

2022Call5.

(1) Q1

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Complete the formulae.

$$\cos x = \boxed{a} + \boxed{b}x + \frac{\boxed{c}}{\boxed{d}}x^2 + \boxed{e}x^3 + \frac{\boxed{f}}{\boxed{g}}x^4 + o(x^4) \text{ as } x \rightarrow 0.$$

$\boxed{a}$ :

NUMERICAL

1 point

1 ✓

$\boxed{b}$ :

NUMERICAL

1 point

0 ✓

$\boxed{c}$ :

NUMERICAL

1 point

-1 ✓

$\boxed{d}$ :

NUMERICAL

1 point

2 ✓

$\boxed{e}$ :

NUMERICAL

1 point

0 ✓

$\boxed{f}$ :

NUMERICAL

1 point

1 ✓

$\boxed{g}$ :

NUMERICAL

2 points

24 ✓

$$x^2\sqrt{1+3x} = \boxed{\text{h}} + \boxed{\text{i}}x + \boxed{\text{j}}x^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}x^3 + \frac{\boxed{\text{m}}}{\boxed{\text{n}}}x^4 + o(x^4) \text{ as } x \rightarrow 0.$$

$\boxed{\text{h}}$ :

NUMERICAL

1 point

0 ✓

$\boxed{\text{i}}$ :

NUMERICAL

1 point

0 ✓

$\boxed{\text{j}}$ :

NUMERICAL

1 point

1 ✓

$\boxed{\text{k}}$ :

NUMERICAL

1 point

3 ✓

$\boxed{\text{l}}$ :

NUMERICAL

1 point

2 ✓

$\boxed{\text{m}}$ :

NUMERICAL

1 point

-9 ✓

$\boxed{\text{n}}$ :

NUMERICAL

2 points

8 ✓

$$(\log(1+x^2))^2 = \boxed{\text{o}} + \boxed{\text{p}}x + \boxed{\text{q}}x^2 + \boxed{\text{r}}x^3 + \boxed{\text{s}}x^4 + o(x^4) \text{ as } x \rightarrow 0.$$

$\boxed{\text{o}}$ :

NUMERICAL

1 point

0 ✓

$\boxed{\text{p}}$ :

NUMERICAL

1 point

0 ✓

$\boxed{\text{q}}$ :

NUMERICAL

1 point

0 ✓

r:

NUMERICAL

1 point

0 ✓

s:

NUMERICAL

4 points

1 ✓

For various  $\alpha \in \mathbb{R}$ , study the limit:

$$\lim_{x \rightarrow 0} \frac{2 \cos x - 2 + x^2 \sqrt{1 + 3x + \alpha x^3}}{(\log(1 + x^2))^2}.$$

This limit converges for  $\alpha = \frac{t}{u}$ .

t:

NUMERICAL

4 points

-3 ✓

u:

NUMERICAL

4 points

2 ✓

In that case, the limit is  $\frac{v}{w}$ .

v:

NUMERICAL

8 points

-25 ✓

w:

NUMERICAL

8 points

24 ✓

(2) Q1

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Complete the formulae.

$$\cos x = \boxed{a} + \boxed{b}x + \frac{\boxed{c}}{\boxed{d}}x^2 + \boxed{e}x^3 + \frac{\boxed{f}}{\boxed{g}}x^4 + o(x^4) \text{ as } x \rightarrow 0.$$

a:	
NUMERICAL	1 point
1 ✓	
b:	
NUMERICAL	1 point
0 ✓	
c:	
NUMERICAL	1 point
-1 ✓	
d:	
NUMERICAL	1 point
2 ✓	
e:	
NUMERICAL	1 point
0 ✓	
f:	
NUMERICAL	1 point
1 ✓	
g:	
NUMERICAL	2 points
24 ✓	

$$x^2\sqrt{1+x} = \boxed{h} + \boxed{i}x + \boxed{j}x^2 + \frac{\boxed{k}}{\boxed{l}}x^3 + \frac{\boxed{m}}{\boxed{n}}x^4 + o(x^4) \text{ as } x \rightarrow 0.$$

h:	
NUMERICAL	1 point
0 ✓	
i:	
NUMERICAL	1 point
0 ✓	
j:	
NUMERICAL	1 point
1 ✓	
k:	
NUMERICAL	1 point

