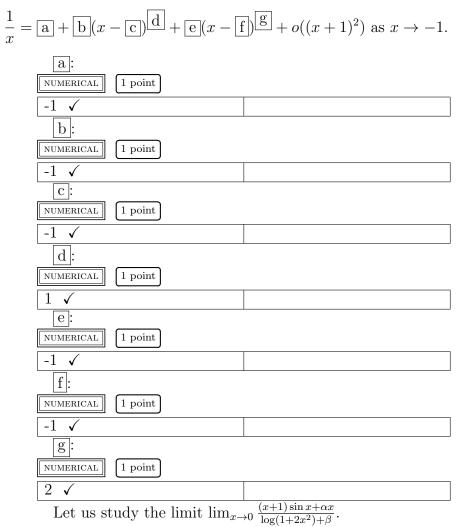
## 2021Call6.

(1) **Q1** 

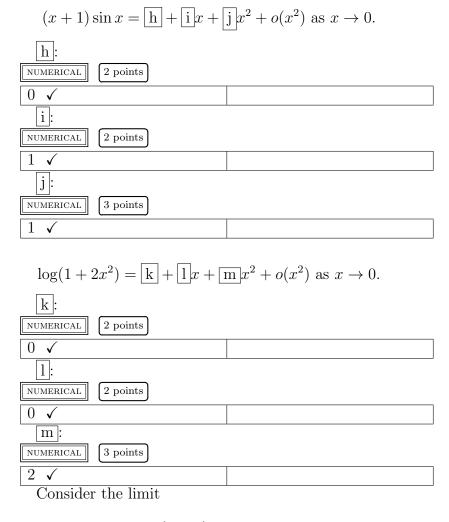
CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (pos-sibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).

Write the **Taylor formula** for the function  $\frac{1}{x}$  around  $x_0 = -1$  to the second degree:

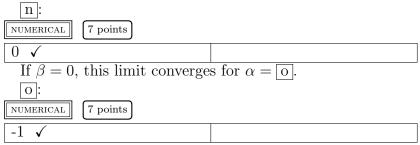


Complete the formulae.

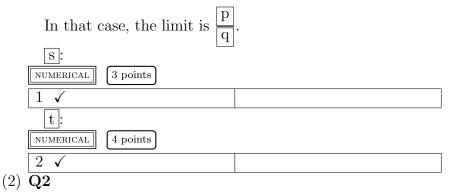


$$\lim_{x \to 0} \frac{(x+1)\sin x + \alpha x}{\log(1+2x^2) + \beta}.$$

If  $\alpha = 1$  there is only one value of  $\beta = \boxed{n}$  such that the limit does not exist.



2



CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (pos-sibly** 0 **or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).

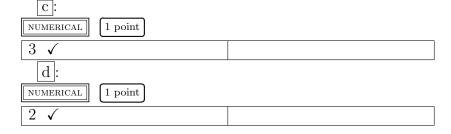
Let us study the following series  $\sum_{n=0}^{\infty} \frac{3^n-1}{2^n} (x+1)^n$ , with various x.

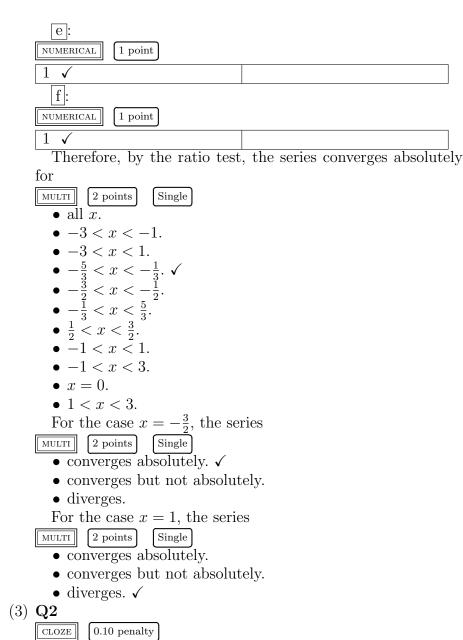
This series makes sense also for  $x \in \mathbb{C}$ . For x = -1 + i, calculate the partial sum  $\sum_{n=0}^{2} \frac{3^n - 1}{2^n} (x+1)^n = \boxed{a} + \boxed{b}i$ .

