ✓ j: NUMERICAL 1 point
1 √       k:       NUMERICAL       2 points
2 ✓ []: NUMERICAL 1 point
1 ✓ m: NUMERICAL 2 points
$3 \checkmark$ $\sin(2x^2) \cdot x = \boxed{o} + \boxed{p}x + \boxed{q}x^2 + \boxed{r}x^3 + o(x^3) \text{ as } x \to 0.$
$ \begin{array}{c} \hline 0 \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \\ \hline \hline \\ \hline \hline \\ \hline \\ \hline \\ \hline \\ \hline \hline \\ \hline \\ \hline \\ \hline \\ \hline \hline \\ \hline \\ \hline \\ \hline \\ \hline \hline \hline \\ \hline \hline \\ \hline \hline \hline \\ \hline \hline \hline \\ \hline \hline \\ \hline \hline \hline \\ \hline \hline \\ \hline \hline \hline \hline \\ \hline \hline \hline \\ \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \hline \hline \hline \\ \hline \hline$
$\begin{array}{c} \hline p: \\ \hline NUMERICAL \end{array} 1 \text{ point} \\ \hline 0 \checkmark \end{array}$
Q:     NUMERICAL     1 point
0 ✓ <u> r</u> : <u> NUMERICAL</u> 3 points
$2 \checkmark$

 $\mathbf{2}$ 

For various  $\alpha, \beta \in \mathbb{R}$ , study the limit:



0.10 penalty CLOZE

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{|\mathbf{a}|}{|\mathbf{b}|}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Complete the formulae.

$$(3x+1)^{\frac{1}{2}} = \boxed{\mathbf{a}} + \frac{\boxed{\mathbf{b}}}{\boxed{\mathbf{c}}}x + \frac{\boxed{\mathbf{d}}}{\boxed{\mathbf{e}}}x^2 + \frac{\boxed{\mathbf{f}}}{\boxed{\mathbf{g}}}x^3 \text{ as } x \to 0.$$

$$(x-1) \exp x = \boxed{h} + \boxed{i}x + \frac{\boxed{j}}{k}x^{2} + \frac{\boxed{l}}{m}x^{3} + o(x^{3}) \text{ as } x \to 0.$$

$$\boxed{h:}$$

$$\boxed{\text{NUMERICAL}} 1 \text{ point}$$

$$\boxed{-1 \checkmark}$$

$$\boxed{i:}$$

$$\boxed{\text{NUMERICAL}} 1 \text{ point}$$

$$\boxed{0 \checkmark}$$

$$\boxed{j:}$$

$$\boxed{\text{NUMERICAL}} 1 \text{ point}$$

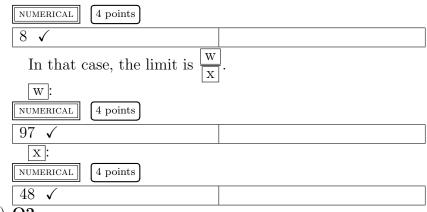
$$\boxed{1 \checkmark}$$

$$\boxed{k:}$$

$$\boxed{\text{NUMERICAL}} 2 \text{ points}$$

a:  
NUMERICAL 1 point  
1 
$$\checkmark$$
  
b:  
NUMERICAL 1 point  
3  $\checkmark$   
C:  
NUMERICAL 1 point  
2  $\checkmark$   
d:  
NUMERICAL 1 point  
-9  $\checkmark$   
e:  
NUMERICAL 1 point  
8  $\checkmark$   
f:  
NUMERICAL 1 point  
27  $\checkmark$   
g:  
NUMERICAL 1 point  
27  $\checkmark$   
d

