2022Call3.

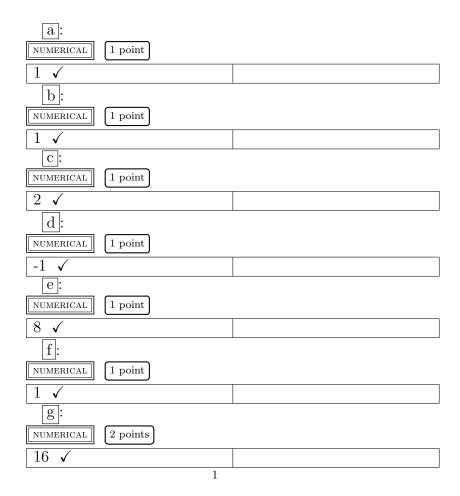
(1) **Q1**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{1}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). log $x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

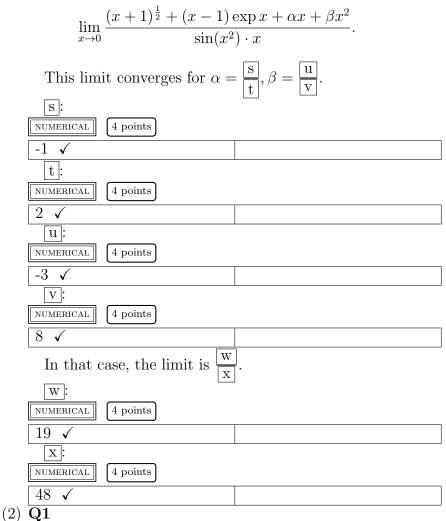
$$(x+1)^{\frac{1}{2}} = \boxed{\mathbf{a}} + \frac{\boxed{\mathbf{b}}}{\boxed{\mathbf{c}}}x + \frac{\boxed{\mathbf{d}}}{\boxed{\mathbf{e}}}x^2 + \frac{\boxed{\mathbf{f}}}{\boxed{\mathbf{g}}}x^3 \text{ as } x \to 0.$$



h:	
NUMERICAL 1 point	
-1 🗸	
i:	
NUMERICAL 1 point	
j:	
NUMERICAL 1 point	
<u>k</u> :	
NUMERICAL 2 points	
NUMERICAL 1 point	
m: NUMERICAL 2 points	
$3 \checkmark$	
J V	
$\sin(x^2) \cdot x = \boxed{\mathbf{o}} + \boxed{\mathbf{p}}x + \boxed{\mathbf{q}}x^2$	$+ [\mathbf{r}]x^3 + o(x^3)$ as $x \to 0$.
NUMERICAL 3 points	
NUMERICAL 1 point	
$\begin{bmatrix} 0 & \checkmark \\ \hline \alpha \end{bmatrix}$	
Q: NUMERICAL 1 point	
$0 \checkmark$	
r :	
NUMERICAL 3 points	
$\boxed{1 \checkmark}$	

$$(x-1) \exp x = \boxed{\mathbf{h}} + \boxed{\mathbf{i}}x + \frac{\boxed{\mathbf{j}}}{\boxed{\mathbf{k}}}x^2 + \frac{\boxed{\mathbf{l}}}{\boxed{\mathbf{m}}}x^3 + o(x^3) \text{ as } x \to 0.$$

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

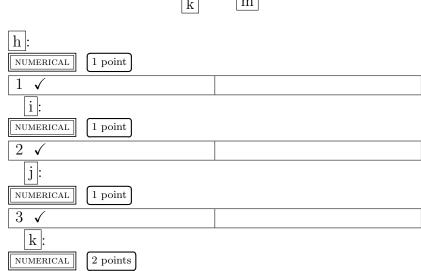


0.10 penalty CLOZE

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|\mathbf{a}|}{|\mathbf{b}|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$(1-x)^{\frac{1}{2}} = \boxed{\mathbf{a}} + \frac{\boxed{\mathbf{b}}}{\boxed{\mathbf{c}}}x + \frac{\boxed{\mathbf{d}}}{\boxed{\mathbf{e}}}x^2 + \frac{\boxed{\mathbf{f}}}{\boxed{\mathbf{g}}}x^3 \text{ as } x \to 0.$$

