

Call4.

(1) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$\sin(-2x) = \boxed{a} + \boxed{b}x + \boxed{c}x^2 + \frac{\boxed{d}}{\boxed{e}}x^3 + o(x^3) \text{ as } x \rightarrow 0.$$

$$\boxed{a}: \boxed{0} \checkmark \quad \boxed{b}: \boxed{-2} \checkmark \quad \boxed{c}: \boxed{0} \checkmark \quad \boxed{d}: \boxed{4} \checkmark \quad \boxed{e}: \boxed{3} \checkmark$$

$$e^x(1+x) = \boxed{g} + \boxed{h}x + \frac{\boxed{i}}{\boxed{j}}x^2 + \frac{\boxed{k}}{\boxed{l}}x^3 \text{ as } x \rightarrow 0.$$

$$\boxed{g}: \boxed{1} \checkmark \quad \boxed{h}: \boxed{2} \checkmark \quad \boxed{i}: \boxed{3} \checkmark \quad \boxed{j}: \boxed{2} \checkmark \quad \boxed{k}: \boxed{2} \checkmark \quad \boxed{l}: \boxed{3} \checkmark$$

$$x \log(1+5x^2) = \boxed{m} + \boxed{n}x + \boxed{o}x^2 + \boxed{p}x^3 + o(x^3) \text{ as } x \rightarrow 0.$$

$$\boxed{m}: \boxed{0} \checkmark \quad \boxed{n}: \boxed{0} \checkmark \quad \boxed{o}: \boxed{0} \checkmark \quad \boxed{p}: \boxed{5} \checkmark$$

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \rightarrow 0} \frac{\sin(-2x) + e^x(1+x) + \alpha + \beta x^2}{x \log(1+5x^2)}.$$

This limit converges for $\alpha = \boxed{q}, \beta = \frac{\boxed{r}}{\boxed{s}}$.

$$\boxed{q}: \boxed{-1} \checkmark \quad \boxed{r}: \boxed{-3} \checkmark \quad \boxed{s}: \boxed{2} \checkmark$$

In that case, the limit is $\frac{\boxed{t}}{\boxed{u}}$.

$$\boxed{y}: \boxed{2} \checkmark \quad \boxed{u}: \boxed{5} \checkmark$$

(2) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign

should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$\sin(2x) = \boxed{a} + \boxed{b}x + \boxed{c}x^2 + \frac{\boxed{d}}{\boxed{e}}x^3 + o(x^3) \text{ as } x \rightarrow 0.$$

$$\boxed{a}: \boxed{0 \quad \checkmark} \quad \boxed{b}: \boxed{2 \quad \checkmark} \quad \boxed{c}: \boxed{0 \quad \checkmark} \quad \boxed{d}: \boxed{-4 \quad \checkmark} \quad \boxed{e}: \boxed{3 \quad \checkmark}$$

$$e^{-x}(1-x) = \boxed{g} + \boxed{h}x + \frac{\boxed{i}}{\boxed{j}}x^2 + \frac{\boxed{k}}{\boxed{l}}x^3 \text{ as } x \rightarrow 0.$$

$$\boxed{g}: \boxed{1 \quad \checkmark} \quad \boxed{h}: \boxed{-2 \quad \checkmark} \quad \boxed{i}: \boxed{3 \quad \checkmark} \quad \boxed{j}: \boxed{2 \quad \checkmark} \quad \boxed{k}: \boxed{-2 \quad \checkmark} \\ \boxed{l}: \boxed{3 \quad \checkmark}$$

$$x \log(1+3x^2) = \boxed{m} + \boxed{n}x + \boxed{o}x^2 + \boxed{p}x^3 + o(x^3) \text{ as } x \rightarrow 0.$$

$$\boxed{m}: \boxed{0 \quad \checkmark} \quad \boxed{n}: \boxed{0 \quad \checkmark} \quad \boxed{o}: \boxed{0 \quad \checkmark} \quad \boxed{p}: \boxed{3 \quad \checkmark}$$

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \rightarrow 0} \frac{\sin(2x) + e^{-x}(1-x) + \alpha + \beta x^2}{x \log(1+3x^2)}.$$

This limit converges for $\alpha = \boxed{q}, \beta = \frac{\boxed{r}}{\boxed{s}}$.

$$\boxed{q}: \boxed{-1 \quad \checkmark} \quad \boxed{r}: \boxed{-3 \quad \checkmark} \quad \boxed{s}: \boxed{2 \quad \checkmark}$$

In that case, the limit is $\frac{\boxed{t}}{\boxed{u}}$.

$$\boxed{y}: \boxed{-2 \quad \checkmark} \quad \boxed{u}: \boxed{3 \quad \checkmark}$$

(3) Q2

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{9^n-1}{n^2+1}(x+1)^{2n}$, with various x .

