2022Call4.

(1) **Q1**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\log(x) = \boxed{a} + \boxed{b}(x-1) + \frac{\boxed{c}}{\boxed{d}}(x-1)^2 + \frac{\boxed{e}}{\boxed{f}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

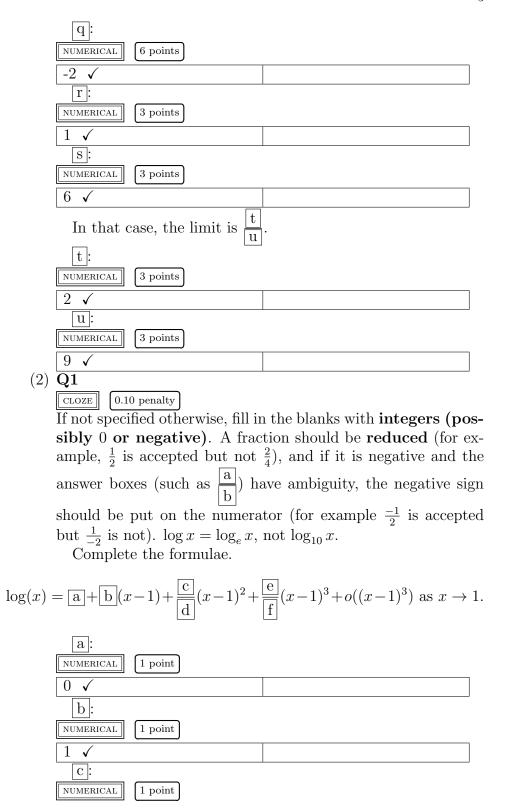
| a: | |
|-------------------|--|
| NUMERICAL 1 point | |
| 0 ✓ | |
| b: | |
| NUMERICAL 1 point | |
| 1 ✓ | |
| <u>c</u> : | |
| NUMERICAL 1 point | |
| -1 ✓ | |
| d: | |
| NUMERICAL 1 point | |
| 2 🗸 | |
| e: | |
| NUMERICAL 1 point | |
| 1 ✓ | |
| <u>f</u> : | |
| NUMERICAL 1 point | |
| 3 / | |

$$(x-1)x^{\frac{1}{3}} = \boxed{g} + \boxed{h}(x-1) + \boxed{\frac{i}{j}}(x-1)^2 + \boxed{\frac{k}{l}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

g:

NUMERICAL 1 point

| | 0 🗸 | | |
|-----------|--|---|----------------|
| | h : | | |
| | NUMERICAL 1 point | | |
| | 1 🗸 | | |
| | i: | | |
| | NUMERICAL 1 point | | |
| | | | |
| | 1 1 | | |
| | <u>j</u> : | | |
| | NUMERICAL 1 point | | |
| | 3 ✓ | | |
| | k: | | |
| | NUMERICAL 1 point | | |
| | <u> </u> | | |
| | 1: | | |
| | | | |
| | NUMERICAL 1 point | | |
| | 9 🗸 | | |
| | | | |
| $(x-1)^2$ | $\sin(x-1) = \boxed{\mathbf{m}} + \boxed{\mathbf{n}}(x-1) + \boxed{\mathbf{o}}$ | $(x-1)^2 + p(x-1)^3 + o((x-1)^3)$ | as $x \to 1$. |
| | | | |
| | m: NUMERICAL 1 point | | |
| | | | |
| | 0 🗸 | | |
| | <u>n</u> : | | |
| | NUMERICAL 1 point | | |
| | 0 🗸 | | |
| | <u>o</u> : | | |
| | NUMERICAL 1 point | | |
| | 0 🗸 | | |
| | p: | | |
| | NUMERICAL 3 points | | |
| | 1 🗸 | | |
| | For various $\alpha, \beta \in \mathbb{R}$, study | the limit: | |
| | ,, , | | |
| | $\log(x) + (x-1)x^{\frac{1}{3}} + \epsilon$ | $\alpha(r-1) + \beta(r-1)^2$ | |
| | $\lim_{x \to 1} \frac{\log(x) + (x-1)x^{\frac{1}{3}} + \epsilon}{(x-1)^2 \sin^2(x-1)}$ | $\frac{\alpha(\alpha-1)+\beta(\alpha-1)}{\alpha(\alpha-1)}$. | |
| | , | | |
| | This limit converges for $\alpha =$ | $= [q], \beta = \boxed{r}.$ | |
| | | S | |



```
d:
            NUMERICAL
                             1 point
             2 √
               e :
             NUMERICAL
                             1 point
             \overline{1} \checkmark
                f :
             NUMERICAL
                             1 point
             3 ✓
(1-x)x^{\frac{1}{3}} = \boxed{g} + \boxed{h}(x-1) + \boxed{\frac{1}{|1|}}(x-1)^2 + \boxed{\frac{k}{|1|}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.
               g:
                             1 point
             NUMERICAL
             0 🗸
               h :
            NUMERICAL
                             1 point
             <u>-1 √</u>
               i:
                             1 point
             NUMERICAL
             <u>-</u>1 ✓
               j :
             NUMERICAL
                             1 point
             3 ✓
               | k |:
            NUMERICAL
                             1 point
             1 √
               1:
             NUMERICAL
                             1 point
             9 🗸
(x-1)^2 \sin(3(x-1)) = \boxed{\mathbf{m}} + \boxed{\mathbf{n}} (x-1) + \boxed{\mathbf{o}} (x-1)^2 + \boxed{\mathbf{p}} (x-1)^3 + o((x-1)^3) \text{ as } x \to 1.
               m:
            NUMERICAL 1 point
```