1 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">1</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px;">1 point</div>
2 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">m</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px;">1 point</div>
-1 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">n</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px;">2 points</div>
8 ✓	

$$(\log(1 + 2x^2))^2 = \boxed{o} + \boxed{p}x + \boxed{q}x^2 + \boxed{r}x^3 + \boxed{s}x^4 + o(x^4) \text{ as } x \rightarrow 0.$$

<div style="border: 1px solid black; display: inline-block; padding: 2px;">o</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px;">1 point</div>
0 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">p</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px;">1 point</div>
0 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">q</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px;">1 point</div>
0 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">r</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px;">1 point</div>
0 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">s</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px;">4 points</div>
4 ✓	

For various  $\alpha \in \mathbb{R}$ , study the limit:

$$\lim_{x \rightarrow 0} \frac{2 \cos x - 2 + x^2 \sqrt{1+x} + \alpha x^3}{(\log(1 + 2x^2))^2}.$$

This limit converges for  $\alpha = \frac{\boxed{t}}{\boxed{u}}$ .

<div style="border: 1px solid black; display: inline-block; padding: 2px;">t</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px;">4 points</div>
-1 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px;">u</div> :	

NUMERICAL	4 points
2 ✓	
In that case, the limit is $\frac{v}{w}$ .	
v:	
NUMERICAL	8 points
-1 ✓	
w:	
NUMERICAL	8 points
96 ✓	

(3) Q2

CLOZE	0.10 penalty
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If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the

answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign

should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us study the following series  $\sum_{n=0}^{\infty} \frac{2^n - 1}{n!} (x + 1)^{2n}$ , with various  $x$ .

This series makes sense also for  $x \in \mathbb{C}$ . For  $x = i$ , calculate the partial sum  $\sum_{n=0}^2 \frac{2^n - 1}{n!} (x + 1)^{2n} = \boxed{a} + \boxed{b}i$ .

a:	
NUMERICAL	1 point
-6 ✓	
b:	
NUMERICAL	1 point
2 ✓	

In order to discuss the convergence using the ratio test for  $x \in \mathbb{R}$ , we put  $a_n = \frac{2^n - 1}{n!} |x + 1|^{2n}$ . Complete the formula.

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \boxed{c}$$

c:	
NUMERICAL	2 points
0 ✓	

Therefore, by the ratio test, the series converges absolutely for

**MULTI** 4 points Single

- all  $x$ . ✓
- $-3 < x < -1$ .
- $-3 < x < 1$ .
- $-\frac{5}{4} < x < -\frac{3}{4}$ .
- $-\frac{3}{2} < x < -\frac{1}{2}$ .
- $\frac{1}{2} < x < \frac{3}{2}$ .
- $\frac{3}{4} < x < \frac{5}{4}$ .
- $-1 < x < 1$ .
- $-1 < x < 3$ .
- $x = 0$ .
- $1 < x < 3$ .

For the case  $x = -\frac{3}{2}$ , the series

**MULTI** 2 points Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

Calculate the infinite sum.

$$\sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n = \frac{\boxed{d}}{\boxed{e}}.$$

**a**:

**NUMERICAL** 1 point

5 ✓

**b**:

**NUMERICAL** 1 point

2 ✓

(4) **Q2**

**CLOZE** 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{\boxed{a}}{\boxed{b}}$ ) have ambiguity, the negative sign

should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us study the following series  $\sum_{n=0}^{\infty} \frac{2^n - 1}{n!} (x + 1)^{2n}$ , with various  $x$ .

This series makes sense also for  $x \in \mathbb{C}$ . For  $x = -i$ , calculate the partial sum  $\sum_{n=0}^2 \frac{2^n - 1}{n!} (x + 1)^{2n} = \boxed{a} + \boxed{b}i$ .

a:

NUMERICAL

1 point

-6 ✓

b:

NUMERICAL

1 point

-2 ✓

In order to discuss the convergence using the ratio test for  $x \in \mathbb{R}$ , we put  $a_n = \frac{2^n - 1}{n!} |x + 1|^{2n}$ . Complete the formula.

