2021Call5.

(1) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$\log(x) = \boxed{a} + \boxed{b}(x-1) + \frac{\boxed{c}}{\boxed{d}}(x-1)^2 + \frac{\boxed{e}}{\boxed{f}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{a}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{e}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{f}$:

NUMERICAL | 1 point

3 ✓	

$$(x-1)x^{\frac{1}{2}} = \boxed{g} + \boxed{h}(x-1) + \frac{\boxed{i}}{\boxed{j}}(x-1)^2 + \frac{\boxed{k}}{\boxed{l}}(x-1)^3 \text{ as } x \to 1.$$

$\boxed{g}$:

NUMERICAL | 1 point

0 ✓	

h:

NUMERICAL 1 point

1 ✓	

i:

NUMERICAL 1 point

1 ✓	

j:

NUMERICAL 1 point

2 ✓	

k:

NUMERICAL 1 point

-1 ✓	

l:

NUMERICAL 1 point

8 ✓	

$(x-1)\sin((x-1)^2) = \boxed{\text{m}} + \boxed{\text{n}}(x-1) + \boxed{\text{o}}(x-1)^2 + \boxed{\text{p}}(x-1)^3 + o((x-1)^3)$ as $x \to 1$.

m:

NUMERICAL 1 point

0 ✓	

n:

NUMERICAL 1 point

0 ✓	

o:

NUMERICAL 1 point

0 ✓	

p:

NUMERICAL 3 points

1 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to 1} \frac{\log(x) + (x-1)x^{\frac{1}{2}} + \alpha(x-1) + \beta(x-1)^2}{(x-1)\sin((x-1)^2)}.$$

This limit converges for $\alpha = \boxed{\text{q}}, \beta = \boxed{\text{r}}$.

q:

NUMERICAL | 6 points

-2 ✓	

r:

NUMERICAL | 6 points

0 ✓	

In that case, the limit is $\frac{\boxed{\text{s}}}{\boxed{\text{t}}}$.

s:

NUMERICAL | 3 points

5 ✓	

t:

NUMERICAL | 3 points

24 ✓	

(2) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$\log(x) = \boxed{\text{a}} + \boxed{\text{b}}(x-1) + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}(x-1)^2 + \frac{\boxed{\text{e}}}{\boxed{\text{f}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

a:

NUMERICAL | 1 point

0 ✓	

b:

NUMERICAL | 1 point

1 ✓	

c:

NUMERICAL | 1 point

-1 ✓	

d:

NUMERICAL | 1 point

2 ✓	

e:

NUMERICAL 1 point

1 ✓	

f:

NUMERICAL 1 point

3 ✓	

$$(x-1)x^{\frac{1}{2}} = \boxed{g} + \boxed{h}(x-1) + \frac{\boxed{i}}{\boxed{j}}(x-1)^2 + \frac{\boxed{k}}{\boxed{l}}(x-1)^3 \text{ as } x \to 1.$$

g:

NUMERICAL 1 point

0 ✓	

h:

NUMERICAL 1 point

1 ✓	

i:

NUMERICAL 1 point

1 ✓	

j:

NUMERICAL 1 point

2 ✓	

k:

NUMERICAL 1 point

-1 ✓	

l:

NUMERICAL 1 point

8 ✓	

$$(x-1)\sin(2(x-1)^2) = \boxed{m} + \boxed{n}(x-1) + \boxed{o}(x-1)^2 + \boxed{p}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

m:

NUMERICAL 1 point

0 ✓	

n:

NUMERICAL 1 point

0 ✓	

o:

NUMERICAL | 1 point

0 ✓	

p:

NUMERICAL | 3 points

2 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 1} \frac{\log(x) + (x-1)x^{\frac{1}{2}} + \alpha(x-1) + \beta(x-1)^2}{(x-1)\sin(2(x-1)^2)}.$$

This limit converges for $\alpha =$ [q], $\beta =$ [r].

q:

NUMERICAL | 6 points

-2 ✓	

r:

NUMERICAL | 6 points

0 ✓	

In that case, the limit is $\frac{[s]}{[t]}$.

s:

NUMERICAL | 3 points

5 ✓	

t:

NUMERICAL | 3 points

48 ✓	

(3) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{[a]}{[b]}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{2^n - 1}{n!}(x+1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{2^n - 1}{n!}(x+1)^n = \boxed{\text{a}} + \boxed{\text{b}}\, i$.

