2021Call6.

(1) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Write the **Taylor formula** for the function $\frac{1}{x}$ around $x_0 = -1$ to the second degree:

$$\frac{1}{x} = \boxed{a} + \boxed{b}(x - \boxed{c})^{\boxed{d}} + \boxed{e}(x - \boxed{f})^{\boxed{g}} + o((x+1)^2) \text{ as } x \to -1.$$

$\boxed{a}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{e}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{f}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{g}$:

NUMERICAL | 1 point

2 ✓	

Let us study the limit $\lim_{x\to 0} \frac{(x+1)\sin x + \alpha x}{\log(1+2x^2)+\beta}$.

Complete the formulae.

$$(x+1)\sin x = \boxed{\text{h}} + \boxed{\text{i}}x + \boxed{\text{j}}x^2 + o(x^2) \text{ as } x \to 0.$$

$\boxed{\text{h}}$:

NUMERICAL | 2 points

0 ✓	

$\boxed{\text{i}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{j}}$:

NUMERICAL | 3 points

1 ✓	

$$\log(1+2x^2) = \boxed{\text{k}} + \boxed{\text{l}}x + \boxed{\text{m}}x^2 + o(x^2) \text{ as } x \to 0.$$

$\boxed{\text{k}}$:

NUMERICAL | 2 points

0 ✓	

$\boxed{\text{l}}$:

NUMERICAL | 2 points

0 ✓	

$\boxed{\text{m}}$:

NUMERICAL | 3 points

2 ✓	

Consider the limit

$$\lim_{x\to 0} \frac{(x+1)\sin x + \alpha x}{\log(1+2x^2)+\beta}.$$

If $\alpha = 1$ there is only one value of $\beta = \boxed{\text{n}}$ such that the limit does not exist.

$\boxed{\text{n}}$:

NUMERICAL | 7 points

0 ✓	

If $\beta = 0$, this limit converges for $\alpha = \boxed{\text{o}}$.

$\boxed{\text{o}}$:

NUMERICAL | 7 points

-1 ✓	

In that case, the limit is $\frac{\boxed{\text{p}}}{\boxed{\text{q}}}$.

$\boxed{\text{s}}$:

NUMERICAL | 3 points

1 ✓	

$\boxed{\text{t}}$:

NUMERICAL | 4 points

2 ✓	

(2) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n-1}{2^n}(x+1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = -1+i$, calculate the partial sum $\sum_{n=0}^{2} \frac{3^n-1}{2^n}(x+1)^n = \boxed{\text{a}} + \boxed{\text{b}}i$.

$\boxed{\text{a}}$:

NUMERICAL | 1 point

-2 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

1 ✓	

In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n-1}{2^n}|x+1|^n$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{\text{c}}}{\boxed{\text{d}}}|x+\boxed{\text{e}}|^{\boxed{\text{f}}}.$$

$\boxed{\text{c}}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

2 ✓	

e:

NUMERICAL | 1 point

1 ✓	

f:

NUMERICAL | 1 point

1 ✓	

Therefore, by the ratio test, the series converges absolutely for

MULTI | 2 points | Single

- all x.
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-\frac{5}{3} < x < -\frac{1}{3}$. ✓
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $-\frac{1}{3} < x < \frac{5}{3}$.
- $\frac{1}{2} < x < \frac{3}{2}$.
- $-1 < x < 1$.
- $-1 < x < 3$.
- $x = 0$.
- $1 < x < 3$.

For the case $x = -\frac{3}{2}$, the series

MULTI | 2 points | Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = 1$, the series

MULTI | 2 points | Single

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(3) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n-1}{2^n}(x-1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = 1-i$, calculate the partial sum $\sum_{n=0}^{2} \frac{3^n-1}{2^n}(x-1)^n = \boxed{\text{a}} + \boxed{\text{b}}i$.

$\boxed{\text{a}}$:

NUMERICAL | 1 point

-2 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

-1 ✓	

In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n-1}{2^n}|x-1|^n$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{\text{c}}}{\boxed{\text{d}}}|x + \boxed{\text{e}}|^{\boxed{\text{f}}}.$$

$\boxed{\text{c}}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{\text{f}}$:

NUMERICAL | 1 point

1 ✓	

Therefore, by the ratio test, the series converges absolutely for

MULTI | 2 points | Single

- all x.
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-\frac{5}{3} < x < -\frac{1}{3}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $\frac{1}{3} < x < \frac{5}{3}$. ✓
- $\frac{1}{2} < x < \frac{3}{2}$.
- $-1 < x < 1$.
- $-1 < x < 3$.

- $x = 0$.
- $1 < x < 3$.

