2022Call1.

(1) **Q1**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the limit $\lim_{x\to 0} \frac{\exp x - x(1+x)^{\frac{1}{3}} + \alpha}{\log(1+2x^2) + \beta}$. Complete the formulae.

$$\exp x = \boxed{\mathbf{a}} + \boxed{\mathbf{b}}x + \frac{\boxed{\mathbf{c}}}{\boxed{\mathbf{d}}}x^2 + o(x^2) \text{ as } x \to 0.$$

a:	
NUMERICAL 2 points	
1 🗸	
b:	
NUMERICAL 2 points	
1 🗸	
<u>c</u> :	
NUMERICAL 1 point	
1 🗸	
d:	
NUMERICAL 1 point	
2 🗸	

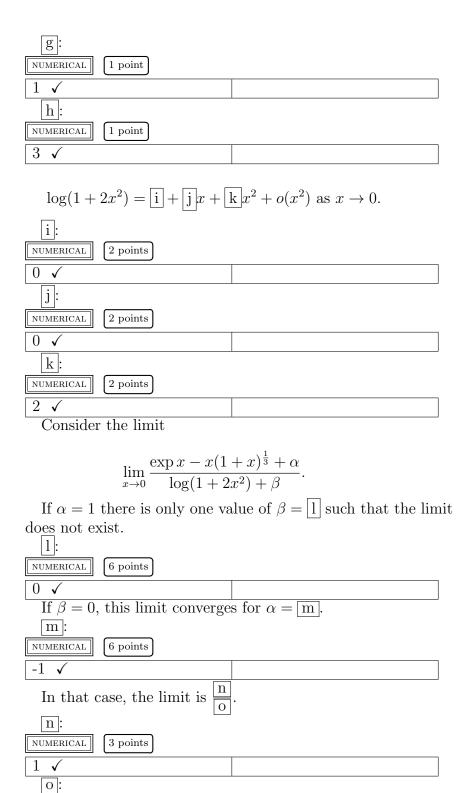
$$x(1+x)^{\frac{1}{3}} = \boxed{e} + \boxed{f}x + \frac{\boxed{g}}{\boxed{h}}x^2 + o(x^2) \text{ as } x \to 0.$$

e:	
NUMERICAL 2 points	
0 🗸	
<u>f</u> :	
NUMERICAL 2 points	
1 🗸	

1

NUMERICAL

3 points



12 $(2) \overline{\mathbf{Q1}}$ CLOZE 0.10 penalty If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). Let us study the limit $\lim_{x\to 0} \frac{\exp(3x) - 3x(1+x)^{\frac{1}{3}} + \alpha}{\log(1+x^2) + \beta}$. Complete the formulae. $\exp(3x) = \boxed{\mathbf{a}} + \boxed{\mathbf{b}}x + \boxed{\frac{\mathbf{c}}{\boxed{\mathbf{d}}}}x^2 + o(x^2) \text{ as } x \to 0.$ a : NUMERICAL 2 points b : NUMERICAL 2 points 3 ✓ c : NUMERICAL 1 point 9 🗸 d: NUMERICAL 1 point 2 **√** $x(1+x)^{\frac{1}{3}} = \boxed{e} + \boxed{f}x + \frac{\boxed{g}}{\boxed{h}}x^2 + o(x^2) \text{ as } x \to 0.$ e : 2 points NUMERICAL 0 🗸

f:

NUMERICAL

2 points

n:
NUMERICAL

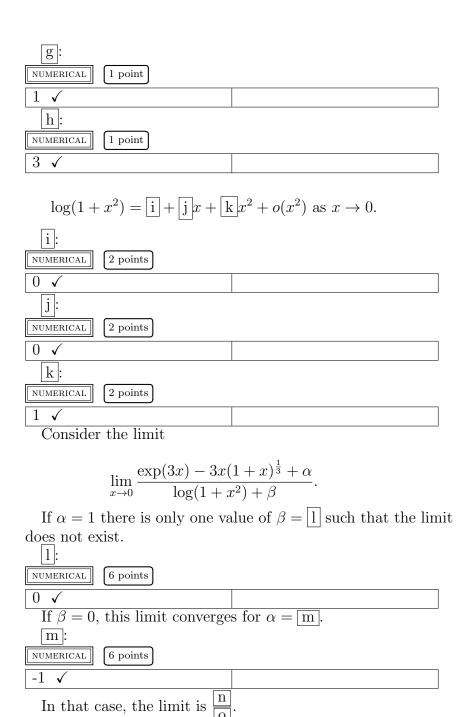
7 √

O:

NUMERICAL

3 points

3 points



2 ✓

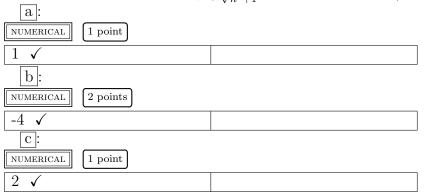
 $(3) \overline{\mathbf{Q2}}$

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

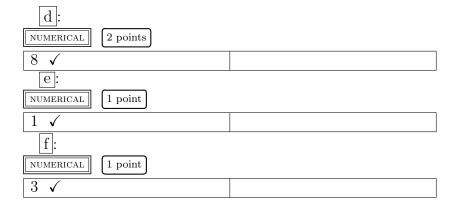
Let us study the following series $\sum_{n=0}^{\infty} \frac{8^n}{\sqrt{n^2+1}} (x+1)^{3n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For x = -1 + i, calculate the partial sum $\sum_{n=0}^{1} \frac{8^n}{\sqrt{n^2+1}} (x+1)^{3n} = \boxed{\mathbf{a}} + \boxed{\mathbf{b}} \sqrt{\boxed{\mathbf{c}}} i$.



In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{8^n}{\sqrt{n^2+1}}|x+1|^{3n}$. Complete the formula.

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \boxed{\mathbf{d}} x + \boxed{\mathbf{e}} \boxed{\mathbf{f}}.$$



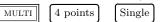
Therefore, by the ratio test, the series converges absolutely for $\frac{|\mathbf{g}|}{|\mathbf{h}|} < x < \frac{|\mathbf{i}|}{|\mathbf{j}|}$: | g |: NUMERICAL 2 points -3 ✓ h |: NUMERICAL 2 points 2 **<** i: 2 points NUMERICAL -1 **√** | j |: 2 points NUMERICAL

For the case $x = -\frac{3}{2}$, the series

MULTI 4 points Single

- converges absolutely.
- converges but not absolutely. \checkmark
- diverges.

For the case x = 1, the series



- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(4) **Q2**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{8^n}{\sqrt{n^2+1}} (x-1)^{3n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For x = 1-i, calculate the partial sum $\sum_{n=0}^{1} \frac{8^n}{\sqrt{n^2+1}} (x-1)^{3n} = \boxed{\mathbf{a}} + \boxed{\mathbf{b}} \sqrt{\boxed{\mathbf{c}}} i$.

a:	
NUMERICAL 1 point	
1 🗸	
b:	_
NUMERICAL 2 points	
4 🗸	
<u>c</u> :	_
NUMERICAL 1 point	
2 🗸	
In order to discuss the convergence using the ratio test $x \in \mathbb{R}$, we put $a_n = \frac{8^n}{\sqrt{n^2+1}} x-1 ^{3n}$. Complete the formula.	for
$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \boxed{\mathbf{d}} x + \boxed{\mathbf{e}} ^{\boxed{\mathbf{f}}}.$	
d:	
NUMERICAL 2 points	
8 🗸	
e:	
NUMERICAL 1 point	_
-1 ✓	
<u>f</u> :	
NUMERICAL 1 point	_
3 🗸	
Therefore, by the ratio test, the series converges absolut	ely
for $\frac{g}{h} < x < \frac{i}{j}$: g :	
NUMERICAL 2 points	
1 🗸	
h:	_
NUMERICAL 2 points	
<u></u>	7
i:	_
NUMERICAL 2 points	
3 ✓	
j:	_
NUMERICAL 2 points	

For the case $x = \frac{3}{2}$, the series

MULTI 4 points Single

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case x = 1, the series

MULTI 4 points Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(5) Q3

0.10 penalty

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|a|}{|b|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x^3}{x^2 + 3x + 2}.$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of f(x).

