2021Call5.

(1) **Q1**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$\log(x) = \boxed{a} + \boxed{b}(x-1) + \boxed{c}(x-1)^2 + \boxed{e}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

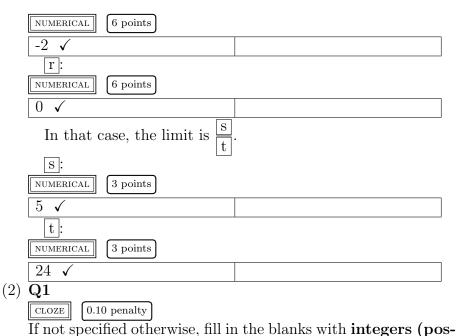
	u	
a:		
NUMERICAL	1 point	
0 🗸		
b:		
NUMERICAL	1 point	
1 🗸		
c:		
NUMERICAL	1 point	
-1 √		
d:		
NUMERICAL	1 point	
2 🗸		
e:		
NUMERICAL	1 point	
1 🗸		
f:		
NUMERICAL	1 point	
3 ✓		

$$(x-1)x^{\frac{1}{2}} = \boxed{g} + \boxed{h}(x-1) + \boxed{\frac{i}{j}}(x-1)^2 + \boxed{\frac{k}{l}}(x-1)^3 \text{ as } x \to 1.$$

g:

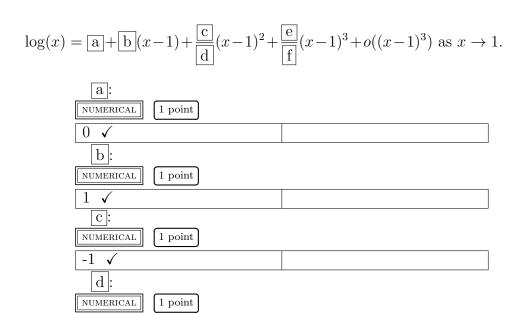
NUMERICAL 1 point

	0 🗸		
	h:		
	NUMERICAL 1 point		
	1 🗸		
	i:		
	NUMERICAL 1 point		
	1 🗸		
	j :		
	NUMERICAL 1 point		
	2 ✓		
	k:		
	NUMERICAL 1 point		
	-1 ✓		
	1:		
	NUMERICAL 1 point		
	8 🗸		
(x-1)	$\sin((x-1)^2) = m + n(x-1) + o(x-1)^2 + p$	$(x-1)^3 + o((x-1)^3)$) as $x \to 1$.
	m:		
	NUMERICAL 1 point		
	0 ✓		
	n:		
	NUMERICAL 1 point		
	0 🗸		
	0:		
	NUMERICAL 1 point		
	<u> </u>		
	NUMERICAL 3 points		
	1 ✓		
	For various $\alpha, \beta \in \mathbb{R}$, study the limit:		
	$\log(x) + (x-1)x^{\frac{1}{2}} + \alpha(x-1) + \beta(x-1)$	$(x-1)^2$	
	$\lim_{x \to 1} \frac{\log(x) + (x-1)x^{\frac{1}{2}} + \alpha(x-1) + \beta(x-1)}{(x-1)\sin((x-1)^2)}$	·	
	This limit converges for $\alpha = [q], \beta = [r]$.		
	q:		



If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.



$\mid 2 \mid \checkmark$		
e:		
NUMERICAL 1 point		
1 ✓		
f:		
NUMERICAL 1 point		
3 🗸		
O V		
$(x-1)x^{\frac{1}{2}} = \boxed{\mathbf{g}} + \boxed{\mathbf{h}}(x-1) + \boxed{\boxed{\mathbf{j}}}(x-1)$	$(x-1)^2 + \frac{k}{1}(x-1)^3 \text{ as } x \to 1.$	
g:		
NUMERICAL 1 point		
0 ✓		
h:		
NUMERICAL 1 point		
1 🗸		
i:		
NUMERICAL 1 point		
1 🗸		
j:		
NUMERICAL 1 point		
2 ✓		
<u>k</u> :		
NUMERICAL 1 point		
<u>-1 √</u>		
NUMERICAL 1 point		
8 🗸		
$(x-1)\sin(2(x-1)^2) = \boxed{m} + \boxed{n}(x-1) + \boxed{n}$	$- o(x-1)^2 + p(x-1)^3 + o((x-1)^3)$	3) as $x \to 1$.
<u>m</u> :		
NUMERICAL 1 point		
0 🗸		
n:		
NUMERICAL 1 point		

0 🗸	
0:	
NUMERICAL 1 point	
0 🗸	
p:	
NUMERICAL 3 points	
2 🗸	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 1} \frac{\log(x) + (x-1)x^{\frac{1}{2}} + \alpha(x-1) + \beta(x-1)^2}{(x-1)\sin(2(x-1)^2)}.$$

This limit converges for $\alpha = \boxed{\mathbf{q}}, \beta = \boxed{\mathbf{r}}$.

q:
NUMERICAL 6 points

-2 √

Γ:
NUMERICAL 6 points

0 √

In that case, the limit is s.

s:
NUMERICAL 3 points

t:
NUMERICAL 3 points
48 ✓

(3) **Q2**CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{2^n-1}{n!} (x+1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For x = i, calculate the partial sum $\sum_{n=0}^{2} \frac{2^{n}-1}{n!} (x+1)^{n} = [a] + [b]i$.