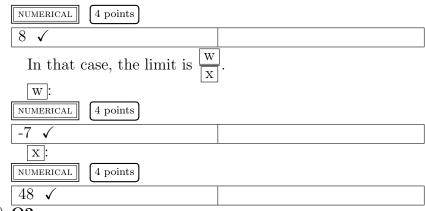


$$(1+x)\exp x = \boxed{\mathbf{h}} + \boxed{\mathbf{i}}x + \frac{\boxed{\mathbf{j}}}{\boxed{\mathbf{k}}}x^2 + \frac{\boxed{\mathbf{l}}}{\boxed{\mathbf{m}}}x^3 + o(x^3) \text{ as } x \to 0.$$

a :
NUMERICAL 1 point
1 🗸
b:
NUMERICAL 1 point
-1 🗸
C:
NUMERICAL 1 point
$2 \checkmark$
d:
NUMERICAL 1 point
-1 🗸
<u>e</u> :
NUMERICAL 1 point
8 🗸
f:
NUMERICAL 1 point
-1 🗸
g:
NUMERICAL 2 points
16 🗸

2 🗸	
1:	
NUMERICAL 1 point	
2 🗸	
:	
NUMERICAL 2 points	
3 🗸	
$\sin(5r^2) \cdot r = 0 + pr + ar^2$	$2 + [r] r^3 + o(r^3)$ as $r \to 0$

$\sin(5x) \cdot x = 0 + px + qx + 1x + o(x) \text{ as } x \to 0.$
<u> </u>
NUMERICAL 3 points
p:
NUMERICAL 1 point
q: NUMERICAL 1 point
r :
NUMERICAL 3 points
5 🗸
For various $\alpha, \beta \in \mathbb{R}$, study the limit:
$\lim_{x \to 0} \frac{(1-x)^{\frac{1}{2}} - (1+x) \exp x + \alpha x + \beta x^2}{\sin(5x^2) \cdot x}.$
$\lim_{x \to 0} \frac{(1-x)^{\frac{1}{2}} - (1+x) \exp x + \alpha x + \beta x^2}{\sin(5x^2) \cdot x}.$ This limit converges for $\alpha = \frac{s}{t}, \beta = \frac{u}{v}.$
This limit converges for $\alpha = \frac{s}{t}, \beta = \frac{u}{v}$.
This limit converges for $\alpha = \frac{s}{t}, \beta = \frac{u}{v}$.
This limit converges for $\alpha = \frac{s}{t}, \beta = \frac{u}{v}$. <u>s</u> : <u>NUMERICAL</u> 4 points
This limit converges for $\alpha = \frac{s}{t}, \beta = \frac{u}{v}$. s: <u>NUMERICAL</u> 4 points 5 \checkmark
This limit converges for $\alpha = \frac{s}{t}, \beta = \frac{u}{v}$. <u>s</u> : <u>NUMERICAL</u> 4 points <u>5 \checkmark</u> <u>t</u> :
This limit converges for $\alpha = \frac{s}{t}, \beta = \frac{u}{v}$. s: <u>NUMERICAL</u> 4 points 5 \checkmark t: <u>NUMERICAL</u> 4 points 2 \checkmark <u>u</u> :
This limit converges for $\alpha = \frac{s}{t}, \beta = \frac{u}{v}$. s: <u>NUMERICAL</u> 4 points 5 \checkmark <u>t</u> : <u>NUMERICAL</u> 4 points 2 \checkmark <u>u</u> : <u>NUMERICAL</u> 4 points
This limit converges for $\alpha = \frac{s}{t}, \beta = \frac{u}{v}$. s: <u>NUMERICAL</u> 4 points 5 \checkmark t: <u>NUMERICAL</u> 4 points 2 \checkmark <u>u</u> :