$\mathbf{O}$	
1:	
NUMERICAL         1 point	
$1 \checkmark$	
m:	
NUMERICAL 2 points	
$3 \checkmark$	
$\sin(x^2) \cdot x = \boxed{\mathbf{o}} + \boxed{\mathbf{p}}x + \boxed{\mathbf{q}}x^2 - \frac{1}{2}x^2 - \frac{1}{2}x$	$+ [\mathbf{r}] x^3 + o(x^3)$ as $x \to 0$ .
0:	
NUMERICAL 3 points	
<b>p</b> :	
NUMERICAL     1 point	
NUMERICAL         1 point	
0 🗸	
r:	
NUMERICAL         3 points	
$1 \checkmark$	
For various $\alpha, \beta \in \mathbb{R}$ , study t	the limit:
$(2 - 1)^{\frac{1}{2}} + (- 1)^{\frac{1}{2}}$	
$\lim_{x \to 0} \frac{(3x+1)^{\frac{1}{2}} + (x-1)e}{\sin(x^2)}$	$\frac{\exp x + \alpha x + \beta x}{2}$
$x \rightarrow 0$ $\sin(x^2)$	$\cdot x$
This limit converges for $\alpha =$	s <sub>c</sub> <u>u</u>
This limit converges for $\alpha =$	$t, \rho = v.$
s:	
NUMERICAL         4 points	
-3 🗸	
t	
NUMERICAL 4 points	
$2 \checkmark$	
NUMERICAL 4 points	
$5 \checkmark$	
V:	



(3) **Q2** 

CLOZE 0.10 penalty

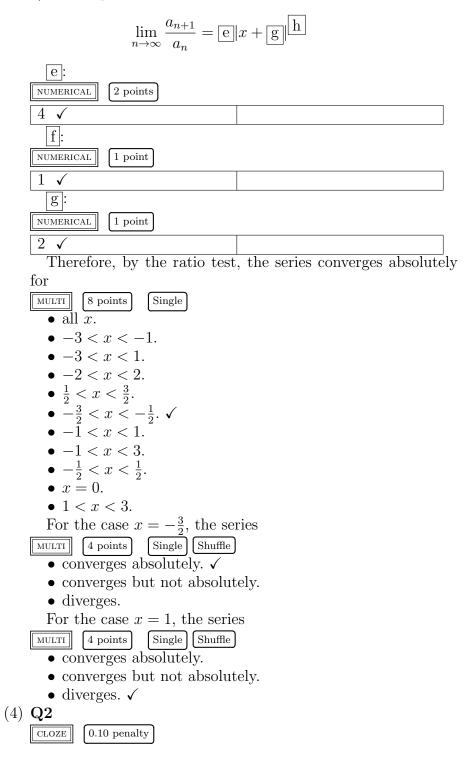
If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us study the following series  $\sum_{n=0}^{\infty} \frac{4^n-1}{n^2+3}(x+1)^{2n}$ , with various x.

This series makes sense also for  $x \in \mathbb{C}$ . For x = i, calculate the partial sum  $\sum_{n=0}^{2} \frac{4^n - 1}{n^2 + 3} (x + 1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} i$ .

a:	
NUMERICAL 1 point	
-60 🗸	
b:	
NUMERICAL 1 point	
7 🗸	
C:	
NUMERICAL 1 point	
3 🗸	
d	
NUMERICAL 1 point	
2 🗸	

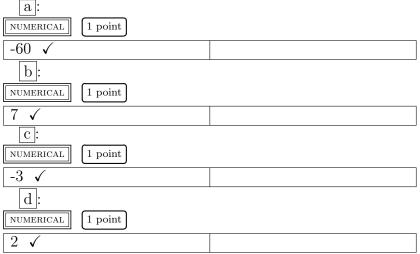
In order to use the ratio test for  $x \in \mathbb{R}$ , we put  $a_n = \frac{4^n - 1}{n^2 + 3}(x + 1)^{2n}$ . Complete the formula.



If not specified otherwise, fill in the blanks with **integers (pos-sibly** 0 **or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not). log  $x = \log_e x$ , not  $\log_{10} x$ .

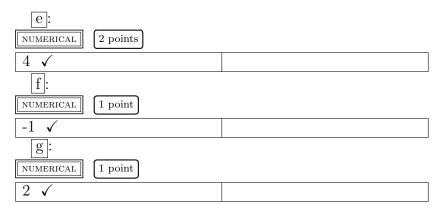
but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ . Let us study the following series  $\sum_{n=0}^{\infty} \frac{4^n - 1}{n^2 + 3} (x - 1)^{2n}$ , with various x.