In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{2^n - 1}{n!} |x + 1|^n$. Complete the formula.

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \boxed{\mathbf{c}}$$



Therefore, by the ratio test, the series converges absolutely for

MULTI 2 points Single

- $\overline{\bullet}$ all \overline{x} . \checkmark
- -3 < x < -1.
- -3 < x < 1.
- -3 < x < 1. $-\frac{5}{4} < x < -\frac{3}{4}$. $-\frac{3}{2} < x < -\frac{1}{2}$. $\frac{1}{2} < x < \frac{3}{2}$. $\frac{3}{4} < x < \frac{5}{4}$. -1 < x < 1.

- -1 < x < 3.
- x = 0.
- 1 < x < 3.

For the case $x = -\frac{3}{2}$, the series

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case x = 1, the series

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

$(4) \ \mathbf{Q3}$ CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as a b have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x^3}{x^2 - 1}.$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of f(x).

$$\begin{array}{c|c}
\hline
\text{MULTI} & 4 \text{ points} \\
\bullet & -2 & (-100\%)
\end{array}$$
Single

- \bullet -1 \checkmark
- $-\frac{1}{2}$ (-100%)
- $0 \ (-100\%)$ $\frac{1}{2} \ (-100\%)$ $1 \ \checkmark$
- 2 (-100%)

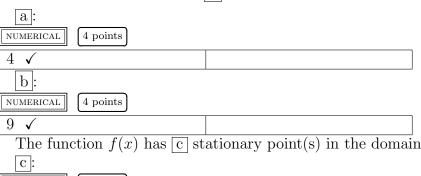
Choose all asymptotes of f(x).

MULTI 4 points Single

- $y = -1 \ (-100\%)$
- $y = -\frac{1}{2} (-100\%)$
- y = 0 (-100%)
- $y = \frac{1}{2} (-100\%)$ y = 2 (-100%)
- $x = -2 \ (-100\%)$
- x = -1
- $x = -\frac{1}{2} (-100\%)$
- x = 0 (-100%) $x = \frac{1}{2}$ (-100%) x = 1 \checkmark
- x = 2 (-100%)
- $y = x \checkmark$
- y = -x (-100%)
- y = 2x (-100%)

One has

$$f'(2) = \frac{\boxed{a}}{\boxed{b}}.$$



4 points NUMERICAL 3 ✓

Choose the behaviour of f(x) in the interval (1, 2).

MULTI 4 points Single

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing \checkmark

(5) **Q3**

0.10 penalty CLOZE

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|a|}{|b|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x^3}{x^2 - 4}.$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of f(x).

$$\begin{array}{c|c}
\hline
\text{MULTI} & 4 \text{ points} \\
\bullet & -2 \checkmark \\
\end{array}$$
Single

- -1 (-100%)
- $-\frac{1}{2}$ (-100%) 0 (-100%) $\frac{1}{2}$ (-100%)

- 1 (−100%)
- 2 √

Choose all asymptotes of f(x).

MULTI 4 points Single

- y = 100%
- $y = -\frac{1}{2} (-100\%)$ y = 0 (-100%)
- $y = \frac{1}{2} (-100\%)$
- y = 2(-100%)
- x = -2
- $x = -1 \ (-100\%)$
- $x = -\frac{1}{2} (-100\%)$ x = 0 (-100%)
- $x = \frac{1}{2} (-100\%)$ x = 1 (-100%)
- x = 2
- $y = x \checkmark$
- y = -x (-100%)
- y = 2x (-100%)

One has

$$f'(1) = \frac{\boxed{\mathbf{a}}}{\boxed{\mathbf{b}}}.$$

a : 4 points NUMERICAL

-11 **√**

b |:

NUMERICAL 4 points

9 🗸

NUMERICAL 4 points

3 ✓

Choose the behaviour of f(x) in the interval (1,2).

MULTI 4 points Single

- monotonically decreasing \checkmark
- monotonically increasing
- neither decreasing nor increasing

$(6) \mathbf{Q4}$

CLOZE 0.10 penalty If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 2x \log(x^2 + 1) dx.$$

We first calculate the primitive of $\log y$ by integration by parts. Fill in the blanks.

$$\int \log(y)dy = [y^{\boxed{\mathbf{a}}}\log(y)] - \int \boxed{\mathbf{b}}dy.$$

J	J
a:	
NUMERICAL 1 point	
1 🗸	
b:	
NUMERICAL 1 point	
1 🗸	

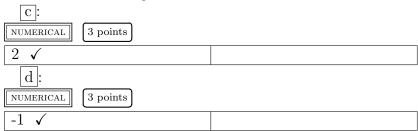
Choose a primitive of the function $\log y$:

- $\bullet y \log y 1$
- $y \log y y \checkmark$
- $y^2 \log y$
- $\log(y^2)$
- $y^2 \log y$
- $y \log y$

Knowing a primitive of $\log y$, the integral

$$\int_0^1 2x \log(x^2 + 1) dx = \boxed{\mathbf{c}} \log 2 + \boxed{\mathbf{d}}$$

can be carried out by substitution.