(3) $Q\bar{2}$

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n-1}{2^n} (x+1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For x = i, calculate the partial sum $\sum_{n=0}^{2} \frac{3^n - 1}{2^n} (x+1)^n = \boxed{a} + \boxed{b}i$.

a NUMERICAL 1 point	
<u> </u>	
b:	
NUMERICAL 1 point	
5 🗸	

In order to discuss the convergence using the root test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n - 1}{2^n} |x + 1|^n$. Complete the formula.

$$\lim_{n \to \infty} (a_n)^{\frac{1}{n}} = \frac{\boxed{c}}{\boxed{d}} |x + \boxed{e}|^{\boxed{f}}.$$





Therefore, by the ratio test, the series converges absolutely for

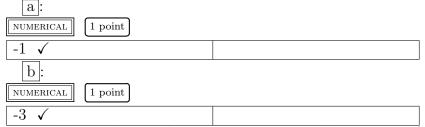
	MULTI 2 points Single
	• all \overline{x} .
	• $-3 < x < -1$.
	• $-3 < x < 1$.
	• $-\frac{5}{2} < x < -\frac{1}{2}$.
	• $-\frac{5}{3} < x < -\frac{1}{3}$. \checkmark • $-\frac{3}{2} < x < -\frac{1}{2}$.
	$\begin{array}{c} 2 \\ 2 \\ 1 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\$
	• $\frac{1}{3} < x < \frac{5}{3}$. • $\frac{1}{2} < x < \frac{3}{2}$.
	• $-1 < x < 1$.
	• $-1 < x < 3.$
	• $x = 0$.
	• $1 < x < 3.$
	For the case $x = -\frac{3}{2}$, the series
	MULTI 2 points Single
	• converges absolutely. \checkmark
	• converges but not absolutely.
	• diverges.
	For the case $x = 1$, the series
	MULTI 2 points Single
	• converges absolutely.
	• converges but not absolutely.
	 diverges. ✓
`	Q2
1	

(4) **Q2**

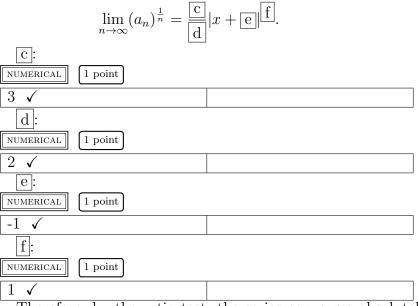
```
CLOZE 0.10 penalty
```

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as \boxed{a}) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$. Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n-1}{2^n} (x-1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For x = i, calculate the partial sum $\sum_{n=0}^{2} \frac{3^{n}-1}{2^{n}} (x-1)^{n} = [a] + [b]i$.



In order to discuss the convergence using the root test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n - 1}{2^n} |x - 1|^n$. Complete the formula.



Therefore, by the ratio test, the series converges absolutely for

MULTI 2 points Single
• all x.
•
$$-3 < x < -1$$
.
• $-3 < x < 1$.
• $-\frac{5}{3} < x < -\frac{1}{3}$.
• $-\frac{3}{2} < x < -\frac{1}{2}$.
• $\frac{1}{3} < x < \frac{5}{3}$. \checkmark
• $\frac{1}{2} < x < \frac{3}{2}$.
• $-1 < x < 1$.
• $-1 < x < 3$.

x = 0.
1 < x < 3.
For the case x = -³/₂, the series
MULTI 2 points Single
converges absolutely.
converges but not absolutely.
diverges. √
For the case x = ¹/₃, the series
MULTI 2 points Single
converges absolutely.
converges but not absolutely.
diverges. √

(5) Q3

If not specified otherwise, fill in the blanks with **integers (pos-sibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log \frac{x^2 + 1}{x + 2}.$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of f(x).

