

2022Call4.

(1) Q1

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Complete the formulae.

$$\log(x) = \boxed{a} + \boxed{b}(x-1) + \frac{\boxed{c}}{\boxed{d}}(x-1)^2 + \frac{\boxed{e}}{\boxed{f}}(x-1)^3 + o((x-1)^3) \text{ as } x \rightarrow 1.$$

$\boxed{a}$ :

NUMERICAL

1 point

0 ✓

$\boxed{b}$ :

NUMERICAL

1 point

1 ✓

$\boxed{c}$ :

NUMERICAL

1 point

-1 ✓

$\boxed{d}$ :

NUMERICAL

1 point

2 ✓

$\boxed{e}$ :

NUMERICAL

1 point

1 ✓

$\boxed{f}$ :

NUMERICAL

1 point

3 ✓

$$(x-1)x^{\frac{1}{3}} = \boxed{g} + \boxed{h}(x-1) + \frac{\boxed{i}}{\boxed{j}}(x-1)^2 + \frac{\boxed{k}}{\boxed{l}}(x-1)^3 + o((x-1)^3) \text{ as } x \rightarrow 1.$$

$\boxed{g}$ :

NUMERICAL

1 point

0 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">h</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">1 point</div>
1 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">i</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">1 point</div>
1 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">j</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">1 point</div>
3 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">k</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">1 point</div>
-1 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">l</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">1 point</div>
9 ✓	

$$(x-1)^2 \sin(x-1) = \boxed{m} + \boxed{n}(x-1) + \boxed{o}(x-1)^2 + \boxed{p}(x-1)^3 + o((x-1)^3) \text{ as } x \rightarrow 1.$$

<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">m</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">1 point</div>
0 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">n</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">1 point</div>
0 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">o</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">1 point</div>
0 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">p</div> :	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">NUMERICAL</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">3 points</div>
1 ✓	

For various  $\alpha, \beta \in \mathbb{R}$ , study the limit:

$$\lim_{x \rightarrow 1} \frac{\log(x) + (x-1)x^{\frac{1}{3}} + \alpha(x-1) + \beta(x-1)^2}{(x-1)^2 \sin(x-1)}.$$

This limit converges for  $\alpha = \boxed{q}, \beta = \frac{\boxed{r}}{\boxed{s}}$ .

q:

NUMERICAL

6 points

-2 ✓

r:

NUMERICAL

3 points

1 ✓

s:

NUMERICAL

3 points

6 ✓

In that case, the limit is  $\frac{t}{u}$ .

t:

NUMERICAL

3 points

2 ✓

u:

NUMERICAL

3 points

9 ✓

(2) Q1

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Complete the formulae.

$$\log(x) = \boxed{a} + \boxed{b}(x-1) + \frac{\boxed{c}}{\boxed{d}}(x-1)^2 + \frac{\boxed{e}}{\boxed{f}}(x-1)^3 + o((x-1)^3) \text{ as } x \rightarrow 1.$$

a:

NUMERICAL

1 point

0 ✓

b:

NUMERICAL

1 point

1 ✓

c:

NUMERICAL

1 point

-1 ✓	
d:	
NUMERICAL	1 point
2 ✓	
e:	
NUMERICAL	1 point
1 ✓	
f:	
NUMERICAL	1 point
3 ✓	

$$(1-x)x^{\frac{1}{3}} = \boxed{g} + \boxed{h}(x-1) + \frac{\boxed{i}}{\boxed{j}}(x-1)^2 + \frac{\boxed{k}}{\boxed{l}}(x-1)^3 + o((x-1)^3) \text{ as } x \rightarrow 1.$$

g:	
NUMERICAL	1 point
0 ✓	
h:	
NUMERICAL	1 point
-1 ✓	
i:	
NUMERICAL	1 point
-1 ✓	
j:	
NUMERICAL	1 point
3 ✓	
k:	
NUMERICAL	1 point
1 ✓	
l:	
NUMERICAL	1 point
9 ✓	

$$(x-1)^2 \sin(3(x-1)) = \boxed{m} + \boxed{n}(x-1) + \boxed{o}(x-1)^2 + \boxed{p}(x-1)^3 + o((x-1)^3) \text{ as } x \rightarrow 1.$$