MULTI 4 points Single

- $\overline{\bullet}$ -2 $\sqrt{}$
- -1 ✓
- \bullet $-\frac{1}{2}$ (-100%)
- 0 (-100%)• $\frac{1}{2}$ (-100%)
- $\bar{1}$ (-100%)
- 2(-100%)

Choose all asymptotes of f(x).

MULTI 4 points Single

- $y = -1 \ (-100\%)$
- $y = -\frac{1}{2} (-100\%)$ y = 0 (-100%)
- $y = \frac{1}{2} (-100\%)$

•
$$y = 2 (-100\%)$$

•
$$x = -2 \checkmark$$

•
$$x = -1$$

•
$$x = -\frac{1}{2} (-100\%)$$

• $x = 0 (-100\%)$
• $x = \frac{1}{2} (-100\%)$
• $x = 1 (-100\%)$

•
$$x = 0 \ (-100\%)$$

•
$$x = \frac{1}{2} (-100\%)$$

•
$$x = \bar{1} (-100\%)$$

•
$$x = 2 (-100\%)$$

•
$$y = x + 3 \ (-100\%)$$

•
$$y = x + 1 \ (-100\%)$$

• $y = x - 1 \ (-100\%)$

$$y = x - 3 \checkmark$$

One has

$$f'(1) = \frac{\boxed{a}}{\boxed{b}}.$$

a : NUMERICAL 4 points 13 ✓ b : NUMERICAL 4 points 36 ✓ The function f(x) has [c] stationary point(s) in the domain c : NUMERICAL 4 points 3 ✓ Choose the behaviour of f(x) in the interval (-5, -4).

MULTI 4 points Single

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing \checkmark

$(6) \ \mathbf{Q3}$ CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as a b have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x^3}{x^2 - 3x + 2}.$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of

MULTI 4 points Single

- \bullet -2 (-100%)
- \bullet -1 (-100%)
- \bullet $-\frac{1}{2}$ (-100%)
- 0 (−100%)
- $\frac{1}{2}$ (-100%)
- 1 √
- 2 √

Choose all asymptotes of f(x).

- $y = -1 \ (-100\%)$
- $y = -\frac{1}{2} (-100\%)$ y = 0 (-100%)

- $y = \frac{1}{2} (-100\%)$ y = 2 (-100%)
- $x = -2 \ (-100\%)$
- $x = -1 \ (-100\%)$
- $x = -\frac{1}{2} (-100\%)$ x = 0 (-100%)• $x = \frac{1}{2} (-100\%)$ $x = 1 \checkmark$
- x = 2
- $y = x + 3 \checkmark$
- $y = x + 1 \ (-100\%)$
- $y = -x 1 \ (-100\%)$
- $y = -x 3 \ (-100\%)$

One has

$$f'(3) = \frac{\boxed{a}}{\boxed{b}}.$$

a : NUMERICAL 4 points -27 b : 4 points NUMERICAL

	4 🗸			
	The function $f(x)$ has \boxed{c} stationary point(s) in the domain			
	<u>c</u> :			
	NUMERICAL 4 points			
	3 ✓			
	Choose the behaviour of $f(x)$ in the interval $(-5, -4)$.			
	MULTI 4 points Single			
	• monotonically decreasing			
	$ullet$ monotonically increasing \checkmark			
	• neither decreasing nor increasing			
(7)	Q4			
	CLOZE 0.10 penalty			
	Let us calculate the following integral.			
	$\int^{\frac{\pi}{2}}$. () (π) ,			
	$\int_0^{\frac{\pi}{2}} \sin(x) \cos\left(x + \frac{\pi}{6}\right) dx.$			
	v			
	Complete the formula			
	$\cos\left(x + \frac{\pi}{6}\right) = \frac{\sqrt{a}}{b}\cos x + \frac{c}{d}\sin x.$			
	$\begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} a & b \end{bmatrix}$			
	<u>a</u> :			
	NUMERICAL 2 points			
	3 🗸			
	b:			
	NUMERICAL 2 points			
	2 ✓			
	[C]:			
	NUMERICAL 2 points			
	-1 🗸			
	<u>d</u> :			
	NUMERICAL 2 points			
	2 ✓			
	Choose a primitive of $\sin(x)\cos(x)$.			
	MULTI 8 points Single			
	$\bullet \ \frac{1}{3}\cos^3(x)\sin(x)$			
	$\bullet \ -\frac{1}{3}\cos^3(x)\sin(x)$			
	$\bullet \ \frac{1}{3}\sin^3(x)$			
	$\bullet \ -\frac{1}{3}\sin^3(x)$			
	\bullet $\frac{1}{2}\cos^2(x)$			