| n: |
|--|
| NUMERICAL 1 point |
| 0 🗸 |
| 0: |
| NUMERICAL 1 point |
| 0 🗸 |
| <u>p</u> : |
| NUMERICAL 3 points |
| 3 ✓ |
| For various $\alpha, \beta \in \mathbb{R}$, study the limit: |
| |
| $\log(x) + (1-x)x^{\frac{1}{3}} + \alpha(x-1) + \beta(x-1)^2$ |
| $\lim_{x \to 1} \frac{\log(x) + (1-x)x^{\frac{1}{3}} + \alpha(x-1) + \beta(x-1)^2}{(x-1)^2 \sin(3(x-1))}.$ |
| |
| This limit converges for $\alpha = \boxed{q}, \beta = \boxed{\frac{r}{s}}$. |
| <u>q</u> : |
| NUMERICAL 6 points |
| 0 ✓ |
| <u>r</u> : |
| NUMERICAL 3 points |
| 5 ✓ |
| s: |
| NUMERICAL 3 points |
| 6 🗸 |
| In that are the limit is t |
| In that case, the limit is u . |
| t: |
| NUMERICAL 3 points |
| 4 🗸 |
| u: |
| NUMERICAL 3 points |
| 27 ✓ |
| $\overline{\mathrm{Q2}}$ |
| CLOZE 0.10 penalty |

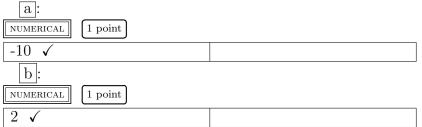
If not specified otherwise, fill in the blanks with **integers** (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the

(3)

answer boxes (such as $\frac{|a|}{|b|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted

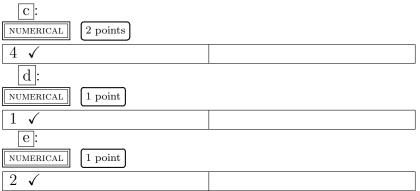
but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$. Let us study the following series $\sum_{n=0}^{\infty} \frac{4^n-1}{n^2+2}(x+1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For x = i, calculate the partial sum $\sum_{n=0}^{2} \frac{4^{n}-1}{n^{2}+2} (x+1)^{2n} = [a] + [b]i$.



In order to discuss the convergence using the root test for $x \in \mathbb{R}$, we put $a_n = \frac{4^n - 1}{n^2 + 2} |x + 1|^{2n}$. Complete the formula.

$$\lim_{n \to \infty} (a_n)^{\frac{1}{n}} = \boxed{\mathbf{c}} x + \boxed{\mathbf{d}} \boxed{\mathbf{e}}.$$



Therefore, by the root test, the series converges absolutely for

- \bullet all \overline{x} .
- -3 < x < -1.

- -3 < x < 1. -3 < x < 1. $-\frac{5}{3} < x < -\frac{1}{3}$. $-\frac{3}{2} < x < -\frac{1}{2}$. ✓ $\frac{1}{3} < x < \frac{5}{3}$. $\frac{1}{2} < x < \frac{3}{2}$. -1 < x < 1.

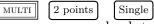
- -1 < x < 3.
- x = 0.
- 1 < x < 3.