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \boxed{\text{c}}$$

c:

NUMERICAL

2 points

0 ✓

Therefore, by the ratio test, the series converges absolutely for

MULTI

4 points

Single

- all  $x$ . ✓
- $-3 < x < -1$ .
- $-3 < x < 1$ .
- $-\frac{5}{4} < x < -\frac{3}{4}$ .
- $-\frac{3}{2} < x < -\frac{1}{2}$ .
- $\frac{1}{2} < x < \frac{3}{2}$ .
- $\frac{3}{4} < x < \frac{5}{4}$ .
- $-1 < x < 1$ .
- $-1 < x < 3$ .
- $x = 0$ .
- $1 < x < 3$ .

For the case  $x = \frac{3}{2}$ , the series

MULTI

2 points

Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

Calculate the infinite sum.

$$\sum_{n=0}^{\infty} \left(-\frac{1}{3}\right)^n = \frac{\boxed{\text{d}}}{\boxed{\text{e}}}.$$

a:

NUMERICAL

1 point

3 ✓



b:

NUMERICAL

1 point

4 ✓

(5) Q3

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us consider the following function

$$f(x) = \frac{(x-1)^3}{x(x+1)}.$$

The function  $f(x)$  is not defined on the whole real line  $\mathbb{R}$ . Choose all the points that are **not** in the natural domain of  $f(x)$ .

MULTI

4 points

Single

- $-2$  (−100%)
- $-1$  ✓
- $-\frac{1}{2}$  (−100%)
- $0$  ✓
- $\frac{1}{2}$  (−100%)
- $1$  (−100%)
- $2$  (−100%)

Choose all asymptotes of  $f(x)$ .

MULTI

4 points

Single

- $y = -1$  (−100%)
- $y = -\frac{1}{2}$  (−100%)
- $y = 0$  (−100%)
- $y = \frac{1}{2}$  (−100%)
- $y = 2$  (−100%)
- $x = -2$  (−100%)
- $x = -1$  ✓
- $x = -\frac{1}{2}$  (−100%)
- $x = 0$  ✓
- $x = \frac{1}{2}$  (−100%)
- $x = 1$  (−100%)
- $x = 2$  (−100%)

- $y = x - 4$  ✓
- $y = x$  (-100%)
- $y = x + 4$  (-100%)

One has

$$f'(2) = \frac{\boxed{a}}{\boxed{b}}.$$

**a**:

NUMERICAL

4 points

13 ✓

**b**:

NUMERICAL

4 points

36 ✓

The function  $f(x)$  has **c** stationary point(s) in the domain

**c**:

NUMERICAL

4 points

3 ✓

Choose the behaviour of  $f(x)$  in the interval  $[3, 4]$ .

MULTI

4 points

Single

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

### (6) Q3

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the

answer boxes (such as  $\frac{\boxed{a}}{\boxed{b}}$ ) have ambiguity, the negative sign

should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us consider the following function

$$f(x) = \frac{(x+1)^3}{x(x-1)}.$$

The function  $f(x)$  is not defined on the whole real line  $\mathbb{R}$ . Choose all the points that are **not** in the natural domain of  $f(x)$ .

MULTI

4 points

Single

- -2 (-100%)
- -1 (-100%)

- $-\frac{1}{2}$  (−100%)
- 0 ✓
- $\frac{1}{2}$  (−100%)
- 1 ✓
- 2 (−100%)

Choose all asymptotes of  $f(x)$ .

☐ MULTI ☐ 4 points ☐ Single

- $y = -1$  (−100%)
- $y = -\frac{1}{2}$  (−100%)
- $y = 0$  (−100%)
- $y = \frac{1}{2}$  (−100%)
- $y = 2$  (−100%)
- $x = -2$  (−100%)
- $x = -1$  (−100%)
- $x = -\frac{1}{2}$  (−100%)
- $x = 0$  ✓
- $x = \frac{1}{2}$  (−100%)
- $x = 1$  ✓
- $x = 2$  (−100%)
- $y = x - 4$  (−100%)
- $y = x$  (−100%)
- $y = x + 4$  ✓

One has

$$f'(2) = \frac{\boxed{a}}{\boxed{b}}.$$

:

☐ NUMERICAL ☐ 4 points

✓

:

☐ NUMERICAL ☐ 4 points

✓

The function  $f(x)$  has  stationary point(s) in the domain

:

☐ NUMERICAL ☐ 4 points

✓

Choose the behaviour of  $f(x)$  in the interval  $[3, 4]$ .

