2022Call5.

(1) **Q1**

If not specified otherwise, fill in the blanks with **integers** (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\cos x = \boxed{\mathbf{a}} + \boxed{\mathbf{b}}x + \frac{\boxed{\mathbf{c}}}{\boxed{\mathbf{d}}}x^2 + \boxed{\mathbf{e}}x^3 + \frac{\boxed{\mathbf{f}}}{\boxed{\mathbf{g}}}x^4 + o(x^4) \text{ as } x \to 0.$$

a:
NUMERICAL 1 point
1 🗸
b:
NUMERICAL 1 point
0 ✓
<u>c</u> :
NUMERICAL 1 point
-1 ✓
d:
NUMERICAL 1 point
2 ✓
e:
NUMERICAL 1 point
0 ✓
f :
NUMERICAL 1 point
1 🗸
g:
NUMERICAL 2 points
24 🗸

$$x^2\sqrt{1+3x} = \boxed{\mathbf{h}} + \boxed{\mathbf{i}}x + \boxed{\mathbf{j}}x^2 + \boxed{\mathbf{k}} \\ 1 \end{bmatrix} x^3 + \boxed{\mathbf{m}} \\ x^4 + o(x^4) \text{ as } x \to 0.$$

$$\boxed{\mathbf{h}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{0}} \checkmark \qquad \boxed{\mathbf{i}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{1}} \checkmark \qquad \boxed{\mathbf{k}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{3}} \checkmark \qquad \boxed{\mathbf{1}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{2}} \checkmark \qquad \boxed{\mathbf{m}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{2}} \checkmark \qquad \boxed{\mathbf{m}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{9}} \checkmark \qquad \boxed{\mathbf{n}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{2}} \text{ points}$$

$$\boxed{\mathbf{8}} \checkmark \qquad \boxed{\mathbf{0}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{0}} \checkmark \qquad \boxed{\mathbf{p}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{0}} \checkmark \qquad \boxed{\mathbf{p}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{0}} \checkmark \qquad \boxed{\mathbf{p}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

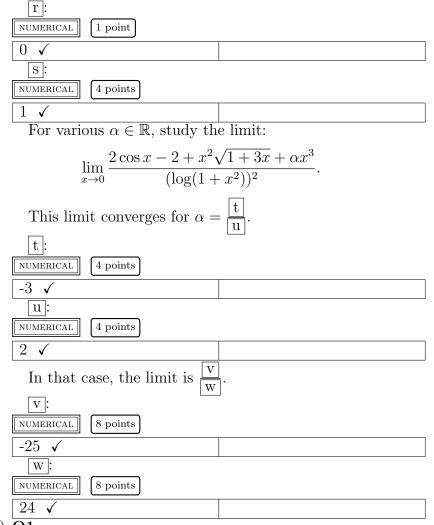
$$\boxed{\mathbf{0}} \checkmark \qquad \boxed{\mathbf{q}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{0}} \checkmark \qquad \boxed{\mathbf{q}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{0}} \checkmark \qquad \boxed{\mathbf{q}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{0}} \checkmark \qquad \boxed{\mathbf{q}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$

$$\boxed{\mathbf{0}} \checkmark \qquad \boxed{\mathbf{q}} : \\ \text{NUMERICAL} \qquad \boxed{\mathbf{1}} \text{ point}$$



(2) **Q1**CLOZE 0.10 penalty

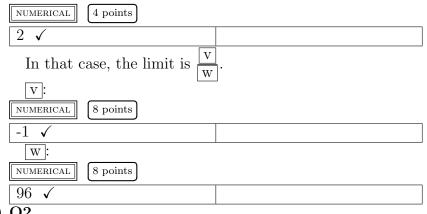
If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\cos x = \boxed{\mathbf{a}} + \boxed{\mathbf{b}}x + \frac{\boxed{\mathbf{c}}}{\boxed{\mathbf{d}}}x^2 + \boxed{\mathbf{e}}x^3 + \frac{\boxed{\mathbf{f}}}{\boxed{\mathbf{g}}}x^4 + o(x^4) \text{ as } x \to 0.$$

a: NUMERICAL 1 point	
1 ✓	
b:	
NUMERICAL 1 point	
0 🗸	
c:	
NUMERICAL 1 point	
-1 🗸	
[d]:	
NUMERICAL 1 point	
2 ✓ e:	
NUMERICAL 1 point	
0 ✓	
f:	
NUMERICAL 1 point	
1 🗸	
g:	
NUMERICAL 2 points	
24 ✓	
$x^{2}\sqrt{1+x} = \boxed{\mathbf{h}} + \boxed{\mathbf{i}}x + \boxed{\mathbf{j}}x^{2} + \frac{\boxed{\mathbf{k}}}{\boxed{1}}$	$x^{3} + \frac{\boxed{m}}{\boxed{n}} x^{4} + o(x^{4}) \text{ as } x \to 0.$
Ė	
<u>h</u> :	
NUMERICAL 1 point	
0 ✓	
<u>i</u> :	
NUMERICAL 1 point	
0 ✓	
j:	
NUMERICAL 1 point	
1.	
k: NUMERICAL 1 point	
TOMBICOTI T POINT	