For the case $x = \frac{2}{3}$, the series

MULTI | 2 points | Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = 1$, the series

MULTI | 2 points | Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(4) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x^3}{x^2 - x}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single

- -2 (-100%)
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0 ✓
- $\frac{1}{2}$ (-100%)
- 1 ✓
- 2 (-100%)

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -1$ (-100%)
- $y = -\frac{1}{2}$ (-100%)
- $y = 0$ (-100%)

- $y = \frac{1}{2}$ (−100%)
- $y = 2$ (−100%)
- $x = -2$ (−100%)
- $x = -1$ (−100%)
- $x = -\frac{1}{2}$ (−100%)
- $x = 0$ (−100%)
- $x = \frac{1}{2}$ (−100%)
- $x = 1$ ✓
- $x = 2$ (−100%)
- $y = x$ (−100%)
- $y = x + 1$ ✓
- $y = -x$ (−100%)
- $y = -x - 1$ (−100%)
- $y = 2x$ (−100%)

One has

$$f'(3) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 4 points

3 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 4 points

4 ✓	

The function $f(x)$ has $\boxed{\text{c}}$ stationary point(s) in the domain

$\boxed{\text{c}}$:

NUMERICAL | 4 points

2 ✓	

Choose the behaviour of $f(x)$ in the interval $(2, 3)$.

MULTI | 4 points | Single

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(5) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x^3}{x - x^2}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single

- -2 (-100%)
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0 ✓
- $\frac{1}{2}$ (-100%)
- 1 ✓
- 2 (-100%)

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -1$ (-100%)
- $y = -\frac{1}{2}$ (-100%)
- $y = 0$ (-100%)
- $y = \frac{1}{2}$ (-100%)
- $y = 2$ (-100%)
- $x = -2$ (-100%)
- $x = -1$ (-100%)
- $x = -\frac{1}{2}$ (-100%)
- $x = 0$ (-100%)
- $x = \frac{1}{2}$ (-100%)
- $x = 1$ ✓
- $x = 2$ (-100%)
- $y = x$ (-100%)
- $y = x + 1$ (-100%)
- $y = -x - 1$ ✓
- $y = 2x$ (-100%)

One has

$$f'(3) = \frac{\boxed{a}}{\boxed{b}}.$$

$\boxed{a}$:

NUMERICAL | 4 points

-3 ✓	

b:

NUMERICAL | 4 points

4 ✓	

The function $f(x)$ has c stationary point(s) in the domain

c:

NUMERICAL | 4 points

2 ✓	

Choose the behaviour of $f(x)$ in the interval $(2, 3)$.

MULTI | 4 points | Single

- monotonically decreasing ✓
- monotonically increasing
- neither decreasing nor increasing

(6) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 x^2 e^{-x} dx.$$

By applying the integration by parts, we have

$$\int_0^1 x^2 e^{-x} dx = [\boxed{a} x^2 e^{-x}]_0^1 - \int_0^1 \boxed{b} x^{\boxed{c}} e^{-x} dx.$$

a:

NUMERICAL | 1 point

-1 ✓	

b:

NUMERICAL | 1 point

-2 ✓	

c:

NUMERICAL | 1 point

1 ✓	

By continuing the calculation, we obtain

$$\int_0^1 x^2 e^{-x} dx = \boxed{\text{d}} + \boxed{\text{e}} e^{\boxed{\text{f}}}.$$

$\boxed{\text{d}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

-5 ✓	

$\boxed{\text{f}}$:

NUMERICAL | 1 point

-1 ✓	

(7) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^2 x^2 e^{-x} dx.$$

By applying the integration by parts, we have

$$\int_0^2 x^2 e^{-x} dx = [\boxed{\text{a}} x^2 e^{-x}]_0^2 - \int_0^2 \boxed{\text{b}} x^{\boxed{\text{c}}} e^{-x} dx.$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

-2 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 1 point

1 ✓	

By continuing the calculation, we obtain

$$\int_0^2 x^2 e^{-x} dx = \boxed{\text{d}} + \boxed{\text{e}} e^{\boxed{\text{f}}}.$$

$\boxed{\text{d}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

-10 ✓	

$\boxed{\text{f}}$:

NUMERICAL | 1 point

-2 ✓	

(8) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Consider the following improper integral for various $\alpha \in \mathbb{R}$.

$$\int_0^\infty x e^{\alpha x^2} dx.$$

Choose all values of α such that the improper integral is convergent.

MULTI | 4 points | Single

- -6 ✓
- $-\pi$ ✓
- $-e$ ✓
- -2 ✓
- -1 ✓
- $-\frac{1}{2}$ ✓
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 (-100%)
- 2 (-100%)
- π (-100%)
- e (-100%)

- 6 (−100%)

Among the correct options above, take the smallest value of α and calculate the improper integral: $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$.

a:

NUMERICAL 2 points

1 ✓	

b:

NUMERICAL 2 points

12 ✓	

Choose the values of β such that the following integral converges.

$$\int_0^\infty x^{\beta-4}e^{-x^3}dx.$$

MULTI 4 points Single

- -5 (−100%)
- $-\pi$ (−100%)
- $-e$ (−100%)
- -2 (−100%)
- -1 (−100%)
- $-\frac{1}{2}$ (−100%)
- 0 (−100%)
- $\frac{1}{2}$ (−100%)
- 1 (−100%)
- 2 (−100%)
- e (−100%)
- π ✓
- 5 ✓

Total of marks: 138