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^2 \frac{9^n-1}{n^2+1}(x+1)^{2n} = \boxed{a} + i\boxed{b}$.

$$\boxed{a}: \boxed{-64 \quad \checkmark} \quad \boxed{b}: \boxed{8 \quad \checkmark}$$

In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{9^n - 1}{n^2 + 1}(x + 1)^{2n}$. Complete the formula.

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \boxed{c} \| x + \boxed{d} \| \boxed{e}$$

$$\boxed{c}: \boxed{9} \quad \checkmark \quad \boxed{d}: \boxed{1} \quad \checkmark \quad \boxed{e}: \boxed{2} \quad \checkmark$$

Therefore, by the root test, the series converges absolutely for

- all x .
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-\frac{4}{3} < x < -\frac{2}{3}$. \checkmark
- $-\frac{8}{9} < x < -\frac{1}{9}$.
- $\frac{1}{9} < x < \frac{8}{9}$.
- $\frac{2}{3} < x < \frac{4}{3}$.
- $-1 < x < 1$.
- $-1 < x < 3$.
- $x = 0$.
- $1 < x < 3$.

For the case $x = -\frac{3}{2}$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. \checkmark

For the case $x = 1$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. \checkmark

(4) Q2

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{9^n - 1}{n^2 + 1}(x - 1)^{2n}$, with various x .

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^2 \frac{9^n - 1}{n^2 + 1}(x - 1)^{2n} = \boxed{a} + i\boxed{b}$.

$$\boxed{a}: \boxed{-64} \quad \checkmark \quad \boxed{b}: \boxed{-8} \quad \checkmark$$

In order to discuss the convergence using the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{9^n - 1}{n^2 + 1}(x - 1)^{2n}$. Complete the formula.

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \boxed{\text{c}} \| x + \boxed{\text{d}} \| \boxed{\text{e}}$$

$\boxed{\text{c}}$: $\boxed{9 \quad \checkmark}$ $\boxed{\text{d}}$: $\boxed{-1 \quad \checkmark}$ $\boxed{\text{e}}$: $\boxed{2 \quad \checkmark}$

Therefore, by the root test, the series converges absolutely for

- all x .
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-\frac{4}{3} < x < -\frac{2}{3}$.
- $-\frac{8}{9} < x < -\frac{1}{9}$.
- $\frac{1}{9} < x < \frac{8}{9}$.
- $\frac{2}{3} < x < \frac{4}{3}$. \checkmark
- $-1 < x < 1$.
- $-1 < x < 3$.
- $x = 0$.
- $1 < x < 3$.

For the case $x = \frac{2}{3}$, the series

- converges absolutely. \checkmark
- converges but not absolutely.
- diverges.

For the case $x = 1$, the series

- converges absolutely. \checkmark
- converges but not absolutely.
- diverges.

(5) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign

should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{xe^x}{1-x}.$$

The function $f(x)$ is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of $f(x)$.

- $-e$
- -1
- $-\frac{1}{e}$
- 0
- $\frac{1}{e}$
- 1 ✓
- e

Choose all asymptotes of $f(x)$.

- $y = -\pi$
- $y = -\frac{\pi}{2}$
- $y = -\frac{\pi}{4}$
- $y = 0$ ✓
- $y = \frac{\pi}{4}$
- $y = \frac{\pi}{2}$
- $y = e$
- $x = -1$
- $x = -\frac{1}{e}$
- $x = 0$
- $x = 1$ ✓
- $x = \frac{1}{e}$
- $y = x$
- $y = -x$
- $y = ex$

The function $f(x)$ has stationary point(s) in the domain

: ✓

Among the stationary point(s), there is a local maximum at

$$x = \frac{\text{b}}{\text{c}} + \frac{\sqrt{\text{d}}}{\text{e}}.$$

: ✓ : ✓ : ✓ : ✓

Choose the behaviour of $f(x)$ in the interval $(1, 2)$..

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(6) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the

answer boxes (such as $\frac{\text{a}}{\text{b}}$) have ambiguity, the negative sign

should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \frac{xe^{-x}}{1+x}.$$

The function $f(x)$ is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of $f(x)$.

- $-e$
- -1 ✓
- $-\frac{1}{e}$
- 0
- $\frac{1}{e}$
- 1
- e

Choose all asymptotes of $f(x)$.