$\boxed{\text{a}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

4 ✓	

In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{2^n - 1}{n!}|x+1|^n$. Complete the formula.

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \boxed{\text{c}}$$

$\boxed{\text{c}}$:

NUMERICAL | 4 points

0 ✓	

Therefore, by the ratio test, the series converges absolutely for

MULTI | 2 points | Single

- all x. ✓
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-\frac{5}{4} < x < -\frac{3}{4}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $\frac{1}{2} < x < \frac{3}{2}$.
- $\frac{3}{4} < x < \frac{5}{4}$.
- $-1 < x < 1$.
- $-1 < x < 3$.
- $x = 0$.
- $1 < x < 3$.

For the case $x = -\frac{3}{2}$, the series

MULTI | 2 points | Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = 1$, the series

MULTI | 2 points | Single

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(4) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x^3}{x^2 - 1}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single

- -2 (-100%)
- -1 ✓
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 ✓
- 2 (-100%)

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -1$ (-100%)
- $y = -\frac{1}{2}$ (-100%)
- $y = 0$ (-100%)
- $y = \frac{1}{2}$ (-100%)
- $y = 2$ (-100%)
- $x = -2$ (-100%)
- $x = -1$ ✓
- $x = -\frac{1}{2}$ (-100%)
- $x = 0$ (-100%)
- $x = \frac{1}{2}$ (-100%)
- $x = 1$ ✓
- $x = 2$ (-100%)
- $y = x$ ✓
- $y = -x$ (-100%)
- $y = 2x$ (-100%)

One has

$$f'(2) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 4 points

4 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 4 points

9 ✓	

The function $f(x)$ has $\boxed{\text{c}}$ stationary point(s) in the domain

$\boxed{\text{c}}$:

NUMERICAL | 4 points

3 ✓	

Choose the behaviour of $f(x)$ in the interval $(1, 2)$.

MULTI | 4 points | Single

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(5) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{x^3}{x^2 - 4}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single

- -2 ✓
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)
- $\frac{1}{2}$ (-100%)

- 1 (−100%)
- 2 ✓

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -1$ (−100%)
- $y = -\frac{1}{2}$ (−100%)
- $y = 0$ (−100%)
- $y = \frac{1}{2}$ (−100%)
- $y = 2$ (−100%)
- $x = -2$ ✓
- $x = -1$ (−100%)
- $x = -\frac{1}{2}$ (−100%)
- $x = 0$ (−100%)
- $x = \frac{1}{2}$ (−100%)
- $x = 1$ (−100%)
- $x = 2$ ✓
- $y = x$ ✓
- $y = -x$ (−100%)
- $y = 2x$ (−100%)

One has

$$f'(1) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

a:

NUMERICAL | 4 points

-11 ✓	

b:

NUMERICAL | 4 points

9 ✓	

The function $f(x)$ has c stationary point(s) in the domain

c:

NUMERICAL | 4 points

3 ✓	

Choose the behaviour of $f(x)$ in the interval $(1, 2)$.

MULTI | 4 points | Single

- monotonically decreasing ✓
- monotonically increasing
- neither decreasing nor increasing

(6) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^1 2x \log(x^2+1)dx.$$

We first calculate the primitive of $\log y$ by integration by parts. Fill in the blanks.

$$\int \log(y)dy = [y^{\boxed{a}} \log(y)] - \int \boxed{b} dy.$$

$\boxed{a}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

1 ✓	

Choose a primitive of the function $\log y$:

MULTI | 4 points | Single

- $y \log y - 1$
- $y \log y - y$ ✓
- $y^2 \log y$
- $\log(y^2)$
- $y^2 - \log y$
- $y - \log y$

Knowing a primitive of $\log y$, the integral

$$\int_0^1 2x \log(x^2+1)dx = \boxed{c} \log 2 + \boxed{d}$$

can be carried out by substitution.

$\boxed{c}$:

NUMERICAL | 3 points

2 ✓	

$\boxed{d}$:

NUMERICAL | 3 points

-1 ✓	

(7) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^2 2x \log(x^1 + 1) dx.$$

We first calculate the primitive of $\log y$ by integration by parts. Fill in the blanks.

$$\int \log(y) dy = [y^{\boxed{a}} \log(y)] - \int \boxed{b} dy.$$

$\boxed{a}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

1 ✓	

Choose a primitive of the function $\log y$:

MULTI | 4 points | Single

- $y \log y - 1$
- $y \log y - y$ ✓
- $y^2 \log y$
- $\log(y^2)$
- $y^2 - \log y$
- $y - \log y$

Knowing a primitive of $\log y$, the integral

$$\int_0^2 2x \log(x^1 + 1) dx = \boxed{c} \log 5 + \boxed{d}$$

can be carried out by substitution.

