

2022Call3.

(1) Q1

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$(x+1)^{\frac{1}{2}} = \boxed{a} + \frac{\boxed{b}}{\boxed{c}}x + \frac{\boxed{d}}{\boxed{e}}x^2 + \frac{\boxed{f}}{\boxed{g}}x^3 \text{ as } x \rightarrow 0.$$

a:

NUMERICAL

1 point

1 ✓

b:

NUMERICAL

1 point

1 ✓

c:

NUMERICAL

1 point

2 ✓

d:

NUMERICAL

1 point

-1 ✓

e:

NUMERICAL

1 point

8 ✓

f:

NUMERICAL

1 point

1 ✓

g:

NUMERICAL

2 points

16 ✓

$$(x - 1) \exp x = \boxed{h} + \boxed{i}x + \frac{\boxed{j}}{\boxed{k}}x^2 + \frac{\boxed{l}}{\boxed{m}}x^3 + o(x^3) \text{ as } x \rightarrow 0.$$

\boxed{h} :

NUMERICAL

1 point

-1 ✓

\boxed{i} :

NUMERICAL

1 point

0 ✓

\boxed{j} :

NUMERICAL

1 point

1 ✓

\boxed{k} :

NUMERICAL

2 points

2 ✓

\boxed{l} :

NUMERICAL

1 point

1 ✓

\boxed{m} :

NUMERICAL

2 points

3 ✓

$$\sin(x^2) \cdot x = \boxed{o} + \boxed{p}x + \boxed{q}x^2 + \boxed{r}x^3 + o(x^3) \text{ as } x \rightarrow 0.$$

\boxed{o} :

NUMERICAL

3 points

0 ✓

\boxed{p} :

NUMERICAL

1 point

0 ✓

\boxed{q} :

NUMERICAL

1 point

0 ✓

\boxed{r} :

NUMERICAL

3 points

1 ✓

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \rightarrow 0} \frac{(x+1)^{\frac{1}{2}} + (x-1) \exp x + \alpha x + \beta x^2}{\sin(x^2) \cdot x}.$$

This limit converges for $\alpha = \frac{s}{t}, \beta = \frac{u}{v}$.

s:

NUMERICAL 4 points

-1 ✓

t:

NUMERICAL 4 points

2 ✓

u:

NUMERICAL 4 points

-3 ✓

v:

NUMERICAL 4 points

8 ✓

In that case, the limit is $\frac{w}{x}$.

w:

NUMERICAL 4 points

19 ✓

x:

NUMERICAL 4 points

48 ✓

(2) **Q1**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$(1-x)^{\frac{1}{2}} = \frac{a}{c} + \frac{b}{c}x + \frac{d}{e}x^2 + \frac{f}{g}x^3 \text{ as } x \rightarrow 0.$$

a:	
NUMERICAL	1 point
1 ✓	
b:	
NUMERICAL	1 point
-1 ✓	
c:	
NUMERICAL	1 point
2 ✓	
d:	
NUMERICAL	1 point
-1 ✓	
e:	
NUMERICAL	1 point
8 ✓	
f:	
NUMERICAL	1 point
-1 ✓	
g:	
NUMERICAL	2 points
16 ✓	

$$(1+x)\exp x = \boxed{h} + \boxed{i}x + \frac{\boxed{j}}{\boxed{k}}x^2 + \frac{\boxed{l}}{\boxed{m}}x^3 + o(x^3) \text{ as } x \rightarrow 0.$$

h:	
NUMERICAL	1 point
1 ✓	
i:	
NUMERICAL	1 point
2 ✓	
j:	
NUMERICAL	1 point
3 ✓	
k:	
NUMERICAL	2 points

2 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">1:</div>	
<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px;">NUMERICAL</div> <div style="border: 1px solid black; padding: 2px 5px;">1 point</div> </div>	
2 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">m:</div>	
<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px;">NUMERICAL</div> <div style="border: 1px solid black; padding: 2px 5px;">2 points</div> </div>	
3 ✓	

$\sin(5x^2) \cdot x = \boxed{o} + \boxed{p}x + \boxed{q}x^2 + \boxed{r}x^3 + o(x^3)$ as $x \rightarrow 0$.