- $y = -\pi$
- $y = -\frac{\pi}{2}$
- $y = -\frac{\pi}{4}$
- $y = 0$ ✓
- $y = \frac{\pi}{4}$
- $y = \frac{\pi}{2}$
- $y = e$
- $x = -1$ ✓
- $x = -\frac{1}{e}$
- $x = 0$
- $x = 1$
- $x = \frac{1}{e}$
- $y = x$
- $y = -x$
- $y = ex$

The function $f(x)$ has stationary point(s) in the domain

: ✓

Among the stationary point(s), there is a local maximum at

$$x = \frac{\text{b}}{\text{c}} + \frac{\sqrt{\text{d}}}{\text{e}}.$$

: ✓ : ✓ : ✓ : ✓

Choose the behaviour of $f(x)$ in the interval $(1, 2)$..

- monotonically decreasing ✓
- monotonically increasing

- neither decreasing nor increasing

(7) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us calculate the following integral.

$$\int_0^{\frac{\pi}{2}} \sin^2(x) \cos\left(x + \frac{\pi}{6}\right) dx.$$

Complete the formula

$$\cos\left(x + \frac{\pi}{6}\right) = \frac{\sqrt{\boxed{a}}}{\boxed{b}} \cos x + \frac{\boxed{c}}{\boxed{d}} \sin x.$$

a: $\boxed{3} \quad \checkmark$ **b**: $\boxed{2} \quad \checkmark$ **c**: $\boxed{-1} \quad \checkmark$ **d**: $\boxed{2} \quad \checkmark$

Choose a primitive of $\sin^2(x) \cos(x)$.

- $\frac{1}{3} \cos^3(x) \sin(x)$
- $-\frac{1}{3} \cos^3(x) \sin(x)$
- $\frac{1}{3} \sin^3(x) \quad \checkmark$
- $-\frac{1}{3} \sin^3(x)$
- $\frac{1}{2} \cos^3(\sin(x))$
- $-\frac{1}{2} \cos^3(\sin(x))$
- $\frac{1}{2} \sin^3(\cos(x))$
- $-\frac{1}{2} \sin^3(\cos(x))$

Choose a primitive of $\sin^3(x)$.

- $-\frac{1}{4} \cos^4(x)$
- $\frac{1}{4} \sin^4(x)$
- $-\cos(x) + \frac{1}{3} \cos^3(x) \quad \checkmark$
- $\cos(x) - \frac{1}{3} \cos^3(x)$
- $-\cos(x) + \frac{1}{3} \cos^3(x)$
- $\sin(x) - \frac{1}{3} \sin^3(x)$
- $x - \frac{1}{3} \sin^3(x)$
- $x - \frac{1}{3} \cos^3(x)$
- $x + \frac{1}{4} \sin^4(x)$
- $x - \frac{1}{4} \cos^4(x)$

By continuing, we get

$$\int_0^{\frac{\pi}{2}} \sin^2(x) \cos\left(x + \frac{\pi}{6}\right) dx = \frac{\boxed{e}}{\boxed{f}} + \frac{\sqrt{\boxed{g}}}{\boxed{h}}$$

e: ☒ f: ☒ g: ☒ h: ☒

(8) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign

should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us calculate the following integral.

$$\int_0^{\frac{\pi}{2}} \sin^2(x) \cos\left(x + \frac{7\pi}{6}\right) dx.$$

Complete the formula

$$\cos\left(x + \frac{7\pi}{6}\right) = -\frac{\sqrt{\boxed{a}}}{\boxed{b}} \cos x + \frac{\boxed{c}}{\boxed{d}} \sin x.$$

a: ☒ b: ☒ c: ☒ d: ☒

Choose a primitive of $\sin^2(x) \cos(x)$.

- $\frac{1}{3} \cos^3(x) \sin(x)$
- $-\frac{1}{3} \cos^3(x) \sin(x)$
- $\frac{1}{3} \sin^3(x)$ ☒
- $-\frac{1}{3} \sin^3(x)$
- $\frac{1}{2} \cos^3(\sin(x))$
- $-\frac{1}{2} \cos^3(\sin(x))$
- $\frac{1}{2} \sin^3(\cos(x))$
- $-\frac{1}{2} \sin^3(\cos(x))$

Choose a primitive of $\sin^3(x)$.