For the case $x = -\frac{3}{2}$, the series

MULTI 2 points Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case x = 1, the series



- converges absolutely.
- converges but not absolutely.
- diverges. ✓

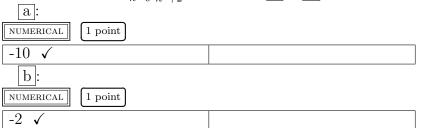
$(4) \ \mathbf{Q2}$

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$. Let us study the following series $\sum_{n=0}^{\infty} \frac{4^n-1}{n^2+2} (x-1)^{2n}$, with various x.

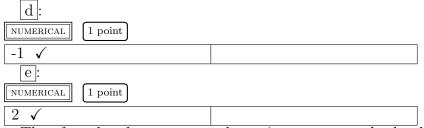
This series makes sense also for $x \in \mathbb{C}$. For x = i, calculate the partial sum $\sum_{n=0}^{2} \frac{4^{n}-1}{n^{2}+2} (x-1)^{2n} = \boxed{\mathbf{a}} + \boxed{\mathbf{b}}i$.



In order to discuss the convergence using the root test for $x \in \mathbb{R}$, we put $a_n = \frac{4^n - 1}{n^2 + 2}|x - 1|^{2n}$. Complete the formula.

$$\lim_{n \to \infty} (a_n)^{\frac{1}{n}} = \boxed{\mathbf{c}} x + \boxed{\mathbf{d}} \boxed{\mathbf{e}}.$$





Therefore, by the root test, the series converges absolutely for

MULTI 2 points Single

- all \overline{x} .
- -3 < x < -1.
- -3 < x < 1.
- -3 < x < 1. $-\frac{5}{3} < x < -\frac{1}{3}$. $-\frac{3}{2} < x < -\frac{1}{2}$. $\frac{1}{3} < x < \frac{5}{3}$. $\frac{1}{2} < x < \frac{3}{2}$. \checkmark -1 < x < 1.

- -1 < x < 3.
- x = 0.
- 1 < *x* < 3.

For the case $x = -\frac{3}{2}$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case x = 1, the series



- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(5) Q3CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|a|}{|b|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log\left(\frac{x}{1 + \log(x)}\right).$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of f(x).

MULTI 4 points Single $\overline{\bullet} - e \overline{\checkmark}$

- -1 √
- $\begin{array}{ccc} \bullet & -\frac{1}{e} \checkmark \\ \bullet & 0 \checkmark \end{array}$

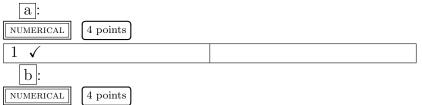
- $\frac{1}{e} \checkmark$ 1 (-100%)
- e (−100%)

Choose all asymptotes of f(x).

- $y = -\pi \ (-100\%)$
- $y = -\frac{\pi}{2} (-100\%)$ $y = -\frac{\pi}{4} (-100\%)$ y = 0 (-100%)
- $y = \frac{\pi}{4} (-100\%)$ $y = \frac{\pi}{2} (-100\%)$
- $y = \pi \ (-100\%)$
- $x = -1 \ (-100\%)$
- $x = 0 \ (-100\%)$
- $x = 1/e \checkmark$
- $x = 1 \ (-100\%)$
- $x = e \ (-100\%)$
- $\bullet \ y = x$ (-100%)
- y = -x (-100%)
- $y = \pi x \ (-100\%)$

One has

$$f'(e) = \frac{\boxed{\mathbf{a}}}{\boxed{\mathbf{b}}e}.$$



The function f(x) has [c] stationary point(s) in the domain c :

4 points NUMERICAL

Choose the behaviour of f(x) in the interval $(\frac{1}{2}, 2)$.

MULTI 4 points Single

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing \checkmark

(6) **Q3**

1

0.10 penalty CLOZE

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log\left(\frac{x}{2 + \log(x)}\right).$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of f(x).

MULTI 4 points Single

- $\overline{\bullet} e \overline{\checkmark}$
- \bullet -1 \checkmark
- $-1/e \checkmark$
- $-1/e^2 \checkmark$
- 0 ✓
- 1/e² ✓
- 1/e ✓
- 1 (−100%)
- e(-100%)

Choose all asymptotes of f(x).