☐ MULTI ☐ 4 points ☐ Single

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

## (7) Q4

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{\boxed{a}}{\boxed{b}}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us calculate the following integral.

$$\int_1^2 \frac{\log x}{x^2} dx.$$

By noting that we can find easily a primitive of  $\frac{1}{x^2}$ , we can apply integration by parts. Fill in the blanks.

$$\int_1^2 \frac{\log x}{x^2} dx = \boxed{a}_1^2 - \int_1^2 \boxed{b} dx.$$

Choose correct functions.

a:

MULTI

4 points

Single

- $-\log x$
- $x \log x - 1$
- $-\log x/x$  ✓
- $\log x/x^3$
- $1/x$
- $-1/x^2$

b:

MULTI

4 points

Single

- $-\log x$
- $x \log x - 1$
- $-\log x/x$
- $\log x/x^3$
- $1/x$
- $-1/x^2$  ✓

Continuing, we get

$$\int_1^2 \frac{\log x}{x^2} dx = \frac{\boxed{c}}{\boxed{d}} + \frac{\boxed{e} \log \boxed{f}}{\boxed{g}}.$$

c:

NUMERICAL

3 points

1 ✓	
d:	
<div style="border: 1px solid black; padding: 2px; display: inline-block;">NUMERICAL</div>	<div style="border: 1px solid black; padding: 2px; display: inline-block;">3 points</div>
2 ✓	
e:	
<div style="border: 1px solid black; padding: 2px; display: inline-block;">NUMERICAL</div>	<div style="border: 1px solid black; padding: 2px; display: inline-block;">3 points</div>
-1 ✓	
f:	
<div style="border: 1px solid black; padding: 2px; display: inline-block;">NUMERICAL</div>	<div style="border: 1px solid black; padding: 2px; display: inline-block;">4 points</div>
2 ✓	
g:	
<div style="border: 1px solid black; padding: 2px; display: inline-block;">NUMERICAL</div>	<div style="border: 1px solid black; padding: 2px; display: inline-block;">3 points</div>
2 ✓	

(8) Q4

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{\boxed{a}}{\boxed{b}}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us calculate the following integral.

$$\int_1^3 \frac{\log x}{x^2} dx.$$

By noting that we can find easily a primitive of  $\frac{1}{x^2}$ , we can apply integration by parts. Fill in the blanks.

$$\int_1^3 \frac{\log x}{x^2} dx = \boxed{a}_1^3 - \int_1^3 \boxed{b} dx.$$

Choose correct functions.

a:

MULTI

4 points

Single

- $-\log x$
- $x \log x - 1$
- $-\log x/x$  ✓
- $\log x/x^3$
- $1/x$
- $-1/x^2$

:

- $-\log x$
- $x \log x - 1$
- $-\log x/x$
- $\log x/x^3$
- $1/x$
- $-1/x^2$  ✓

Continuing, we get

$$\int_1^3 \frac{\log x}{x^2} dx = \frac{\text{c}}{\text{d}} + \frac{\text{e} \log \text{f}}{\text{g}}.$$

:

✓

:

✓

:

✓

:

✓

:

✓

(9) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{\text{a}}{\text{b}}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us calculate the following improper integral based on definition.

$$\int_0^{\infty} x \exp(\alpha x^2) dx$$

We split the integral into two parts:  $\int_0^1 x \exp(\alpha x^2) dx + \int_1^{\infty} x \exp(\alpha x^2) dx$   
 $\int_0^1 x \exp(\alpha x^2) dx$  converges for the following  $\alpha$ .

☐ MULTI ☐ 1 point ☐ Single

- $\alpha > -\frac{1}{4}$
- $\alpha < -\frac{1}{4}$
- $\alpha > -1$
- $\alpha < -1$
- $\alpha > 0$
- $\alpha < 0$
- $\alpha > 1$
- $\alpha < 1$
- $\alpha > \frac{1}{4}$
- $\alpha < \frac{1}{4}$
- all  $\alpha \in \mathbb{R}$  ✓

$\int_1^{\infty} x \exp(\alpha x^2) dx$  converges for the following  $\alpha$ .