$\boxed{c}$:

NUMERICAL | 3 points

5 ✓	

$\boxed{d}$:

NUMERICAL | 3 points

-4 ✓	

(8) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = 4x\exp(x^2)y^{\frac{1}{2}}.$$

MULTI | 2 points | Single

- $y(x) = C\exp(x^2)$
- $y(x) = \exp(x^2) + C$
- $y(x) = 2\exp(x^2) + C$
- $y(x) = C\exp(x^4)$
- $y(x) = \exp(x^4) + C$
- $y(x) = 2\exp(x^4) + C$
- $y(x) = (\exp(x^2) + C)^2$ ✓
- $y(x) = 2(\exp(x^2) + C)^2$

Determine $C = \boxed{a} > 0$ with the initial condition $y(0) = 9$

$\boxed{a} > 0$:

NUMERICAL | 2 points

2 ✓	

Choose the general solution of the following differential equation.

$$y''(x) + y'(x) - 6y(x) = 0.$$

MULTI | 2 points | Single

- $y(x) = C_1\exp(2x) + C_2\exp(-3x)$ ✓
- $y(x) = C_1\exp(-2x) + C_2\exp(3x)$
- $y(x) = C_1\exp(-x) + C_2\exp(6x)$
- $y(x) = C_1\exp(-6x) + C_2\exp(1x)$
- $y(x) = C_1\sin(-3x) + C_2\cos(2x)$
- $y(x) = C_1\sin(-2x) + C_2\cos(3x)$
- $y(x) = C_1\sin(-x) + C_2\cos(6x)$
- $y(x) = C_1\sin(-6x) + C_2\cos(1x)$

Find a solution $y(x)$ such that $y(0) = 3$ and $\lim_{x\to\infty} y(x) = 0$. $C_1 = \boxed{\text{a}}, C_2 = \boxed{\text{b}}$.

$\boxed{\text{b}}$:

NUMERICAL | 2 points

0 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

3 ✓	

With this solution, find a value $a = \boxed{\text{d}}$ such that $\lim_{x\to-\infty} \frac{y(x)}{e^{ax}}$ converges to a non-zero limit.

$\boxed{\text{d}}$:

NUMERICAL | 2 points

-3 ✓	

(9) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = 4x \exp(x^2) y^{\frac{1}{2}}.$$

MULTI | 2 points | Single

- $y(x) = C \exp(x^2)$
- $y(x) = \exp(x^2) + C$
- $y(x) = 2\exp(x^2) + C$
- $y(x) = C \exp(x^4)$
- $y(x) = \exp(x^4) + C$
- $y(x) = 2\exp(x^4) + C$
- $y(x) = (\exp(x^2) + C)^2$ ✓
- $y(x) = 2(\exp(x^2) + C)^2$

Determine $C = \boxed{\text{a}} > 0$ with the initial condition $y(0) = 4$

$\boxed{\text{a}} > 0$:

NUMERICAL | 2 points

1 ✓	

Choose the general solution of the following differential equation.

$$y''(x) + 5y'(x) - 6y(x) = 0.$$

MULTI | 2 points | Single

- $y(x) = C_1 \exp(-3x) + C_2 \exp(2x)$
- $y(x) = C_1 \exp(-2x) + C_2 \exp(3x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(6x)$
- $y(x) = C_1 \exp(-6x) + C_2 \exp(1x)$ ✓
- $y(x) = C_1 \sin(-3x) + C_2 \cos(2x)$
- $y(x) = C_1 \sin(-2x) + C_2 \cos(3x)$
- $y(x) = C_1 \sin(-x) + C_2 \cos(6x)$
- $y(x) = C_1 \sin(-6x) + C_2 \cos(1x)$

Find a solution $y(x)$ such that $y(0) = 3$ and $\lim_{x\to\infty} y(x) = 0$. $C_1 =$ [a], $C_2 =$ [b].

[b]:

NUMERICAL | 2 points

3 ✓	

[c]:

NUMERICAL | 2 points

0 ✓	

With this solution, find a value $a =$ [d] such that $\lim_{x\to-\infty} \frac{y(x)}{e^{ax}}$ converges to a non-zero limit.

[d]:

NUMERICAL | 2 points

-6 ✓	

Total of marks: 180