NUMERICAL 1 point	
-2 🗸	
b:	
NUMERICAL         1 point	
1 🗸	

In order to discuss the convergence using the ratio test for  $x \in \mathbb{R}$ , we put  $a_n = \frac{3^n - 1}{2^n} |x + 1|^n$ . Complete the formula.

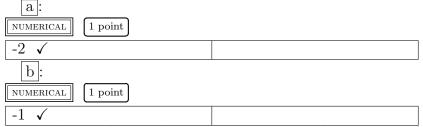
$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{c}}{\boxed{d}} |x + \boxed{e}|^{\boxed{f}}.$$



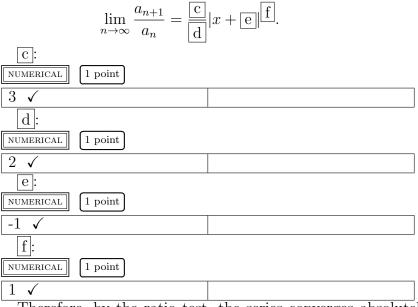


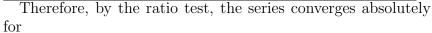
If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not). Let us study the following series  $\sum_{n=0}^{\infty} \frac{3^n-1}{2^n} (x-1)^n$ , with various x.

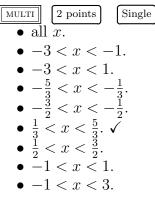
This series makes sense also for  $x \in \mathbb{C}$ . For x = 1-i, calculate the partial sum  $\sum_{n=0}^{2} \frac{3^n-1}{2^n} (x-1)^n = \boxed{a} + \boxed{b}i$ .



In order to discuss the convergence using the ratio test for  $x \in \mathbb{R}$ , we put  $a_n = \frac{3^n - 1}{2^n} |x - 1|^n$ . Complete the formula.







x = 0.
1 < x < 3. For the case x = <sup>2</sup>/<sub>3</sub>, the series
MULTI 2 points Single
converges absolutely. ✓
converges but not absolutely.
diverges.
For the case x = 1, the series
MULTI 2 points Single
converges absolutely. ✓
converges absolutely. ✓
converges but not absolutely.
diverges.

(4) **Q3** 

If not specified otherwise, fill in the blanks with **integers (pos-sibly** 0 **or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

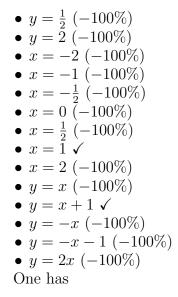
Let us consider the following function

$$f(x) = \frac{x^3}{x^2 - x}.$$

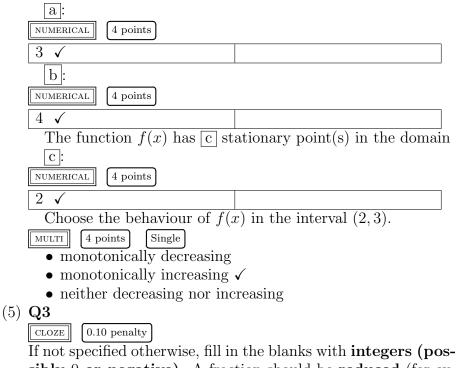
The function f(x) is not defined on the whole real line  $\mathbb{R}$ . Choose all the points that are **not** in the natural domain of f(x).