This series makes sense also for  $x \in \mathbb{C}$ . For x = i, calculate the partial sum  $\sum_{n=0}^{2} \frac{4^n - 1}{n^2 + 3} (x - 1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} i$ .



In order to use the ratio test for  $x \in \mathbb{R}$ , we put  $a_n = \frac{4^n - 1}{n^2 + 3}(x - 1)^{2n}$ . Complete the formula.

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \boxed{e} x + \boxed{g}^{h}$$



Therefore, by the ratio test, the series converges absolutely for

MULTI 8 points Single  
• all x.  
• 
$$-3 < x < -1$$
.  
•  $-3 < x < 1$ .  
•  $-2 < x < 2$ .  
•  $\frac{1}{2} < x < \frac{3}{2}$ .  $\checkmark$   
•  $-\frac{3}{2} < x < -\frac{1}{2}$ .  
•  $-1 < x < 1$ .  
•  $-1 < x < 3$ .  
•  $-\frac{1}{2} < x < \frac{1}{2}$ .  
•  $x = 0$ .  
•  $1 < x < 3$ .  
For the case  $x = -\frac{3}{2}$ , the series  
MULTI 4 points Single Shuffle  
• converges absolutely.  
• diverges.  $\checkmark$   
For the case  $x = 1$ , the series  
MULTI 4 points Single Shuffle  
• converges but not absolutely.  
• diverges.  $\checkmark$ 

(5) **Q3** 

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us consider the following function

$$f(x) = \log\left(\frac{x}{x^2 + 1}\right).$$

The function f(x) is not defined on the whole real line  $\mathbb{R}$ . Choose all the points that are **not** in the natural domain of f(x).

−2 ✓ • -1 ✓ •  $-\frac{1}{2}$ • 0 <del>v</del> •  $\frac{1}{2}$  (-100%) • 1 (-100%)• 2(-100%)Choose all asymptotes of f(x). MULTI 4 points Single •  $y = -1 \ (-100\%)$ •  $y = -\frac{1}{2} (-100\%)$ • y = 0 (-100%) •  $y = \frac{1}{2} (-100\%)$ • y = 2 (-100%)•  $x = -2 \ (-100\%)$ •  $x = -1 \ (-100\%)$ •  $x = -\frac{1}{2} (-100\%)$ • x = 0  $\sqrt{}$ •  $x = \frac{1}{2} (-100\%)$ •  $x = \hat{1} (-100\%)$ • x = 2 (-100%)•  $y = -x \ (-100\%)$ • y = x (-100%)One has  $f'(2) = \frac{\boxed{a}}{\boxed{b}}.$ a: NUMERICAL 4 points -3 🗸 b : NUMERICAL 4 points 10 🗸 The function f(x) has c stationary point(s) in the domain **c**:

Choose the behaviour of f(x) in the interval (0, 2).

MULTI 4 points Single

NUMERICAL

1 🗸

• monotonically decreasing

4 points

• monotonically increasing

- neither decreasing nor increasing  $\checkmark$
- (6) **Q3**

CLOZE 0.10 penalty

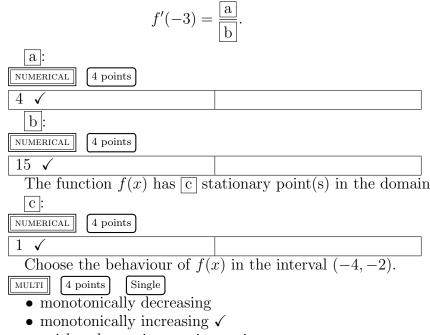
If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ . Let us consider the following function

$$f(x) = \log\left(\frac{-x}{x^2 + 1}\right).$$

The function f(x) is not defined on the whole real line  $\mathbb{R}$ . Choose all the points that are **not** in the natural domain of f(x).