•
$$\frac{1}{2}\sin^2(x)$$
 \checkmark

- $\frac{1}{2}\sin^2(x) \checkmark$ $-\frac{1}{2}\sin^2(x)$ $\frac{1}{3}\sin^2\cos(x)$ $-\frac{1}{3}\sin^2\cos(x)$

Choose a primitive of $\sin^2(x)$.

MULTI 8 points Single

- $\bullet \sin^3(x)/3$
- $\cos^3(x)/3$
- $x\sin^2(x)$
- $x \cos^2(x)$
- $\sin(x)\cos(x)$
- $\sin^2(x)\cos(x)/2$
- $\sin(x)\cos^2(x)/2$
- $x/2 + \cos(2x)/4$
- $x/2 \sin(2x)/4$

By continuing, we get

$$\int_0^{\frac{\pi}{2}} \sin(x) \cos\left(x + \frac{\pi}{6}\right) dx = \frac{\sqrt{\boxed{e}}}{\boxed{\boxed{f}}} + \frac{\boxed{g}}{\boxed{h}} \pi$$

e: 6 points NUMERICAL 3 ✓ f: NUMERICAL 6 points 4 ✓ g : NUMERICAL 3 points **-1 √** h : NUMERICAL 3 points

 $(8) \mathbf{Q4}$

0.10 penalty CLOZE

Let us calculate the following integral.

$$\int_0^{\frac{\pi}{6}} \sin(x) \cos\left(x + \frac{\pi}{6}\right) dx.$$

Complete the formula

$$\cos\left(x + \frac{\pi}{6}\right) = \frac{\sqrt{\boxed{a}}}{\boxed{\boxed{b}}}\cos x + \frac{\boxed{\boxed{c}}}{\boxed{\boxed{d}}}\sin x.$$

a:	
NUMERICAL 2 points	
3 🗸	
b:	
NUMERICAL 2 points	
2 🗸	
c:	
NUMERICAL 2 points	
-1 ✓	
d:	
NUMERICAL 2 points	
2 🗸	
Choose a primitive of $\sin^2(x)$.)

Choose a primitive of $\sin^2(x)$.

MULTI 8 points $\bullet \sin^3(x)/3$ Single

- $\cos^3(x)/3$
- $x\sin^2(x)$
- $x\cos^2(x)$
- $\sin(x)\cos(x)$
- $\sin^2(x)\cos(x)/2$
- $\sin(x)\cos^2(x)/2$
- $x/2 + \cos(2x)/4$ • $x/2 - \sin(2x)/4$

Choose a primitive of sin(x) cos(x).

MULTI 8 points Single • $\frac{1}{3}\cos^3(x)\sin(x)$ • $-\frac{1}{3}\cos^3(x)\sin(x)$

- $\begin{array}{l}
 3 \cos^{3}(x) \\
 \hline
 \frac{1}{3} \sin^{3}(x) \\
 \hline
 -\frac{1}{3} \sin^{3}(x) \\
 \hline
 \frac{1}{2} \cos^{2}(x) \\
 \hline
 \frac{1}{2} \sin^{2}(x) \checkmark \\
 \hline
 -\frac{1}{2} \sin^{2}(x) \\
 \hline
 \end{array}$
- $\bullet \frac{1}{3}\sin^2\cos(x)$ $\bullet -\frac{1}{3}\sin^2\cos(x)$