<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">o:</div>	
<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px;">NUMERICAL</div> <div style="border: 1px solid black; padding: 2px 5px;">3 points</div> </div>	
0 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">p:</div>	
<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px;">NUMERICAL</div> <div style="border: 1px solid black; padding: 2px 5px;">1 point</div> </div>	
0 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">q:</div>	
<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px;">NUMERICAL</div> <div style="border: 1px solid black; padding: 2px 5px;">1 point</div> </div>	
0 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">r:</div>	
<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px;">NUMERICAL</div> <div style="border: 1px solid black; padding: 2px 5px;">3 points</div> </div>	
5 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \rightarrow 0} \frac{(1-x)^{\frac{1}{2}} - (1+x) \exp x + \alpha x + \beta x^2}{\sin(5x^2) \cdot x}.$$

This limit converges for $\alpha = \frac{\boxed{s}}{\boxed{t}}, \beta = \frac{\boxed{u}}{\boxed{v}}$.

<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">s:</div>	
<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px;">NUMERICAL</div> <div style="border: 1px solid black; padding: 2px 5px;">4 points</div> </div>	
5 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">t:</div>	
<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px;">NUMERICAL</div> <div style="border: 1px solid black; padding: 2px 5px;">4 points</div> </div>	
2 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">u:</div>	
<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px;">NUMERICAL</div> <div style="border: 1px solid black; padding: 2px 5px;">4 points</div> </div>	
13 ✓	
<div style="border: 1px solid black; display: inline-block; padding: 2px 5px;">v:</div>	

NUMERICAL 4 points

8 ✓

In that case, the limit is $\frac{W}{X}$.

W:

NUMERICAL 4 points

-7 ✓

X:

NUMERICAL 4 points

48 ✓

(3) Q2

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the

answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign

should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n - 1}{2^n} (x + 1)^n$, with various x .

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^2 \frac{3^n - 1}{2^n} (x + 1)^n = \boxed{a} + \boxed{b}i$.

a:

NUMERICAL 1 point

1 ✓

b:

NUMERICAL 1 point

5 ✓

In order to discuss the convergence using the root test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n - 1}{2^n} |x + 1|^n$. Complete the formula.

$$\lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}} = \frac{\boxed{c}}{\boxed{d}} |x + \boxed{e}|^{\boxed{f}}.$$

c:

NUMERICAL 1 point

3 ✓

d:

NUMERICAL 1 point

2 ✓	
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e:

NUMERICAL	1 point
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1 ✓	
-----	--

f:

NUMERICAL	1 point
-----------	---------

1 ✓	
-----	--

Therefore, by the ratio test, the series converges absolutely for

MULTI	2 points	Single
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- all x .
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-\frac{5}{3} < x < -\frac{1}{3}$. ✓
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $\frac{1}{3} < x < \frac{5}{3}$.
- $\frac{1}{2} < x < \frac{3}{2}$.
- $-1 < x < 1$.
- $-1 < x < 3$.
- $x = 0$.
- $1 < x < 3$.

For the case $x = -\frac{3}{2}$, the series

MULTI	2 points	Single
-------	----------	--------

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = 1$, the series

MULTI	2 points	Single
-------	----------	--------

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(4) **Q2**

CLOZE	0.10 penalty
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If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the

answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign

should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n - 1}{2^n} (x - 1)^n$, with various x .

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^2 \frac{3^n - 1}{2^n} (x - 1)^n = \boxed{a} + \boxed{b}i$.