	1 ✓
	1:
	NUMERICAL 1 point
	2 ✓
	m:
	NUMERICAL 1 point
	<u>-1 √</u>
	n:
	NUMERICAL 2 points
	8 🗸
$(\log(1$	$(x^2 + 2x^2)^2 = 0 + px + qx^2 + rx^3 + sx^4 + o(x^4) \text{ as } x \to 0.$
	O:
	NUMERICAL 1 point
	0 ✓
	<u> </u>
	NUMERICAL 1 point
	0 ✓
	<u>q</u> :
	NUMERICAL 1 point
	0 ✓
	NUMERICAL 1 point
	S: NUMERICAL 4 points
	4 V
	For various $\alpha \in \mathbb{R}$, study the limit:
	$\lim_{x \to 0} \frac{2\cos x - 2 + x^2\sqrt{1+x} + \alpha x^3}{(\log(1+2x^2))^2}.$
	$x \rightarrow 0 \qquad (\log(1+2x^2))^2$
	This limit converges for $\alpha = \frac{t}{u}$.
	t:
	NUMERICAL 4 points
	-1 🗸
	u:

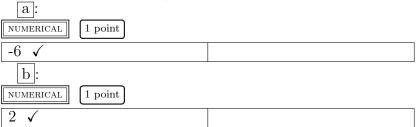


(3) **Q2**[CLOZE] 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{2^{n}-1}{n!} (x+1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For x = i, calculate the partial sum $\sum_{n=0}^{2} \frac{2^{n}-1}{n!} (x+1)^{2n} = \boxed{\mathbf{a}} + \boxed{\mathbf{b}} i$.



In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{2^n - 1}{n!} |x + 1|^{2n}$. Complete the formula.

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \boxed{\mathbf{c}}$$



Therefore, by the ratio test, the series converges absolutely for

MULTI 4 points Single

- all x. \checkmark
- -3 < x < -1.
- -3 < x < 1.
- -3 < x < 1. $-\frac{5}{4} < x < -\frac{3}{4}$. $-\frac{3}{2} < x < -\frac{1}{2}$. $\frac{1}{2} < x < \frac{3}{2}$. $\frac{3}{4} < x < \frac{5}{4}$.

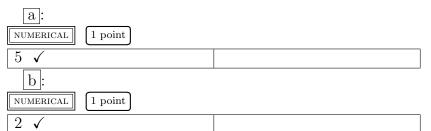
- -1 < x < 1
- -1 < x < 3.
- x = 0.
- 1 < x < 3.

For the case $x = -\frac{3}{2}$, the series MULTI 2 points Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

Calculate the infinite sum.

$$\sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n = \frac{\boxed{d}}{\boxed{e}}.$$

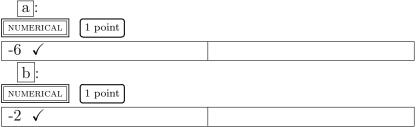


$(4) \ \mathbf{Q2}$ 0.10 penalty CLOZE

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|a|}{|b|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted

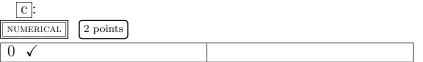
but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$. Let us study the following series $\sum_{n=0}^{\infty} \frac{2^n-1}{n!} (x+1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For x = -i, calculate the partial sum $\sum_{n=0}^{2} \frac{2^{n}-1}{n!} (x+1)^{2n} = \boxed{\mathbf{a}} + \boxed{\mathbf{b}} i$.



In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{2^n - 1}{n!} |x + 1|^{2n}$. Complete the formula.

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\boxed{\mathtt{c}}$$



Therefore, by the ratio test, the series converges absolutely for

MULTI 4 points Single

- $\overline{\bullet}$ all \overline{x} . \checkmark
- -3 < x < -1.
- -3 < x < 1.
- -3 < x < 1. $-\frac{5}{4} < x < -\frac{3}{4}$. $-\frac{3}{2} < x < -\frac{1}{2}$. $\frac{1}{2} < x < \frac{3}{2}$. $\frac{3}{4} < x < \frac{5}{4}$. -1 < x < 1.

- -1 < x < 3.
- x = 0.
- 1 < x < 3.

For the case $x = \frac{3}{2}$, the series

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

Calculate the infinite sum.

$$\sum_{n=0}^{\infty} \left(-\frac{1}{3} \right)^n = \boxed{\frac{\mathbf{d}}{\mathbf{e}}}.$$





(5) Q3

0.10 penalty CLOZE

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|a|}{|b|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{(x-1)^3}{x(x+1)}.$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of f(x).