- $-\frac{1}{4} \cos^4(x)$
- $\frac{1}{4} \sin^4(x)$
- $-\cos(x) + \frac{1}{3} \cos^3(x)$ ☒
- $\cos(x) - \frac{1}{3} \cos^3(x)$
- $-\cos(x) + \frac{1}{3} \cos^3(x)$
- $\sin(x) - \frac{1}{3} \sin^3(x)$
- $x - \frac{1}{3} \sin^3(x)$

- $x - \frac{1}{3} \cos^3(x)$
- $x + \frac{1}{4} \sin^4(x)$
- $x - \frac{1}{4} \cos^4(x)$

By continuing, we get

$$\int_0^{\frac{\pi}{2}} \sin^2(x) \cos\left(x + \frac{7\pi}{6}\right) dx = \frac{\boxed{e}}{\boxed{f}} - \frac{\sqrt{\boxed{g}}}{\boxed{h}}$$

\boxed{e} : $\boxed{1 \quad \checkmark}$ \boxed{f} : $\boxed{3 \quad \checkmark}$ \boxed{g} : $\boxed{3 \quad \checkmark}$ \boxed{h} : $\boxed{6 \quad \checkmark}$

(9) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us consider the improper integral $\int_0^\infty f(x) dx$. Choose all function(s) $f(x)$ for which this improper integral converges.

- $f(x) = \exp(x)$
- $f(x) = \exp(-x)$ ✓
- $f(x) = x^2 \exp(x)$
- $f(x) = x^2 \exp(-x)$ ✓
- $f(x) = \frac{1}{x} \exp(x)$
- $f(x) = \frac{1}{x} \exp(-x)$
- $f(x) = \exp(x^2)$
- $f(x) = \exp(-x^2)$ ✓
- $f(x) = x^2 \exp(x^2)$
- $f(x) = x^2 \exp(-x^2)$ ✓
- $f(x) = \frac{1}{x} \exp(x^2)$
- $f(x) = \frac{1}{x} \exp(-x^2)$

Determine whether the following improper integral converges, and if so, calculate the value. If it does not converge, write $\frac{1}{0}$.

$$\int_0^\infty x^{-\frac{7}{4}} e^{-x^{-\frac{3}{4}}} dx = \frac{\boxed{a}}{\boxed{b}}.$$

\boxed{a} : $\boxed{4 \quad \checkmark}$ \boxed{b} : $\boxed{3 \quad \checkmark}$

Among the following improper integrals, choose the largest (and convergent) one and give its value \boxed{c} .

- $\int_0^\infty x \exp(-x/6) dx$

- $\int_0^\infty \exp(-x/6)$
- $\int_0^\infty x \exp(-x) dx$
- $\int_0^\infty \exp(-x)$
- $\int_0^\infty \frac{1}{x} \exp(-x)$
- $\int_0^\infty x \exp(-3x) dx$
- $\int_0^\infty \exp(-3x) dx$
- $\int_2^\infty \exp(-3x) dx$
- $\int_2^\infty \frac{1}{x} \exp(-3x)$
- $\int_2^\infty x \exp(-3x) dx$
- $\int_2^\infty x^2 \exp(-3x) dx$

\boxed{c} : $\boxed{36 \quad \checkmark}$

(10) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us consider the improper integral $\int_{-\infty}^0 f(x) dx$. Choose all function(s) $f(x)$ for which this improper integral converges.

- $f(x) = \exp(x)$ \checkmark
- $f(x) = \exp(-x)$
- $f(x) = x^2 \exp(x)$ \checkmark
- $f(x) = x^2 \exp(-x)$
- $f(x) = \frac{1}{x} \exp(x)$
- $f(x) = \frac{1}{x} \exp(-x)$
- $f(x) = \exp(x^2)$
- $f(x) = \exp(-x^2)$ \checkmark
- $f(x) = x^2 \exp(x^2)$
- $f(x) = x^2 \exp(-x^2)$ \checkmark
- $f(x) = \frac{1}{x} \exp(x^2)$
- $f(x) = \frac{1}{x} \exp(-x^2)$

Determine whether the following improper integral converges, and if so, calculate the value. If it does not converge, write $\frac{1}{0}$.

$$\int_0^\infty x^{-\frac{7}{5}} e^{-x^{-\frac{2}{5}}} dx = \frac{\boxed{a}}{\boxed{b}}.$$

\boxed{a} : $\boxed{5 \quad \checkmark}$ \boxed{b} : $\boxed{2 \quad \checkmark}$

Among the following improper integrals, choose the largest (and convergent) one and give its value $\boxed{\text{c}}$.

- $\int_0^\infty x \exp(-x/5) dx$
- $\int_0^\infty \exp(-x/5)$
- $\int_0^\infty x \exp(-x) dx$
- $\int_0^\infty \exp(-x)$
- $\int_0^\infty \frac{1}{x} \exp(-x)$
- $\int_0^\infty x \exp(-3x) dx$
- $\int_0^\infty \exp(-3x) dx$
- $\int_2^\infty \exp(-3x) dx$
- $\int_2^\infty \frac{1}{x} \exp(-3x)$
- $\int_2^\infty x \exp(-3x) dx$
- $\int_2^\infty x^2 \exp(-3x) dx$

$\boxed{\text{c}}$: $\boxed{25 \quad \checkmark}$