☐ MULTI ☐ 1 point ☐ Single

- $\alpha > -\frac{1}{4}$
- $\alpha < -\frac{1}{4}$
- $\alpha > -1$
- $\alpha < -1$
- $\alpha > 0$
- $\alpha < 0$  ✓
- $\alpha > 1$
- $\alpha < 1$
- $\alpha > \frac{1}{4}$
- $\alpha < \frac{1}{4}$
- all  $\alpha \in \mathbb{R}$

Take  $\alpha = -5$ . In this case,  $\int_0^{\infty} x \exp(\alpha x^2) dx = \frac{a}{b}$  (if the

integral is divergent, write  $\frac{1}{0}$ ).

a:

☐ NUMERICAL ☐ 1 point

1 ✓

b:

☐ NUMERICAL ☐ 1 point

10 ✓	
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Choose all improper integrals that are convergent.

MULTI	2 points	Single
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- $\int_0^\infty \exp(-x)/x dx$  (-100%)
- $\int_0^\infty \exp(x)/x dx$  (-100%)
- $\int_1^\infty \exp(-x)/x^2 dx$  ✓
- $\int_1^\infty \exp(x)/x^2 dx$  (-100%)
- $\int_0^\infty \log x/x dx$  (-100%)
- $\int_0^\infty \log x/x^2 dx$  (-100%)
- $\int_1^\infty \log x/x dx$  (-100%)
- $\int_1^\infty \log x/x^2 dx$  ✓

(10) Q5

CLOZE	0.10 penalty
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If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the

answer boxes (such as  $\frac{\boxed{a}}{\boxed{b}}$ ) have ambiguity, the negative sign

should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us calculate the following improper integral based on definition.

$$\int_0^\infty x \exp(\alpha x^2) dx$$

We split the integral into two parts:  $\int_0^1 x \exp(\alpha x^2) dx + \int_1^\infty x \exp(\alpha x^2) dx$ .  $\int_1^\infty x \exp(\alpha x^2) dx$  converges for the following  $\alpha$ .

MULTI	1 point	Single
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- $\alpha > -\frac{1}{4}$
- $\alpha < -\frac{1}{4}$
- $\alpha > -1$
- $\alpha < -1$
- $\alpha > 0$
- $\alpha < 0$  ✓
- $\alpha > 1$
- $\alpha < 1$
- $\alpha > \frac{1}{4}$
- $\alpha < \frac{1}{4}$
- all  $\alpha \in \mathbb{R}$

$\int_0^1 x \exp(\alpha x^2) dx$  converges for the following  $\alpha$ .



☐ MULTI ☐ 1 point ☐ Single

- $\alpha > -\frac{1}{4}$
- $\alpha < -\frac{1}{4}$
- $\alpha > -1$
- $\alpha < -1$
- $\alpha > 0$
- $\alpha < 0$
- $\alpha > 1$
- $\alpha < 1$
- $\alpha > \frac{1}{4}$
- $\alpha < \frac{1}{4}$
- all  $\alpha \in \mathbb{R}$  ✓

Take  $\alpha = -4$ . In this case,  $\int_0^\infty x \exp(\alpha x^2) dx = \frac{a}{b}$  (if the integral is divergent, write  $\frac{1}{0}$ ).

☐ a:

☐ NUMERICAL ☐ 1 point

1 ✓

☐ b:

☐ NUMERICAL ☐ 1 point

8 ✓

Choose all improper integrals that are convergent.

☐ MULTI ☐ 2 points ☐ Single

- $\int_0^\infty \log x / x dx$  (−100%)
- $\int_0^\infty \log x / x^2 dx$  (−100%)
- $\int_1^\infty \log x / x dx$  (−100%)
- $\int_1^\infty \log x / x^2 dx$  ✓
- $\int_0^\infty \exp(-x) / x dx$  (−100%)
- $\int_0^\infty \exp(x) / x dx$  (−100%)
- $\int_1^\infty \exp(-x) / x^2 dx$  ✓
- $\int_1^\infty \exp(x) / x^2 dx$  (−100%)

Total of marks: 228