MULTI 4 points Single \bullet -2 $\overline{(-100\%)}$

- -1 ✓
- $-\frac{1}{2}$ (-100%)

- $\frac{1}{2}$ (-100%) 1 (-100%)
- 2 (-100%)

Choose all asymptotes of f(x).

MULTI 4 points Single

- $y = -1 \ (-100\%)$
- $y = -\frac{1}{2} (-100\%)$
- y = 0 (-100%) $y = \frac{1}{2}$ (-100%) y = 2 (-100%)
- $x = -2 \ (-100\%)$
- $\bullet \ x = -1 \checkmark$
- $\begin{array}{l} \bullet \ x = -\frac{1}{2} \ (-100\%) \\ \bullet \ x = 0 \ \checkmark \end{array}$
- $x = \frac{1}{2} (-100\%)$ x = 1 (-100%)
- x = 2 (-100%)

- $y = x 4 \checkmark$
- y = x (-100%)
- y = x + 4 (-100%)

One has

$$f'(2) = \frac{\boxed{a}}{\boxed{b}}.$$

a:

NUMERICAL 4 points

13 ✓

b:

NUMERICAL 4 points

36 ✓

The function f(x) has \boxed{c} stationary point(s) in the domain \boxed{c} :

NUMERICAL 4 points

3 🗸

Choose the behaviour of f(x) in the interval [3, 4].

MULTI 4 points Single

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(6) **Q3**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{(x+1)^3}{x(x-1)}.$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of f(x).

MULTI 4 points Single

- \bullet -2 (-100%)
- -1 (-100%)

•
$$-\frac{1}{2}$$
 (-100%)
• 0 \checkmark
• $\frac{1}{2}$ (-100%)
• 1 \checkmark
• 2 (-100%)
Choose all asymptotes of $f(x)$.

MULTI [4 points] Single
• $y = -1$ (-100%)
• $y = -\frac{1}{2}$ (-100%)
• $y = 0$ (-100%)
• $y = \frac{1}{2}$ (-100%)
• $y = \frac{1}{2}$ (-100%)
• $y = 2$ (-100%)
• $x = -2$ (-100%)
• $x = -1$ (-100%)
• $x = -1$ (-100%)
• $x = -\frac{1}{2}$ (-100%)
• $x = 0$ \checkmark
• $x = \frac{1}{2}$ (-100%)
• $x = 1$ \checkmark
• $x = 2$ (-100%)
• $y = x - 4$ (-100%)
• $y = x - 4$ (-100%)
• $y = x + 4$ \checkmark
One has

$$f'(2) = \frac{a}{b}$$

NUMERICAL [4 points]

**The function $f(x)$ has c stationary point(s) in the domain c :

NUMERICAL [4 points]

**The function $f(x)$ has c stationary point(s) in the domain c :

NUMERICAL [4 points]

**Choose the behaviour of $f(x)$ in the interval [3, 4].

monotonically decreasingmonotonically increasing

• neither decreasing nor increasing \checkmark

(7) **Q4**

If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

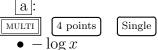
Let us calculate the following integral.

$$\int_{1}^{2} \frac{\log x}{x^2} dx.$$

By noting that we can find easily a primitive of $\frac{1}{x^2}$, we can apply integration by parts. Fill in the blanks.

$$\int_{1}^{2} \frac{\log x}{x^{2}} dx = [a]_{1}^{2} - \int_{1}^{2} b dx.$$

Choose correct functions.



- \bullet $\log x$
- $x \log x 1$
- $-\log x/x \checkmark$
- $\log x/x^3$
- 1/x
- \bullet $-1/x^2$

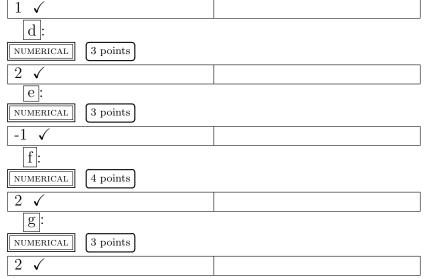
$$\begin{array}{c|c} b : \\ \hline \text{MULTI} & 4 \text{ points} \\ \hline \bullet & -\log x \end{array}$$
 Single

- $x \log x 1$
- $\bullet \log x/x$
- $\log x/x^3$
- 1/x
- $-1/x^2 \checkmark$

Continuing, we get

$$\int_{1}^{2} \frac{\log x}{x^{2}} dx = \frac{\boxed{c}}{\boxed{d}} + \frac{\boxed{e} \log \boxed{f}}{\boxed{g}}.$$





$(8) \mathbf{Q4}$ CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|a|}{|b|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{1}^{3} \frac{\log x}{x^2} dx.$$