\boxed{a} :

NUMERICAL 1 point

-1 ✓

\boxed{b} :

NUMERICAL 1 point

-3 ✓

In order to discuss the convergence using the root test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n - 1}{2^n} |x - 1|^n$. Complete the formula.

$$\lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}} = \frac{\boxed{c}}{\boxed{d}} |x + \boxed{e}|^{\boxed{f}}.$$

\boxed{c} :

NUMERICAL 1 point

3 ✓

\boxed{d} :

NUMERICAL 1 point

2 ✓

\boxed{e} :

NUMERICAL 1 point

-1 ✓

\boxed{f} :

NUMERICAL 1 point

1 ✓

Therefore, by the ratio test, the series converges absolutely for

MULTI 2 points Single

- all x .
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-\frac{5}{3} < x < -\frac{1}{3}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $\frac{1}{3} < x < \frac{5}{3}$. ✓
- $\frac{1}{2} < x < \frac{3}{2}$.
- $-1 < x < 1$.
- $-1 < x < 3$.

- $x = 0$.
- $1 < x < 3$.

For the case $x = -\frac{3}{2}$, the series

☐ MULTI ☐ 2 points ☐ Single

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = \frac{1}{3}$, the series

☐ MULTI ☐ 2 points ☐ Single

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(5) Q3

☐ CLOZE ☐ 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign

should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log \frac{x^2 + 1}{x + 2}.$$

The function $f(x)$ is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of $f(x)$.

☐ MULTI ☐ 4 points ☐ Single

- -2 ✓
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 (-100%)
- 2 (-100%)

Choose all asymptotes of $f(x)$.

☐ MULTI ☐ 4 points ☐ Single

- $y = -1$ (-100%)
- $y = -\frac{1}{2}$ (-100%)
- $y = 0$ (-100%)

- $y = \frac{1}{2}$ (−100%)
- $y = 2$ (−100%)
- $x = -2$ ✓
- $x = -1$ (−100%)
- $x = -\frac{1}{2}$ (−100%)
- $x = 0$ (−100%)
- $x = \frac{1}{2}$ (−100%)
- $x = 1$ (−100%)
- $x = 2$ (−100%)
- $y = x + 3$ (−100%)
- $y = x + 1$ (−100%)
- $y = x - 1$ (−100%)
- $y = x - 3$ (−100%)

One has

$$f'(1) = \frac{\boxed{a}}{\boxed{b}}.$$

a:

NUMERICAL 4 points

2 ✓

b:

NUMERICAL 4 points

3 ✓

The function $f(x)$ has **c** stationary point(s) in the domain

c:

NUMERICAL 4 points

1 ✓

Choose the behaviour of $f(x)$ in the interval $(0, 2)$.

MULTI 4 points Single

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(6) Q3

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log \frac{x^2 + 1}{x - 2}.$$

The function $f(x)$ is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of $f(x)$.

☐ MULTI ☐ 4 points ☐ Single

- -2 (−100%)
- -1 (−100%)
- $-\frac{1}{2}$ (−100%)
- 0 (−100%)
- $\frac{1}{2}$ (−100%)
- 1 (−100%)
- 2 ✓

Choose all asymptotes of $f(x)$.

☐ MULTI ☐ 4 points ☐ Single

- $y = -1$ (−100%)
- $y = -\frac{1}{2}$ (−100%)
- $y = 0$ (−100%)
- $y = \frac{1}{2}$ (−100%)
- $y = 2$ (−100%)
- $x = -2$ (−100%)
- $x = -1$ (−100%)
- $x = -\frac{1}{2}$ (−100%)
- $x = 0$ (−100%)
- $x = \frac{1}{2}$ (−100%)
- $x = 1$ (−100%)
- $x = 2$ ✓
- $y = x + 3$ (−100%)
- $y = x + 1$ (−100%)
- $y = x - 1$ (−100%)
- $y = x - 3$ (−100%)

One has

$$f'(3) = \frac{\boxed{a}}{\boxed{b}}.$$

☐ a:

☐ NUMERICAL ☐ 4 points

☐ -2 ✓

☐ b:

☐ NUMERICAL ☐ 4 points

5 ✓	
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The function $f(x)$ has stationary point(s) in the domain

:

NUMERICAL	4 points
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1 ✓	
-----	--

Choose the behaviour of $f(x)$ in the interval $(2, 3)$.