$$\begin{array}{c} \hline \textbf{MULTI} & 4 \text{ points} & \textbf{Single} \\ \hline \bullet -2 & (-100\%) \\ \bullet -1 & (-100\%) \\ \bullet -\frac{1}{2} & (-100\%) \\ \bullet 0 \checkmark \\ \bullet \frac{1}{2} \checkmark \\ \bullet 1 \checkmark \\ \bullet 2 \checkmark \\ \hline \textbf{Choose all asymptotes of } f(x). \\ \hline \textbf{MULTI} & 4 \text{ points} & \textbf{Single} \\ \hline \bullet y = -1 & (-100\%) \\ \bullet y = -\frac{1}{2} & (-100\%) \\ \bullet y = \frac{1}{2} & (-100\%) \\ \bullet y = \frac{1}{2} & (-100\%) \\ \bullet y = 2 & (-100\%) \\ \bullet x = -2 & (-100\%) \\ \bullet x = -1 & (-100\%) \\ \bullet x = -\frac{1}{2} & (-100\%) \\ \bullet x = 1 & (-100\%) \\ \bullet x = 2 & (-100\%) \\ \bullet x = 2 & (-100\%) \\ \bullet x = 2 & (-100\%) \\ \bullet y = -x & (-100\%) \\ \bullet y = x & (-100\%) \\ \bullet y = x & (-100\%) \end{array}$$

One has



- neither decreasing nor increasing
- (7) **Q4**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us calculate the following integral.

$$\int_0^{\pi} x^2 \sin x dx$$

By noting that we can find easily a primitive of sin(x), we can apply integration by parts. Fill in the blanks.

$$\int_0^{\pi} x^2 \sin x dx. = [a]_0^{\pi} - \int_0^{\pi} b dx.$$

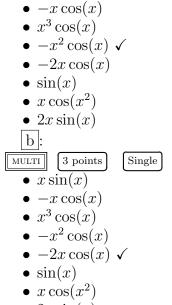
Choose correct functions.

a:

 MULTI

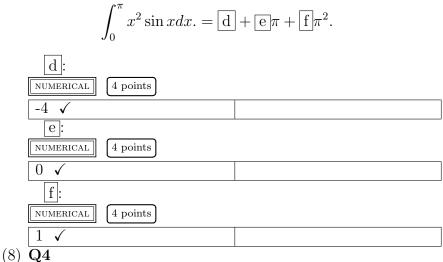
 3 points

 
$$\cdot x \sin(x)$$



• 
$$2x\sin(x)$$

By continuing the calculation, we obtain



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CLOZE
        0.10 penalty
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If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us calculate the following integral.

$$\int_0^{\frac{\pi}{2}} x^2 \sin x dx.$$

By noting that we can find easily a primitive of sin(x), we can apply integration by parts. Fill in the blanks.

$$\int_0^{\frac{\pi}{2}} x^2 \sin x dx. = [a]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} b dx.$$

Choose correct functions.

a:MULTI6 points
$$\cdot x \sin(x)$$
 $-x \cos(x)$  $\cdot x^3 \cos(x)$  $-x^2 \cos(x) \checkmark$  $-2x \cos(x)$  $\cdot -2x \cos(x)$  $\cdot \sin(x)$  $\cdot x \cos(x^2)$  $\cdot 2x \sin(x)$ b:MULTI6 pointsSingle $\cdot x \sin(x)$  $-x \cos(x)$  $\cdot x \sin(x)$  $-x \cos(x)$  $\cdot x^3 \cos(x)$  $-2x \cos(x)$  $-2x \cos(x)$  $\cdot -2x \cos(x) \checkmark$  $\cdot x \cos(x^2)$  $\cdot 2x \sin(x)$ 

By continuing the calculation, we obtain

$$\int_{0}^{\frac{\pi}{2}} x^{2} \sin x dx. = \mathbf{d} + \mathbf{e}\pi + \mathbf{f}\pi^{2}.$$

$$\mathbf{d}:$$

$$\mathbf{u} = \mathbf{d} + \mathbf{e}\pi + \mathbf{f}\pi^{2}.$$

$$\mathbf{d}:$$

$$\mathbf{u} = \mathbf{d} + \mathbf{e}\pi + \mathbf{f}\pi^{2}.$$

$$\mathbf{d} = \mathbf{d} + \mathbf{e}\pi + \mathbf{f}\pi^{2}.$$

NUMERICAL	4 points
0 🗸	

 $(9) \overline{\mathbf{Q5}}$ 

CLOZE 0.10 penalty

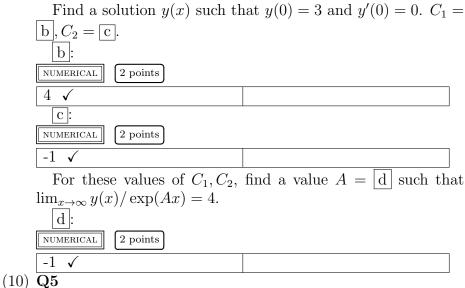
If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not). log  $x = \log_e x$ , not  $\log_{10} x$ .