MULTI 4 points Single

- $y = -\pi (-100\%)$
- $y = -\frac{\pi}{2} (-100\%)$ $y = -\frac{\pi}{4} (-100\%)$

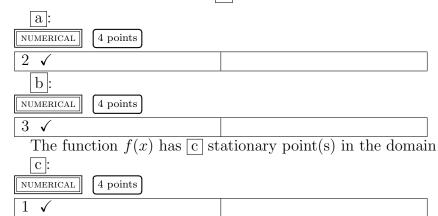
•
$$y = 0 \ (-100\%)$$

- $y = \frac{\pi}{4} (-100\%)$ $y = \frac{\pi}{2} (-100\%)$ $y = \pi (-100\%)$

- $x = -1 \ (-100\%)$
- $x = 0 \ (-100\%)$
- $x = 1/e^2 \checkmark$
- $x = 1/e \ (-100\%)$
- $x = 1 \ (-100\%)$
- $x = e \ (-100\%)$
- $x = e^2 (-100\%)$
- $\bullet \ y = x$ (-100%)
- y = -x (-100%)
- $y = \pi x \ (-100\%)$

One has

$$f'(e) = \frac{\boxed{\mathbf{a}}}{\boxed{\mathbf{b}} e}.$$



Choose the behaviour of f(x) in the interval (2,3).

MULTI 4 points Single

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

$(7) \mathbf{Q4}$ CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|a|}{|b|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 x^2 \exp(-x) dx.$$

It is easy to find a primitive of $\exp(-x)$. By integration by parts, one has

$$\int_0^1 x^2 \exp(-x) = \left[\boxed{\mathbf{a}}\right]_0^1 - \int_0^1 \boxed{\mathbf{b}} dx.$$

Choose the function a,

MULTI 2 points Single

- $\bullet \exp(-x)$
- $2x \exp(-x)$
- $x^2 \exp(-x)$
- \bullet $-\exp(-x)$
- \bullet $-2x \exp(-x)$
- \bullet $-x^2 \exp(-x)$ \checkmark

MULTI 2 points Single

- $\bullet \exp(-x)$
- $2x \exp(-x)$
- $x^2 \exp(-x)$
- \bullet $-\exp(-x)$
- $-2x \exp(-x) \checkmark$
- $\bullet -x^2 \exp(-x)$

Continuing, one has $\int_0^1 x^2 \exp(-x) = \boxed{c} + \boxed{d} e^{-1}$.

C:

NUMERICAL 2 points

NUMERICAL 2 points

2

V

d:

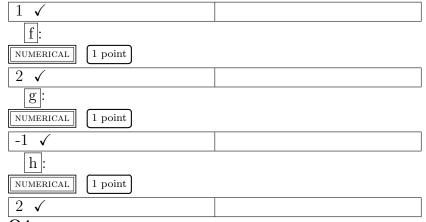
NUMERICAL 2 points

-5 ✓ Instead,

$$\int_0^1 x e^{-x^2} dx = \frac{\boxed{e}}{\boxed{f}} + \frac{\boxed{g}}{\boxed{h}} e^{-1}$$

, one can calculate this by substition.

e:
NUMERICAL 1 point



(8) **Q4**CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^2 x^2 \exp(-x) dx.$$

It is easy to find a primitive of $\exp(-x)$. By integration by parts, one has

$$\int_0^2 x^2 \exp(-x) = \left[\boxed{\mathbf{a}}\right]_0^2 - \int_0^2 \boxed{\mathbf{b}} dx.$$

Choose the function a,

MULTI 2 points Single

- $\bullet \exp(-x)$
- $2x \exp(-x)$
- $x^2 \exp(-x)$
- \bullet $-\exp(-x)$
- \bullet $-2x \exp(-x)$
- $-x^2 \exp(-x)$

and b

MULTI 2 points Single

- $\bullet \exp(-x)$
- $2x \exp(-x)$

•
$$x^2 \exp(-x)$$

• $-\exp(-x)$
• $-2x \exp(-x)$ \checkmark
• $-x^2 \exp(-x)$
Continuing, one has $\int_0^2 x^2 \exp(-x) = ce^{-2} + d$.
c:

NUMERICAL 2 points

-10 \checkmark

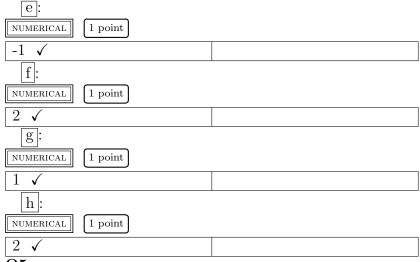
d:

NUMERICAL 2 points

2 \checkmark

$$\int_0^1 x e^{-x^2} dx = \frac{\boxed{e}}{\boxed{f}} e^{-1} + \frac{\boxed{g}}{\boxed{h}}$$

, one can calculate this by substition.