MULTI	4 points	Single
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- monotonically decreasing ✓
- monotonically increasing
- neither decreasing nor increasing

(7) Q4

CLOZE	0.10 penalty
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If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\text{a}}{\text{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_1^2 \frac{1}{2^x + 3 + 2(2^{-x})} dx.$$

Let us change the variables $2^x = t$. Complete the formula

$$\int_1^2 \frac{1}{2^x + 3 + 2(2^{-x})} dx = \int_{\text{a}}^{\text{b}} \frac{1}{\log \text{c} (t^2 + \text{d}t + \text{e})} dt$$

:

NUMERICAL	1 point
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2 ✓	
-----	--

:

NUMERICAL	1 point
-----------	---------

4 ✓	
-----	--

:

NUMERICAL	2 points
-----------	----------

2 ✓	
-----	--

:

NUMERICAL	1 point
-----------	---------

3 ✓	
-----	--

e:

NUMERICAL

1 point

2 ✓

By continuing, we get

$$\int_1^2 \frac{1}{2^x + 3 + 2(2^{-x})} dx = \frac{\log \frac{f}{g}}{\log \frac{i}{j}}.$$

f:

NUMERICAL

2 points

10 ✓

g:

NUMERICAL

2 points

9 ✓

h:

NUMERICAL

2 points

2 ✓

(8) Q4

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_1^3 \frac{1}{2^x + 3 + 2(2^{-x})} dx.$$

Let us change the variables $2^x = t$. Complete the formula

$$\int_1^3 \frac{1}{2^x + 3 + 2(2^{-x})} dx = \int_{\frac{a}{b}}^{\frac{b}{a}} \frac{1}{\log \frac{c}{d} (t^2 + \frac{d}{c} t + \frac{e}{c})} dt$$

a:

NUMERICAL

1 point

2 ✓

b:

NUMERICAL	1 point
8 ✓	
c:	
NUMERICAL	2 points
2 ✓	
d:	
NUMERICAL	1 point
3 ✓	
e:	
NUMERICAL	1 point
2 ✓	

By continuing, we get

$$\int_1^3 \frac{1}{2^x + 3 + 2(2^{-x})} dx = \frac{\log \frac{f}{g}}{\log \frac{i}{j}}.$$

f:	
NUMERICAL	2 points
6 ✓	
g:	
NUMERICAL	2 points
5 ✓	
h:	
NUMERICAL	2 points
2 ✓	

(9) Q5

CLOZE	0.10 penalty
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If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the

answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign

should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = -y(x)^2 \cos(x^2)x.$$

MULTI	2 points	Single
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- $y(x) = \sin(x^2)/2 + C$
- $y(x) = \cos(x^2)/2 + C$
- $y(x) = \sin(x^2 + C)/2$
- $y(x) = \cos(x^2 + C)/2$
- $y(x) = \log(\sin(x^2)/2) + C$
- $y(x) = \log(\cos(x^2)/2) + C$
- $y(x) = \log(\sin(x^2)/2 + C)$
- $y(x) = \log(\cos(x^2)/2 + C)$
- $y(x) = 1/(\sin(x^2)/2 + C)$ ✓
- $y(x) = 1/(\sin(x^2))/2 + C$

Determine $C = \boxed{a}$ with the initial condition $y(0) = 1$

\boxed{a} :

NUMERICAL

2 points

1 ✓

Choose the general solution of the following differential equation.