$$\begin{array}{c} \begin{array}{c} \text{MULTI} & 4 \text{ points} & \text{Single} \\ \hline \bullet & -2 & (-100\%) \\ \bullet & -1 & (-100\%) \\ \bullet & -\frac{1}{2} & (-100\%) \\ \bullet & 0 \checkmark \\ \bullet & \frac{1}{2} & (-100\%) \\ \bullet & 1 \checkmark \\ \bullet & 2 & (-100\%) \\ \hline \text{Choose all asymptotes of } f(x) \\ \hline \begin{array}{c} \text{MULTI} & 4 \text{ points} & \text{Single} \\ \bullet & y = -1 & (-100\%) \\ \bullet & y = -\frac{1}{2} & (-100\%) \\ \bullet & y = 0 & (-100\%) \\ \bullet & y = 0 & (-100\%) \end{array}$$

6



$$f'(3) = \frac{\boxed{a}}{\boxed{b}}$$



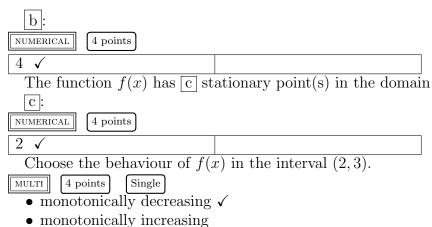
If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ . Let us consider the following function

$$f(x) = \frac{x^3}{x - x^2}.$$

The function f(x) is not defined on the whole real line  $\mathbb{R}$ . Choose all the points that are **not** in the natural domain of f(x).

$$\begin{array}{c} \hline \text{MULTI} & \underline{4 \text{ points}} & \text{Single} \\ \hline & -2 & (-100\%) \\ \hline & -1 & (-100\%) \\ \hline & -\frac{1}{2} & (-100\%) \\ \hline & 0 & \checkmark \\ \hline & \frac{1}{2} & (-100\%) \\ \hline & 0 & \checkmark \\ \hline & \frac{1}{2} & (-100\%) \\ \hline & 1 & \checkmark \\ \hline & 2 & (-100\%) \\ \hline & 1 & \checkmark \\ \hline & 2 & (-100\%) \\ \hline & y & = -\frac{1}{2} & (-100\%) \\ \hline & y & = \frac{1}{2} & (-100\%) \\ \hline & y & = \frac{1}{2} & (-100\%) \\ \hline & y & = 2 & (-100\%) \\ \hline & x & = -2 & (-100\%) \\ \hline & x & = -\frac{1}{2} & (-100\%) \\ \hline & x & = -\frac{1}{2} & (-100\%) \\ \hline & x & = \frac{1}{2} & (-100\%) \\ \hline & x & = \frac{1}{2} & (-100\%) \\ \hline & x & = 1 & \checkmark \\ \hline & x & = 2 & (-100\%) \\ \hline & y & = x & (-100\%) \\ \hline & y & = x & (-100\%) \\ \hline & y & = x & (-100\%) \\ \hline & y & = x & (-100\%) \\ \hline & y & = x & (-100\%) \\ \hline & y & = x & (-100\%) \\ \hline & y & = 2x & (-100\%) \\ \hline & 0 & \text{ne has} \\ \hline & f'(3) & = \begin{array}{|} \underline{a} \\ \hline \\ \hline & \underline{b} \\ \hline \\ \hline \\ \hline & \\ \hline \end{array}$$

8



• neither decreasing nor increasing

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Let us calculate the following integral.

$$\int_0^1 x^2 e^{-x} dx$$

By applying the integration by parts, we have

$$\int_{0}^{1} x^{2} e^{-x} dx = [a] x^{2} e^{-x}]_{0}^{1} - \int_{0}^{1} [b] x^{C} e^{-x} dx.$$