m:	
NUMERICAL	1 point

0 ✓	
-----	--

n:	
----	--

NUMERICAL	1 point
-----------	---------

0 ✓	
-----	--

o:	
----	--

NUMERICAL	1 point
-----------	---------

0 ✓	
-----	--

p:	
----	--

NUMERICAL	3 points
-----------	----------

3 ✓	
-----	--

For various  $\alpha, \beta \in \mathbb{R}$ , study the limit:

$$\lim_{x \rightarrow 1} \frac{\log(x) + (1-x)x^{\frac{1}{3}} + \alpha(x-1) + \beta(x-1)^2}{(x-1)^2 \sin(3(x-1))}.$$

This limit converges for  $\alpha = \boxed{q}, \beta = \frac{\boxed{r}}{\boxed{s}}$ .

q:	
----	--

NUMERICAL	6 points
-----------	----------

0 ✓	
-----	--

r:	
----	--

NUMERICAL	3 points
-----------	----------

5 ✓	
-----	--

s:	
----	--

NUMERICAL	3 points
-----------	----------

6 ✓	
-----	--

In that case, the limit is  $\frac{\boxed{t}}{\boxed{u}}$ .

t:	
----	--

NUMERICAL	3 points
-----------	----------

4 ✓	
-----	--

u:	
----	--

NUMERICAL	3 points
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27 ✓	
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(3) **Q2**

CLOZE	0.10 penalty
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If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the

answer boxes (such as  $\frac{\boxed{a}}{\boxed{b}}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us study the following series  $\sum_{n=0}^{\infty} \frac{4^n - 1}{n^2 + 2} (x + 1)^{2n}$ , with various  $x$ .

This series makes sense also for  $x \in \mathbb{C}$ . For  $x = i$ , calculate the partial sum  $\sum_{n=0}^2 \frac{4^n - 1}{n^2 + 2} (x + 1)^{2n} = \boxed{a} + \boxed{b}i$ .

$\boxed{a}$ :

NUMERICAL	1 point
-10 ✓	

$\boxed{b}$ :

NUMERICAL	1 point
2 ✓	

In order to discuss the convergence using the root test for  $x \in \mathbb{R}$ , we put  $a_n = \frac{4^n - 1}{n^2 + 2} |x + 1|^{2n}$ . Complete the formula.

$$\lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}} = \boxed{c}|x + \boxed{d}|\boxed{e}.$$

$\boxed{c}$ :

NUMERICAL	2 points
4 ✓	

$\boxed{d}$ :

NUMERICAL	1 point
1 ✓	

$\boxed{e}$ :

NUMERICAL	1 point
2 ✓	

Therefore, by the root test, the series converges absolutely for

MULTI	2 points	Single
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- all  $x$ .
- $-3 < x < -1$ .
- $-3 < x < 1$ .
- $-\frac{5}{3} < x < -\frac{1}{3}$ .
- $-\frac{3}{2} < x < -\frac{1}{2}$ . ✓
- $\frac{1}{3} < x < \frac{5}{3}$ .
- $\frac{1}{2} < x < \frac{3}{2}$ .
- $-1 < x < 1$ .

- $-1 < x < 3$ .
- $x = 0$ .
- $1 < x < 3$ .

For the case  $x = -\frac{3}{2}$ , the series

MULTI 2 points Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case  $x = 1$ , the series

MULTI 2 points Single

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(4) Q2

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{\boxed{a}}{\boxed{b}}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us study the following series  $\sum_{n=0}^{\infty} \frac{4^n - 1}{n^2 + 2} (x - 1)^{2n}$ , with various  $x$ .

This series makes sense also for  $x \in \mathbb{C}$ . For  $x = i$ , calculate the partial sum  $\sum_{n=0}^2 \frac{4^n - 1}{n^2 + 2} (x - 1)^{2n} = \boxed{a} + \boxed{b}i$ .