$$y''(x) - 2y'(x) - 8y(x) = 0.$$

MULTI

2 points

Single

- $y(x) = C_1 \exp(-4x) + C_2 \exp(2x)$
- $y(x) = C_1 \exp(-2x) + C_2 \exp(4x)$ ✓
- $y(x) = C_1 \exp(-2x) + C_2 \exp(-8x)$
- $y(x) = C_1 \exp(-8x) + C_2 \exp(1x)$
- $y(x) = C_1 \sin(-4x) + C_2 \cos(2x)$
- $y(x) = C_1 \sin(-2x) + C_2 \cos(4x)$
- $y(x) = C_1 \sin(-2x) + C_2 \cos(8x)$
- $y(x) = C_1 \sin(-8x) + C_2 \cos(1x)$

Find a solution $y(x)$ such that $y(0) = 3$ and $\lim_{x \rightarrow \infty} y(x) = 0$.

$C_1 = \boxed{a}, C_2 = \boxed{b}$.

\boxed{b} :

NUMERICAL

2 points

3 ✓

\boxed{c} :

NUMERICAL

2 points

0 ✓

For general values of C_1, C_2 , choose a correct statement.

MULTI

2 points

Single

- As $x \rightarrow \infty$, the solution converges to 0 for all C_1, C_2 .
- As $x \rightarrow \infty$, the solution converges to 0 only for some C_1, C_2 . ✓
- As $x \rightarrow \infty$, the solution never converges to 0.

(10) Q5

CLOZE

0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = -y(x)^2 \cos(x^2)x.$$

MULTI

2 points

Single

- $y(x) = \sin(x^2)/2 + C$
- $y(x) = \cos(x^2)/2 + C$
- $y(x) = \sin(x^2 + C)/2$
- $y(x) = \cos(x^2 + C)/2$
- $y(x) = \log(\sin(x^2)/2) + C$
- $y(x) = \log(\cos(x^2)/2) + C$
- $y(x) = \log(\sin(x^2)/2 + C)$
- $y(x) = \log(\cos(x^2)/2 + C)$
- $y(x) = 1/(\sin(x^2)/2 + C)$ ✓
- $y(x) = 1/(\sin(x^2))/2 + C$

Determine $C = \boxed{a}$ with the initial condition $y(0) = \frac{1}{2}$

a:

NUMERICAL

2 points

2 ✓

Choose the general solution of the following differential equation.

$$y''(x) + 2y'(x) - 8y(x) = 0.$$

MULTI

2 points

Single

- $y(x) = C_1 \exp(2x) + C_2 \exp(-4x)$ ✓
- $y(x) = C_1 \exp(-2x) + C_2 \exp(4x)$
- $y(x) = C_1 \exp(-2x) + C_2 \exp(-8x)$
- $y(x) = C_1 \exp(-8x) + C_2 \exp(1x)$
- $y(x) = C_1 \sin(-4x) + C_2 \cos(2x)$
- $y(x) = C_1 \sin(-2x) + C_2 \cos(4x)$
- $y(x) = C_1 \sin(-2x) + C_2 \cos(8x)$
- $y(x) = C_1 \sin(-8x) + C_2 \cos(1x)$

Find a solution $y(x)$ such that $y(0) = 3$ and $\lim_{x \rightarrow \infty} y(x) = 0$.
 $C_1 = \boxed{\text{a}}, C_2 = \boxed{\text{b}}.$

$\boxed{\text{b}}:$

NUMERICAL

2 points

0 ✓

$\boxed{\text{c}}:$

NUMERICAL

2 points

3 ✓

For general values of C_1, C_2 , choose a correct statement.

MULTI

2 points

Single

- As $x \rightarrow \infty$, the solution converges to 0 for all C_1, C_2 .
- As $x \rightarrow \infty$, the solution converges to 0 only for some C_1, C_2 . ✓
- As $x \rightarrow \infty$, the solution never converges to 0.

Total of marks: 216