By noting that we can find easily a primitive of $\frac{1}{x^2}$, we can apply integration by parts. Fill in the blanks.

$$\int_1^3 \frac{\log x}{x^2} dx = \begin{bmatrix} \mathbf{a} \end{bmatrix}_1^3 - \int_1^3 \boxed{\mathbf{b}} dx.$$

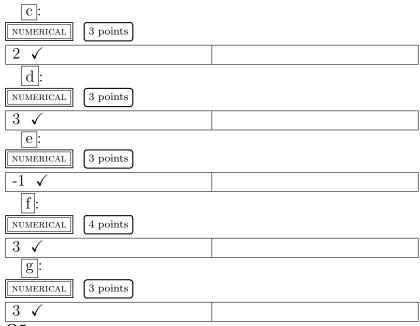
Choose correct functions.

- $\bullet \log x$
- $x \log x 1$
- $-\log x/x \checkmark$ $\log x/x^3$
- 1/*x*
- $-1/x^2$

- \bullet $-\log x$
- $x \log x 1$
- $\bullet \log x/x$
- $\log x/x^3$

Continuing, we get

$$\int_{1}^{3} \frac{\log x}{x^{2}} dx = \frac{\boxed{c}}{\boxed{d}} + \frac{\boxed{e} \log \boxed{f}}{\boxed{g}}.$$



$(9) \overline{\mathbf{Q5}}$ CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral based on definition.

$$\int_0^\infty x \exp(\alpha x^2) dx$$

We split the integral into two parts: $\int_0^1 x \exp(\alpha x^2) dx + \int_1^\infty x \exp(\alpha x^2) dx$ $\int_0^1 x \exp(\alpha x^2) dx$ converges for the following α .

MULTI 1 point Single

- \bullet $\alpha > -\frac{1}{4}$

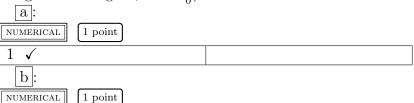
- $\alpha < -1$
- $\alpha > 0$
- α < 0
- $\alpha > 1$
- α < 1
- $\bullet \ \alpha > \frac{1}{4}$ $\bullet \ \alpha < \frac{1}{4}$

• all $\alpha \in \mathbb{R} \checkmark$ $\int_{1}^{\infty} x \exp(\alpha x^{2}) dx \text{ converges for the following } \alpha.$ MULTI 1 point Single

- $\bullet \ \alpha > -\frac{1}{4}$ $\bullet \ \alpha < -\frac{1}{4}$
- $\bullet \ \alpha > -1$
- $\alpha < -1$
- $\alpha > 0$
- α < 0 ✓
- $\alpha > 1$
- α < 1
- $\alpha > \frac{1}{4}$ $\alpha < \frac{1}{4}$
- all $\alpha \in \mathbb{R}$

Take $\alpha = -5$. In this case, $\int_0^\infty x \exp(\alpha x^2) dx = \boxed{\frac{a}{b}}$ (if the

integral is divergent, write $\frac{1}{0}$).



Choose all improper integrals that are convergent.

MULTI 2 points

 $(10) \mathbf{Q5}$

0.10 penalty CLOZE

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|a|}{|b|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral based on definition.

$$\int_0^\infty x \exp(\alpha x^2) dx$$

We split the integral into two parts: $\int_0^1 x \exp(\alpha x^2) dx + \int_1^\infty x \exp(\alpha x^2) dx$ $\int_{1}^{\infty} x \exp(\alpha x^{2}) dx$ converges for the following α .

MULTI 1 point Single

- $\alpha < -\frac{\overline{1}}{4}$
- $\bullet \ \alpha > -1$
- $\alpha < -1$
- $\alpha > 0$
- α < 0 ✓
- $\alpha > 1$
- α < 1
- $\alpha > \frac{1}{4}$ $\alpha < \frac{1}{4}$

• all $\alpha \in \mathbb{R}$ $\int_0^1 x \exp(\alpha x^2) dx$ converges for the following α .

```
MULTI 1 point
                        Single
   \bullet \ \alpha > -\frac{1}{4}
   • \alpha < -\frac{1}{4}
    • \alpha > -1
    • \alpha < -1
    • \alpha > 0

    α < 0</li>

    • \alpha > 1

    α < 1</li>

   • \alpha > \frac{1}{4}
   • \alpha < \frac{1}{4}
   • all \alpha \in \mathbb{R} \checkmark
   Take \alpha = -4. In this case, \int_0^\infty x \exp(\alpha x^2) dx = \boxed{\frac{a}{b}} (if the
integral is divergent, write \frac{1}{0}).
    a :
 NUMERICAL
                 1 point
    b |:
 NUMERICAL
                1 point
   Choose all improper integrals that are convergent.
 MULTI 2 points
```

Total of marks: 228