$$a:$$

$$\boxed{\text{NUMERICAL}} \quad 1 \text{ point}$$

$$\boxed{-1 \checkmark}$$

$$\boxed{b}:$$

$$\boxed{\text{NUMERICAL}} \quad 1 \text{ point}$$

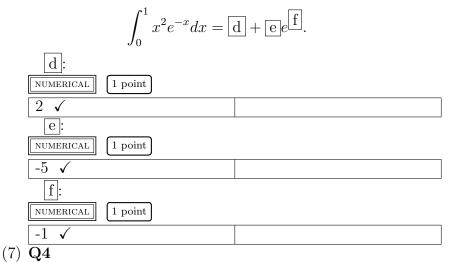
$$\boxed{-2 \checkmark}$$

$$\boxed{c}:$$

$$\boxed{\text{NUMERICAL}} \quad 1 \text{ point}$$

$$\boxed{1 \checkmark}$$

By continuing the calculation, we obtain



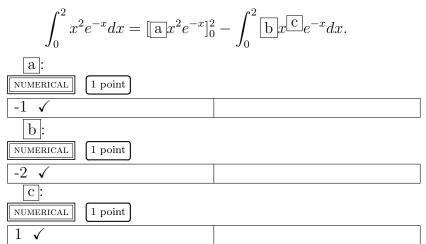
CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\frac{a}{b}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not). log  $x = \log_e x$ , not  $\log_{10} x$ .

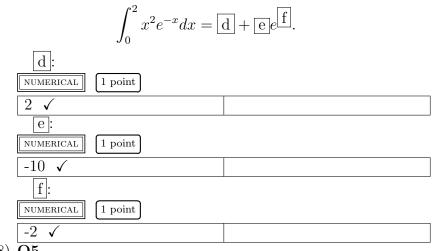
Let us calculate the following integral.

$$\int_0^2 x^2 e^{-x} dx$$

By applying the integration by parts, we have



By continuing the calculation, we obtain



 $(8) \overline{\mathbf{Q5}}$ 

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example,  $\frac{1}{2}$  is accepted but not  $\frac{2}{4}$ ), and if it is negative and the answer boxes (such as  $\boxed{a}$ ) have ambiguity, the negative sign should be put on the numerator (for example  $\frac{-1}{2}$  is accepted but  $\frac{1}{-2}$  is not).  $\log x = \log_e x$ , not  $\log_{10} x$ .

Consider the following improper integral for various  $\alpha \in \mathbb{R}$ .

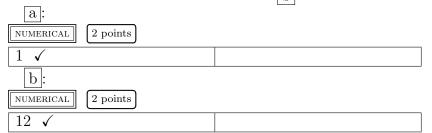
$$\int_0^\infty x e^{\alpha x^2} dx.$$

Choose all values of  $\alpha$  such that the improper integral is convergent.

$$\begin{array}{c|c} \hline \text{MULTI} & 4 \text{ points} & \text{Single} \\ \hline \bullet & -6 \checkmark & \\ \bullet & -\pi \checkmark & \\ \bullet & -e \checkmark & \\ \bullet & -2 \checkmark & \\ \bullet & -1 \checkmark & \\ \bullet & -\frac{1}{2} \checkmark & \\ \bullet & 0 \ (-100\%) & \\ \bullet & \frac{1}{2} \ (-100\%) & \\ \bullet & 1 \ (-100\%) & \\ \bullet & \pi \ (-100\%) & \\ \bullet & e \ (-100\%) & \\ \bullet & e \ (-100\%) & \\ \end{array}$$

● 6 (-100%)

Among the correct options above, take the smallest value of  $\alpha$  and calculate the improper integral:  $\begin{bmatrix} a \\ b \end{bmatrix}$ .



Choose the values of  $\beta$  such that the following integral converges.

$$\int_{0}^{\infty} x^{\beta-4} e^{-x^{3}} dx.$$

$$\boxed{\text{MULTI}} \underbrace{4 \text{ points}}_{0} \underbrace{\text{Single}}_{0}$$

$$= -5 (-100\%)$$

$$= -\pi (-100\%)$$

$$= -2 (-100\%)$$

$$= -1 (-100\%)$$

$$= -\frac{1}{2} (-100\%)$$

$$= \frac{1}{2} (-100\%)$$

$$= 1 (-100\%)$$

$$= 2 (-100\%)$$

$$= e (-100\%)$$

$$= \pi \checkmark$$

$$= 5 \checkmark$$

Total of marks: 138