(7) **Q4**CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers** (**possibly** 0 **or negative**). A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^2 2x \log(x^1 + 1) dx.$$

We first calculate the primitive of $\log y$ by integration by parts. Fill in the blanks.

$$\int \log(y)dy = [y^{\boxed{\mathbf{a}}}\log(y)] - \int \boxed{\mathbf{b}}dy.$$

a:

NUMERICAL 1 point

1
b:

NUMERICAL 1 point

1

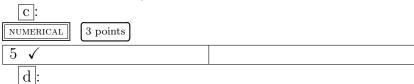
Choose a primitive of the function $\log y$:

- MULTI 4 points Single
 - $\bullet \ y \log y 1$ $\bullet \ y \log y - y \checkmark$
 - $y^2 \log y$
 - $\log(y^2)$
 - $y^2 \log y$
 - $y \log y$

Knowing a primitive of $\log y$, the integral

$$\int_{0}^{2} 2x \log(x^{1} + 1) dx = \boxed{c} \log 5 + \boxed{d}$$

can be carried out by substitution.





 $(8) \mathbf{Q5}$

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = 4x \exp(x^2) y^{\frac{1}{2}}.$$

MULTI 2 points Single

- $\bullet \ y(x) = C \exp(x^2)$
- $y(x) = \exp(x^2) + C$
- $y(x) = 2\exp(x^2) + C$
- $y(x) = C \exp(x^4)$
- $y(x) = \exp(x^4) + C$
- $\bullet \ y(x) = 2\exp(x^4) + C$
- $\bullet \ y(x) = (\exp(x^2) + C)^2 \checkmark$
- $y(x) = 2(\exp(x^2) + C)^2$

Determine $C = \boxed{a} > 0$ with the initial condition y(0) = 9

a > 0:

NUMERICAL 2 points

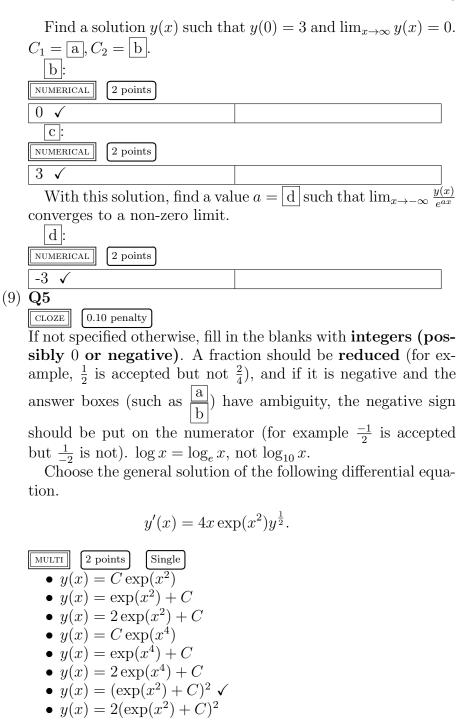
2 **√**

Choose the general solution of the following differential equation.

$$y''(x) + y'(x) - 6y(x) = 0.$$

MULTI 2 points Single

- $\bullet y(x) = C_1 \exp(2x) + C_2 \exp(-3x) \checkmark$
- $y(x) = C_1 \exp(-2x) + C_2 \exp(3x)$
- $\bullet \ y(x) = C_1 \exp(-x) + C_2 \exp(6x)$
- $y(x) = C_1 \exp(-6x) + C_2 \exp(1x)$
- $y(x) = C_1 \sin(-3x) + C_2 \cos(2x)$ • $y(x) = C_1 \sin(-2x) + C_2 \cos(3x)$
- $y(x) = C_1 \sin(-x) + C_2 \cos(6x)$
- $y(x) = C_1 \sin(-6x) + C_2 \cos(1x)$



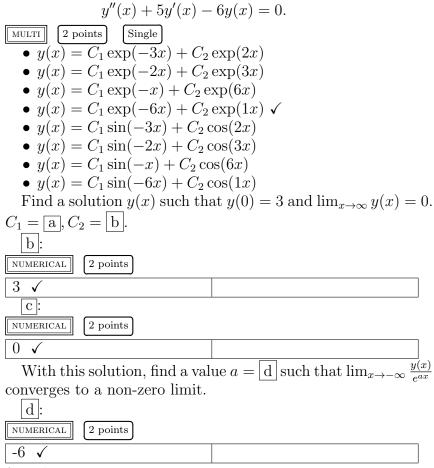
Determine $C = \boxed{a} > 0$ with the initial condition y(0) = 4

|a| > 0:

NUMERICAL

2 points

Choose the general solution of the following differential equation.



Total of marks: 180