Choose the general solution of the following differential equation.

 $y'(x) = 2\exp(y(x))\sin(x^2)x.$ 2 points Single MULTI •  $y(x) = \exp(\cos(x^2)) + C$ •  $y(x) = \log(\cos(x^2)) + C$ •  $y(x) = \exp(-\cos(x^2) + C)$ •  $y(x) = -\log(\cos(x^2) + C) \checkmark$ •  $y(x) = \exp(-\cos(x^2)) + C$ •  $y(x) = -\log(\sin(x^2)) + C$ •  $y(x) = -\log(\sin(x^2) + C)$ •  $y(x) = \exp(-\sin(x^2)) + C$ •  $y(x) = -\log(\sin(x^2)) + C$ Determine C = [a] with the initial condition y(0) = 0a: NUMERICAL 2 points 0 🗸

Choose the general solution of the following differential equation.

y''(x) + 5y'(x) + 4y(x) = 0.MULTI
2 points
Single
•  $y(x) = C_1 \exp(2x) + C_2 \exp(-2x)$ •  $y(x) = C_1 \exp(-x) + C_2 \exp(-4x)$ •  $y(x) = C_1 \exp(x) + C_2 \exp(-4x)$ •  $y(x) = C_1 \exp(-x) + C_2 \exp(-4x) \checkmark$ •  $y(x) = C_1 \sin(-4x) + C_2 \cos(x)$ •  $y(x) = C_1 \sin(-2x) + C_2 \cos(2x)$ •  $y(x) = C_1 \sin(2x) + C_2 \cos(-2x)$ •  $y(x) = C_1 \sin(x) + C_2 \cos(4x)$ 



0.10 penalty CLOZE

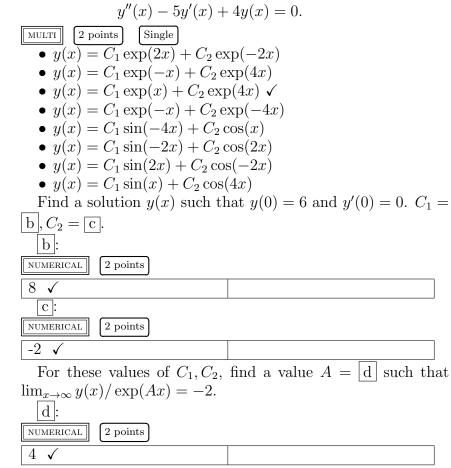
If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{|\mathbf{a}|}{|\mathbf{b}|}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Choose the general solution of the following differential equation.

$$y'(x) = 2 \exp(y(x)) \sin(x^2)x.$$

$$\boxed{\text{MULTI}} \begin{array}{c} 2 \text{ points} & \text{Single} \\ \bullet y(x) = \exp(\cos(x^2)) + C \\ \bullet y(x) = \log(\cos(x^2)) + C \\ \bullet y(x) = \exp(-\cos(x^2) + C) \\ \bullet y(x) = -\log(\cos(x^2) + C) \\ \bullet y(x) = \exp(-\cos(x^2)) + C \\ \bullet y(x) = -\log(\sin(x^2)) + C \\ \bullet y(x) = -\log(\sin(x^2)) + C \\ \bullet y(x) = \exp(-\sin(x^2)) + C \\ \bullet y(x) = -\log(\sin(x^2)) + C \\ \text{Determine } C = \boxed{a} \text{ with the initial condition } y(\sqrt{\frac{\pi}{2}}) = 0 \\ \boxed{a}: \\ \boxed{\text{NUMERICAL}} \begin{array}{c} 2 \text{ points} \end{array}$$

Choose the general solution of the following differential equation.



Total of marks: 258