$$\begin{array}{c} f(x). \\ \hline \text{MULTI} & 4 \text{ points} & \text{Single} \\ \bullet & -2 \checkmark \\ \bullet & -1 \ (-100\%) \\ \bullet & -\frac{1}{2} \ (-100\%) \\ \bullet & 0 \ (-100\%) \\ \bullet & 1 \ (-100\%) \\ \bullet & 1 \ (-100\%) \\ \bullet & 2 \ (-100\%) \\ \bullet & 2 \ (-100\%) \\ \hline \text{Choose all asymptotes of } f(x) \\ \hline \begin{array}{c} \hline \text{MULTI} & 4 \text{ points} & \text{Single} \\ \bullet & y = -1 \ (-100\%) \\ \bullet & y = -\frac{1}{2} \ (-100\%) \\ \bullet & y = 0 \ (-100\%) \\ \bullet & y = 0 \ (-100\%) \end{array}$$

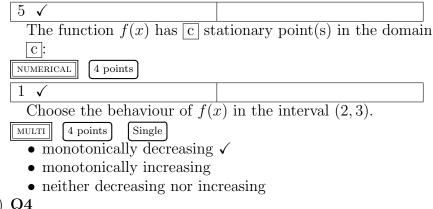
• $y = \frac{1}{2} (-100\%)$ • y = 2 (-100%)• $x = -2 \checkmark$ • $x = -1 \ (-100\%)$ • $x = -\frac{1}{2}(-100\%)$ • x = 0 (-100%) • $x = \frac{1}{2} (-100\%)$ • x = 1 (-100%)• $x = 2 \ (-100\%)$ • y = x + 3 (-100%)• $y = x + 1 \ (-100\%)$ • $y = x - 1 \ (-100\%)$ • y = x - 3 (-100%)One has $f'(1) = \frac{\underline{a}}{\underline{b}}.$ a : NUMERICAL 4 points 2 \checkmark b NUMERICAL 4 points 3 🗸 The function f(x) has c stationary point(s) in the domain c : NUMERICAL 4 points 1 🗸 Choose the behaviour of f(x) in the interval (0, 2). MULTI 4 points Single • monotonically decreasing • monotonically increasing • neither decreasing nor increasing \checkmark (6) **Q3** CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (pos-sibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log \frac{x^2 + 1}{x - 2}.$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of f(x). MULTI 4 points Single $\bullet -2 (-100\%)$ • -1 (-100%)• $-\frac{1}{2}$ (-100%) ● 0 (-100%) • $\frac{1}{2}$ (-100%) • 1 (-100%) • 2 ✓ Choose all asymptotes of f(x). MULTI 4 points Single • $y = -1 \ (-100\%)$ • $y = -\frac{1}{2} (-100\%)$ • y = 0 (-100%)• $y = \frac{1}{2} (-100\%)$ • y = 2 (-100%)• $x = -2 \ (-100\%)$ • $x = -1 \ (-100\%)$ • $x = -\frac{1}{2} (-100\%)$ • x = 0 (-100%)• $x = \frac{1}{2} (-100\%)$ • x = 1 (-100%)• $x = 2 \checkmark$ • y = x + 3 (-100%)• $y = x + 1 \ (-100\%)$ • $y = x - 1 \ (-100\%)$ • y = x - 3 (-100%)One has $f'(3) = \frac{\boxed{a}}{\boxed{b}}.$ a : NUMERICAL 4 points -2 🗸 b 4 points NUMERICAL