$\boxed{a}$ :

NUMERICAL 1 point

-10 ✓

$\boxed{b}$ :

NUMERICAL 1 point

-2 ✓

In order to discuss the convergence using the root test for  $x \in \mathbb{R}$ , we put  $a_n = \frac{4^n - 1}{n^2 + 2} |x - 1|^{2n}$ . Complete the formula.

$$\lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}} = \boxed{c}|x + \boxed{d}|^{\boxed{e}}.$$

$\boxed{c}$ :

NUMERICAL 2 points

4 ✓

d:

NUMERICAL

1 point

-1 ✓

e:

NUMERICAL

1 point

2 ✓

Therefore, by the root test, the series converges absolutely for

MULTI

2 points

Single

- all  $x$ .
- $-3 < x < -1$ .
- $-3 < x < 1$ .
- $-\frac{5}{3} < x < -\frac{1}{3}$ .
- $-\frac{3}{2} < x < -\frac{1}{2}$ .
- $\frac{1}{3} < x < \frac{5}{3}$ .
- $\frac{1}{2} < x < \frac{3}{2}$ . ✓
- $-1 < x < 1$ .
- $-1 < x < 3$ .
- $x = 0$ .
- $1 < x < 3$ .

For the case  $x = -\frac{3}{2}$ , the series

MULTI

2 points

Single

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case  $x = 1$ , the series

MULTI

2 points

Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(5) Q3

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .



Let us consider the following function

$$f(x) = \log \left( \frac{x}{1 + \log(x)} \right).$$

The function  $f(x)$  is not defined on the whole real line  $\mathbb{R}$ . Choose all the points that are **not** in the natural domain of  $f(x)$ .

☐ MULTI ☐ 4 points ☐ Single

- $-e$  ✓
- $-1$  ✓
- $-\frac{1}{e}$  ✓
- $0$  ✓
- $\frac{1}{e}$  ✓
- $1$  (−100%)
- $e$  (−100%)

Choose all asymptotes of  $f(x)$ .

☐ MULTI ☐ 4 points ☐ Single

- $y = -\pi$  (−100%)
- $y = -\frac{\pi}{2}$  (−100%)
- $y = -\frac{\pi}{4}$  (−100%)
- $y = 0$  (−100%)
- $y = \frac{\pi}{4}$  (−100%)
- $y = \frac{\pi}{2}$  (−100%)
- $y = \pi$  (−100%)
- $x = -1$  (−100%)
- $x = 0$  (−100%)
- $x = 1/e$  ✓
- $x = 1$  (−100%)
- $x = e$  (−100%)
- $y = x$  (−100%)
- $y = -x$  (−100%)
- $y = \pi x$  (−100%)

One has

$$f'(e) = \frac{\boxed{a}}{\boxed{b}e}.$$

☐ a:

☐ NUMERICAL ☐ 4 points

1 ✓

☐ b:

☐ NUMERICAL ☐ 4 points

2 ✓	
-----	--

The function  $f(x)$  has  stationary point(s) in the domain

:

NUMERICAL	4 points
-----------	----------

1 ✓	
-----	--

Choose the behaviour of  $f(x)$  in the interval  $(\frac{1}{2}, 2)$ .

MULTI	4 points	Single
-------	----------	--------

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(6) Q3

CLOZE	0.10 penalty
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If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us consider the following function

$$f(x) = \log \left( \frac{x}{2 + \log(x)} \right).$$

The function  $f(x)$  is not defined on the whole real line  $\mathbb{R}$ . Choose all the points that are **not** in the natural domain of  $f(x)$ .

MULTI	4 points	Single
-------	----------	--------

- $-e$  ✓
- $-1$  ✓
- $-1/e$  ✓
- $-1/e^2$  ✓
- $0$  ✓
- $1/e^2$  ✓
- $1/e$  ✓
- $1$  (−100%)
- $e$  (−100%)

Choose all asymptotes of  $f(x)$ .

MULTI	4 points	Single
-------	----------	--------

- $y = -\pi$  (−100%)
- $y = -\frac{\pi}{2}$  (−100%)
- $y = -\frac{\pi}{4}$  (−100%)

- $y = 0$  (−100%)
- $y = \frac{\pi}{4}$  (−100%)
- $y = \frac{\pi}{2}$  (−100%)
- $y = \pi$  (−100%)
- $x = -1$  (−100%)
- $x = 0$  (−100%)
- $x = 1/e^2$  ✓
- $x = 1/e$  (−100%)
- $x = 1$  (−100%)
- $x = e$  (−100%)
- $x = e^2$  (−100%)
- $y = x$  (−100%)
- $y = -x$  (−100%)
- $y = \pi x$  (−100%)

One has

$$f'(e) = \frac{\boxed{a}}{\boxed{b}e}.$$

**a**:

NUMERICAL

4 points

2 ✓

**b**:

NUMERICAL

4 points

3 ✓

The function  $f(x)$  has **c** stationary point(s) in the domain

**c**:

NUMERICAL

4 points

1 ✓

Choose the behaviour of  $f(x)$  in the interval  $(2, 3)$ .