By continuing, we get

$$\int_0^{\frac{\pi}{6}} \sin(x) \cos\left(x + \frac{\pi}{6}\right) dx = \frac{\sqrt{[e]}}{[f]} + \frac{[g]}{[h]} \pi$$

e:	
NUMERICAL 6 points	
3 🗸	
f:	
NUMERICAL 6 points	
8 🗸	
g:	
NUMERICAL 3 points	
-1 ✓	
h:	
NUMERICAL 3 points	
24 🗸	

(9) **Q5**[CLOZE] [0.10 penalty]

If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = y(x)\cos(\log x)/x.$$

MULTI 2 points Single

- $y(x) = \sin(\log x) + C$
- $y(x) = \cos(1/x) + C$
- $y(x) = \log \cos(x) + C$
- $y(x) = \exp(\cos(\log x)) + C$
- $y(x) = \sin(\log x + C)$
- $y(x) = \cos(1/x + C)$
- $y(x) = \log(\cos(x) + C)$
- $y(x) = \exp(\sin(\log x) + C) \checkmark$

Determine $C = \boxed{\mathbf{a}}$ with the initial condition y(1) = 1 $\boxed{\mathbf{a}}$:

NUMERICAL 2 points

0 ✓

Choose the general solution of the following differential equation.

$$y''(x) + y'(x) - 6y(x) = 0.$$

MULTI 2 points Single

- $y(x) = C_1 \exp(-2x) + C_2 \exp(3x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(6x)$
- $y(x) = C_1 \exp(-6x) + C_2 \exp(1x)$
- $y(x) = C_1 \sin(-3x) + C_2 \cos(2x)$
- $y(x) = C_1 \sin(-2x) + C_2 \cos(3x)$
- $\bullet \ y(x) = C_1 \sin(-x) + C_2 \cos(6x)$
- $y(x) = C_1 \sin(-6x) + C_2 \cos(1x)$

Find a solution y(x) such that y(0) = 3 and $\lim_{x \to \infty} y(x) = 0$.

$$C_1 = \boxed{\mathbf{a}}, C_2 = \boxed{\mathbf{b}}$$
 $\boxed{\mathbf{b}}$:

NUMERICAL 2 points

3 \(\tag{c} \)

NUMERICAL 2 points

For general values of C_1, C_2 , choose a correct statement.

MULTI 2 points Single

- \bullet As $x \to \infty$, the solution converges to 0 for all C_1, C_2 .
- As $x \to \infty$, the solution converges to 0 only for some C_1, C_2 .
- As $x \to \infty$, the solution never converges to 0.

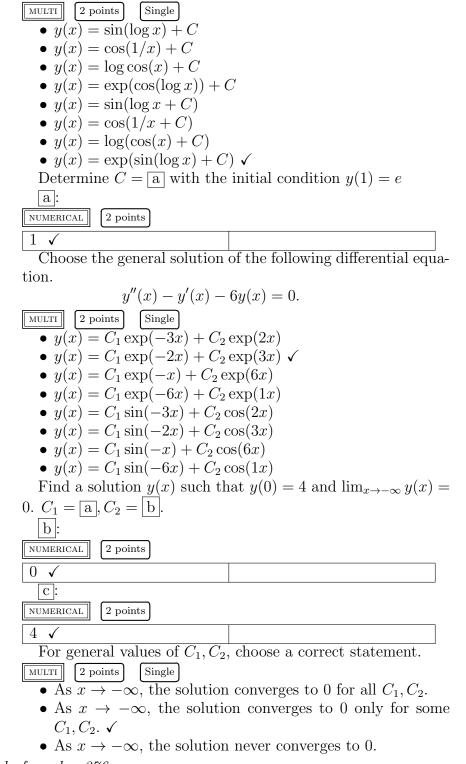
$(10) \ \mathbf{Q5}$

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = y(x)\cos(\log x)/x.$$



Total of marks: 276