(9) **Q5**[CLOZE] [0.10 penalty]

Instead,

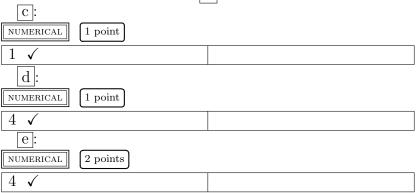
If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral with various $\alpha,\beta\in\mathbb{R},$ as follows

$$\int_{\beta}^{\infty} x^3 \exp(\alpha x^4) dx$$

| . a $\exp(\alpha x^4)$ is a primitive of $x^3 \exp(\alpha x^4)$ if $\alpha \neq 0$. |
|--|
| NUMERICAL 2 points |
| 1 🗸 |
| b: |
| NUMERICAL 2 points |
| 4 🗸 |
| The above improper integral converges for the following α, β |
| MULTI 8 points Single Shuffle |
| $\bullet \alpha > -\frac{1}{4}$ |
| $\bullet \ \alpha < -\frac{1}{4}$ |
| $\bullet \ \alpha > -1$ |
| $\bullet \ \alpha < -1$ |
| $\bullet \ \alpha > 0$ |
| • α < 0 ✓ |
| $ \begin{array}{l} \bullet \ \alpha > 1 \\ \bullet \ \alpha < 1 \end{array} $ |
| $\begin{array}{c} \alpha < 1 \\ \bullet \ \alpha > \frac{1}{4} \end{array}$ |
| $\bullet \ \alpha < \frac{1}{4}$ |
| • all $\alpha \in \mathbb{R}$ |
| and |
| MULTI 4 points Single Shuffle |
| $\beta > -e$ |
| $\bullet \beta < -e$ |
| \bullet $\beta > -1$ |
| $ \begin{array}{l} \bullet \ \beta < -1 \\ \bullet \ \beta > 0 \end{array} $ |
| β > 0β < 0 |
| β < 0β > 1 |
| $\bullet \beta < 1$ |
| \bullet $\beta > e$ |
| \bullet $\beta < e$ |
| • all $\beta \in \mathbb{R} \checkmark$ |

Take α, β for which the integral converges. In such a case, the value of the integral is $-\frac{\boxed{c}}{\boxed{d}_{\alpha}} \exp(\alpha \beta^{\boxed{e}})$.



For α as above, si ottiene

$$\int_{-\infty}^{\infty} x^3 \exp(\alpha x^4) dx = \boxed{\text{f}}.$$

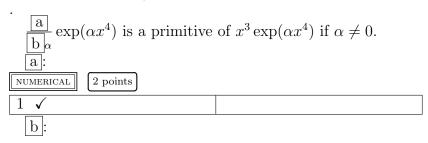


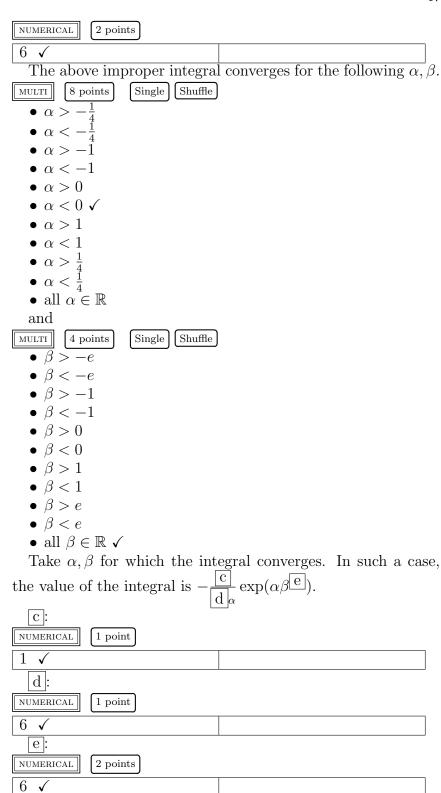
(10) **Q5**CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following improper integral with various $\alpha, \beta \in \mathbb{R}$, as follows

$$\int_{\beta}^{\infty} x^5 \exp(\alpha x^6) dx$$





For α as above, si ottiene

$$\int_{-\infty}^{\infty} x^3 \exp(\alpha x^4) dx = \boxed{\mathbf{f}}.$$



Total of marks: 216