MULTI

4 points

Single

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(7) **Q4**

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{\boxed{a}}{\boxed{b}}$ ) have ambiguity, the negative sign

should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us calculate the following integral.

$$\int_0^1 x^2 \exp(-x) dx.$$

It is easy to find a primitive of  $\exp(-x)$ . By integration by parts, one has

$$\int_0^1 x^2 \exp(-x) = \left[ \boxed{a} \right]_0^1 - \int_0^1 \boxed{b} dx.$$

Choose the function  $\boxed{a}$ ,

☒ MULTI ☐ 2 points ☐ Single

- $\exp(-x)$
- $2x \exp(-x)$
- $x^2 \exp(-x)$
- $-\exp(-x)$
- $-2x \exp(-x)$
- $-x^2 \exp(-x)$  ✓

and  $\boxed{b}$ .

☒ MULTI ☐ 2 points ☐ Single

- $\exp(-x)$
- $2x \exp(-x)$
- $x^2 \exp(-x)$
- $-\exp(-x)$
- $-2x \exp(-x)$  ✓
- $-x^2 \exp(-x)$

Continuing, one has  $\int_0^1 x^2 \exp(-x) = \boxed{c} + \boxed{d} e^{-1}$ .

$\boxed{c}$ :

☒ NUMERICAL ☐ 2 points

2 ✓

$\boxed{d}$ :

☒ NUMERICAL ☐ 2 points

-5 ✓

Instead,

$$\int_0^1 x e^{-x^2} dx = \frac{\boxed{e}}{\boxed{f}} + \frac{\boxed{g}}{\boxed{h}} e^{-1}$$

, one can calculate this by substitution.

$\boxed{e}$ :

☒ NUMERICAL ☐ 1 point

1 ✓	
f:	
NUMERICAL	1 point
2 ✓	
g:	
NUMERICAL	1 point
-1 ✓	
h:	
NUMERICAL	1 point
2 ✓	

(8) Q4

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{\boxed{a}}{\boxed{b}}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us calculate the following integral.

$$\int_0^2 x^2 \exp(-x) dx.$$

It is easy to find a primitive of  $\exp(-x)$ . By integration by parts, one has

$$\int_0^2 x^2 \exp(-x) = \left[ \boxed{a} \right]_0^2 - \int_0^2 \boxed{b} dx.$$

Choose the function  $\boxed{a}$ ,

MULTI

2 points

Single

- $\exp(-x)$
- $2x \exp(-x)$
- $x^2 \exp(-x)$
- $-\exp(-x)$
- $-2x \exp(-x)$
- $-x^2 \exp(-x)$  ✓

and  $\boxed{b}$ .

MULTI

2 points

Single

- $\exp(-x)$
- $2x \exp(-x)$

- $x^2 \exp(-x)$
- $-\exp(-x)$
- $-2x \exp(-x)$  ✓
- $-x^2 \exp(-x)$

Continuing, one has  $\int_0^2 x^2 \exp(-x) = \boxed{\text{c}} e^{-2} + \boxed{\text{d}}$ .

$\boxed{\text{c}}$ :

NUMERICAL 2 points

-10 ✓

$\boxed{\text{d}}$ :

NUMERICAL 2 points

2 ✓

Instead,

$$\int_0^1 x e^{-x^2} dx = \frac{\boxed{\text{e}}}{\boxed{\text{f}}} e^{-1} + \frac{\boxed{\text{g}}}{\boxed{\text{h}}}$$

, one can calculate this by substitution.