$$(7) \, {f Q} 4$$

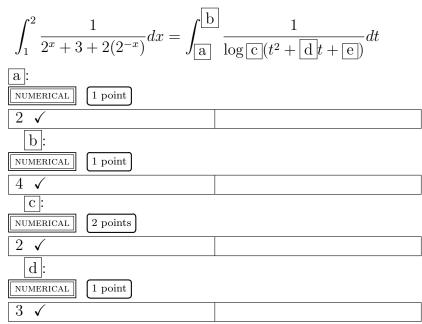
CLOZE 0.10 penalty

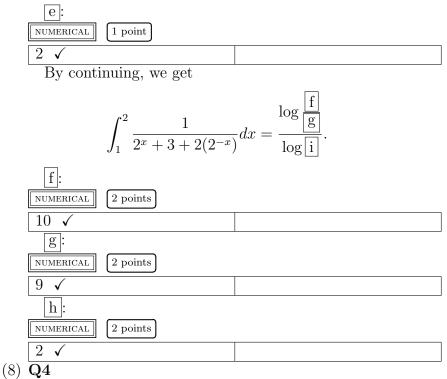
If not specified otherwise, fill in the blanks with **integers (pos-sibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{1}^{2} \frac{1}{2^{x} + 3 + 2(2^{-x})} dx$$

Let us change the variables $2^x = t$. Complete the formula





0.10 penalty CLOZE

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|\mathbf{a}|}{|\mathbf{b}|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{1}^{3} \frac{1}{2^{x} + 3 + 2(2^{-x})} dx.$$

Let us change the variables $2^x = t$. Complete the formula

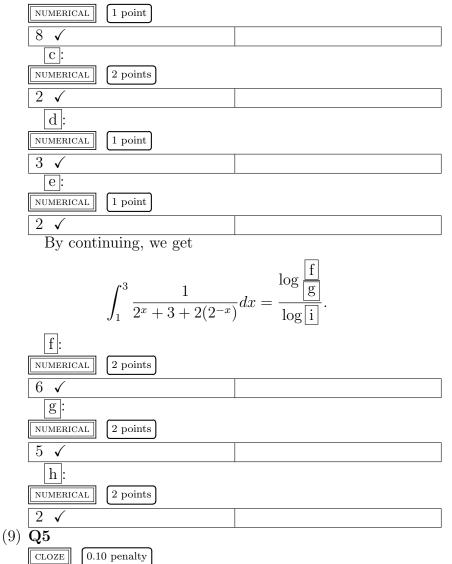
$$\int_{1}^{3} \frac{1}{2^{x} + 3 + 2(2^{-x})} dx = \int_{\boxed{a}}^{\boxed{b}} \frac{1}{\log \boxed{c}(t^{2} + \boxed{d}t + \boxed{e})} dt$$

$$\boxed{a:}$$

$$\boxed{1 \text{ point}}$$

$$\boxed{2 \checkmark}$$

$$\boxed{b:}$$



If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

 $y'(x) = -y(x)^2 \cos(x^2) x.$ MULTI 2 points Single

•
$$y(x) = \sin(x^2)/2 + C$$

• $y(x) = \cos(x^2)/2 + C$
• $y(x) = \sin(x^2 + C)/2$
• $y(x) = \cos(x^2 + C)/2$
• $y(x) = \log(\sin(x^2)/2) + C$
• $y(x) = \log(\cos(x^2)/2) + C$
• $y(x) = \log(\cos(x^2)/2 + C)$
• $y(x) = \log(\cos(x^2)/2 + C)$
• $y(x) = 1/(\sin(x^2)/2 + C)$
• $y(x) = 1/(\sin(x^2))/2 + C$
Determine $C = a$ with the initial condition $y(0) = 1$
a:
NUMERICAL 2 points
1 \checkmark
Choose the general solution of the following differential equa-

tion.