$\boxed{\text{e}}$ :

NUMERICAL 1 point

-1 ✓

$\boxed{\text{f}}$ :

NUMERICAL 1 point

2 ✓

$\boxed{\text{g}}$ :

NUMERICAL 1 point

1 ✓

$\boxed{\text{h}}$ :

NUMERICAL 1 point

2 ✓

(9) Q5

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us calculate the following improper integral with various  $\alpha, \beta \in \mathbb{R}$ , as follows

$$\int_{\beta}^{\infty} x^3 \exp(\alpha x^4) dx$$

a  $\exp(\alpha x^4)$  is a primitive of  $x^3 \exp(\alpha x^4)$  if  $\alpha \neq 0$ .  
 b  $\alpha$   
 a:

NUMERICAL 2 points

1 ✓

b:

NUMERICAL 2 points

4 ✓

The above improper integral converges for the following  $\alpha, \beta$ .

MULTI 8 points Single Shuffle

- $\alpha > -\frac{1}{4}$
- $\alpha < -\frac{1}{4}$
- $\alpha > -1$
- $\alpha < -1$
- $\alpha > 0$
- $\alpha < 0$  ✓
- $\alpha > 1$
- $\alpha < 1$
- $\alpha > \frac{1}{4}$
- $\alpha < \frac{1}{4}$
- all  $\alpha \in \mathbb{R}$

and

MULTI 4 points Single Shuffle

- $\beta > -e$
- $\beta < -e$
- $\beta > -1$
- $\beta < -1$
- $\beta > 0$
- $\beta < 0$
- $\beta > 1$
- $\beta < 1$
- $\beta > e$
- $\beta < e$
- all  $\beta \in \mathbb{R}$  ✓

Take  $\alpha, \beta$  for which the integral converges. In such a case, the value of the integral is  $-\frac{\boxed{\text{c}}}{\boxed{\text{d}}^\alpha} \exp(\alpha\beta\boxed{\text{e}})$ .

$\boxed{\text{c}}$ :

NUMERICAL

1 point

1 ✓

$\boxed{\text{d}}$ :

NUMERICAL

1 point

4 ✓

$\boxed{\text{e}}$ :

NUMERICAL

2 points

4 ✓

For  $\alpha$  as above, si ottiene

$$\int_{-\infty}^{\infty} x^3 \exp(\alpha x^4) dx = \boxed{\text{f}}.$$

$\boxed{\text{f}}$ :

NUMERICAL

4 points

0 ✓

(10) Q5

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us calculate the following improper integral with various  $\alpha, \beta \in \mathbb{R}$ , as follows

$$\int_{\beta}^{\infty} x^5 \exp(\alpha x^6) dx$$

$\boxed{\text{a}}$

$\boxed{\text{b}}_\alpha$

$\boxed{\text{a}}$ :

NUMERICAL

2 points

1 ✓

$\boxed{\text{b}}$ :



NUMERICAL 2 points

6 ✓

The above improper integral converges for the following  $\alpha, \beta$ .

MULTI 8 points Single Shuffle

- $\alpha > -\frac{1}{4}$
- $\alpha < -\frac{1}{4}$
- $\alpha > -1$
- $\alpha < -1$
- $\alpha > 0$
- $\alpha < 0$  ✓
- $\alpha > 1$
- $\alpha < 1$
- $\alpha > \frac{1}{4}$
- $\alpha < \frac{1}{4}$
- all  $\alpha \in \mathbb{R}$

and

MULTI 4 points Single Shuffle

- $\beta > -e$
- $\beta < -e$
- $\beta > -1$
- $\beta < -1$
- $\beta > 0$
- $\beta < 0$
- $\beta > 1$
- $\beta < 1$
- $\beta > e$
- $\beta < e$
- all  $\beta \in \mathbb{R}$  ✓

Take  $\alpha, \beta$  for which the integral converges. In such a case, the value of the integral is  $-\frac{c}{d} \exp(\alpha \beta \frac{e}{d})$ .

c:

NUMERICAL 1 point

1 ✓

d:

NUMERICAL 1 point

6 ✓

e:

NUMERICAL 2 points

6 ✓

For  $\alpha$  as above, si ottiene

$$\int_{-\infty}^{\infty} x^3 \exp(\alpha x^4) dx = \boxed{f}.$$

$\boxed{f}$ :

NUMERICAL

4 points

0 ✓	
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*Total of marks: 216*