$$y''(x) - 2y'(x) - 8y(x) = 0.$$

$$\boxed{\text{MULTI}} \begin{array}{c} 2 \text{ points} & \underline{\text{Single}} \\ \bullet \ y(x) = C_1 \exp(-4x) + C_2 \exp(2x) \\ \bullet \ y(x) = C_1 \exp(-2x) + C_2 \exp(4x) \checkmark \\ \bullet \ y(x) = C_1 \exp(-2x) + C_2 \exp(-8x) \\ \bullet \ y(x) = C_1 \exp(-8x) + C_2 \exp(1x) \\ \bullet \ y(x) = C_1 \sin(-4x) + C_2 \cos(2x) \\ \bullet \ y(x) = C_1 \sin(-2x) + C_2 \cos(4x) \\ \bullet \ y(x) = C_1 \sin(-8x) + C_2 \cos(1x) \\ \hline \text{Find a solution } y(x) \text{ such that } y(0) = 3 \text{ and } \lim_{x \to \infty} y(x) = 0. \\ C_1 = \boxed{a}, C_2 = \boxed{b}. \\ \hline \boxed{b}: \\ \hline \text{NUMERICAL}} \begin{array}{c} 2 \text{ points} \\ \hline 3 \checkmark \\ \hline \text{C}: \\ \hline \text{NUMERICAL}} \begin{array}{c} 2 \text{ points} \\ \hline 0 \checkmark \\ \hline \text{For general values of } C_1, C_2, \text{ choose a correct statement.} \\ \hline \text{MULTI}} \begin{array}{c} 2 \text{ points} \\ \hline \text{Single} \\ \bullet \text{ As } x \to \infty, \text{ the solution converges to 0 for all } C_1, C_2. \\ \bullet \text{ As } x \to \infty, \text{ the solution converges to 0 only for some} \\ \end{array}$$

- е C_1, C_2 . ✓ • As $x \to \infty$, the solution never converges to 0.

(10) **Q5**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (pos-sibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

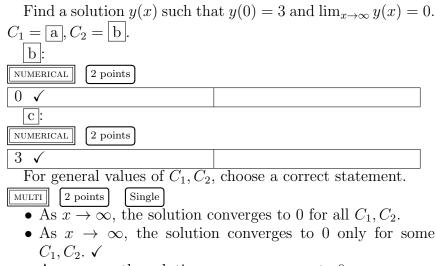
$$y'(x) = -y(x)^2 \cos(x^2)x.$$

MULTI 2 points Single • $y(x) = \sin(x^2)/2 + C$ • $y(x) = \cos(x^2)/2 + C$ • $y(x) = \sin(x^2 + C)/2$ • $y(x) = \cos(x^2 + C)/2$ • $y(x) = \log(\sin(x^2)/2) + C$ • $y(x) = \log(\cos(x^2)/2) + C$ • $y(x) = \log(\sin(x^2)/2 + C)$ • $y(x) = \log(\cos(x^2)/2 + C)$ • $y(x) = 1/(\sin(x^2)/2 + C)$ • $y(x) = 1/(\sin(x^2))/2 + C$ Determine C = [a] with the initial condition $y(0) = \frac{1}{2}$ |a|: NUMERICAL 2 points 2 🗸

Choose the general solution of the following differential equation.

$$y''(x) + 2y'(x) - 8y(x) = 0$$

 $\underbrace{MULTI}_{2 \text{ points}} \underbrace{\text{Single}}_{1 \text{ single}} \\ \bullet \ y(x) = C_1 \exp(2x) + C_2 \exp(-4x) \checkmark \\ \bullet \ y(x) = C_1 \exp(-2x) + C_2 \exp(-4x) \\ \bullet \ y(x) = C_1 \exp(-2x) + C_2 \exp(-8x) \\ \bullet \ y(x) = C_1 \exp(-8x) + C_2 \exp(1x) \\ \bullet \ y(x) = C_1 \sin(-4x) + C_2 \cos(2x) \\ \bullet \ y(x) = C_1 \sin(-2x) + C_2 \cos(4x) \\ \bullet \ y(x) = C_1 \sin(-2x) + C_2 \cos(8x) \\ \bullet \ y(x) = C_1 \sin(-8x) + C_2 \cos(1x) \\ \end{aligned}$



• As $x \to \infty$, the solution never converges to 0.

Total of marks: 216