Call3.

(1) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$e^{-x} = \boxed{a} + \boxed{b}x + \frac{\boxed{c}}{\boxed{d}}x^2 + \frac{\boxed{e}}{\boxed{f}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{a}$: $\boxed{1\ \checkmark}$ $\boxed{b}$: $\boxed{-1\ \checkmark}$ $\boxed{c}$: $\boxed{1\ \checkmark}$ $\boxed{d}$: $\boxed{2\ \checkmark}$ $\boxed{e}$: $\boxed{-1\ \checkmark}$
$\boxed{f}$: $\boxed{6\ \checkmark}$

$$x(1+x)^{\frac{1}{3}} = \boxed{g} + \boxed{h}x + \frac{\boxed{i}}{\boxed{j}}x^2 + \frac{\boxed{k}}{\boxed{l}}x^3 \text{ as } x \to 0.$$

$\boxed{g}$: $\boxed{0\ \checkmark}$ $\boxed{h}$: $\boxed{1\ \checkmark}$ $\boxed{i}$: $\boxed{1\ \checkmark}$ $\boxed{j}$: $\boxed{3\ \checkmark}$ $\boxed{k}$: $\boxed{-1\ \checkmark}$
$\boxed{l}$: $\boxed{9\ \checkmark}$

$$x\sin(x^2) = \boxed{m} + \boxed{n}x + \boxed{o}x^2 + \boxed{p}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{m}$: $\boxed{0\ \checkmark}$ $\boxed{n}$: $\boxed{0\ \checkmark}$ $\boxed{o}$: $\boxed{0\ \checkmark}$ $\boxed{p}$: $\boxed{1\ \checkmark}$

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{e^{-x} + x(1+x)^{\frac{1}{3}} + \alpha + \beta x^2}{x\sin(x^2)}.$$

This limit converges for $\alpha = \boxed{q}$, $\beta = \frac{\boxed{r}}{\boxed{s}}$.

$\boxed{q}$: $\boxed{-1\ \checkmark}$ $\boxed{r}$: $\boxed{-5\ \checkmark}$ $\boxed{s}$: $\boxed{6\ \checkmark}$

In that case, the limit is $\frac{\boxed{t}}{\boxed{u}}$.

$\boxed{y}$: $\boxed{-5\ \checkmark}$ $\boxed{u}$: $\boxed{18\ \checkmark}$

(2) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$e^x = \boxed{\text{a}} + \boxed{\text{b}}x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}x^2 + \frac{\boxed{\text{e}}}{\boxed{\text{f}}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{\text{a}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{b}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{c}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{d}}$: $\boxed{2\ \checkmark}$ $\boxed{\text{e}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{f}}$: $\boxed{6\ \checkmark}$

$$x(1+x)^{\frac{1}{4}} = \boxed{\text{g}} + \boxed{\text{h}}x + \frac{\boxed{\text{i}}}{\boxed{\text{j}}}x^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}x^3 \text{ as } x \to 0.$$

$\boxed{\text{g}}$: $\boxed{0\ \checkmark}$ $\boxed{\text{h}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{i}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{j}}$: $\boxed{4\ \checkmark}$ $\boxed{\text{k}}$: $\boxed{-3\ \checkmark}$ $\boxed{\text{l}}$: $\boxed{32\ \checkmark}$

$$x\sin(5x^2) = \boxed{\text{m}} + \boxed{\text{n}}x + \boxed{\text{o}}x^2 + \boxed{\text{p}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{\text{m}}$: $\boxed{0\ \checkmark}$ $\boxed{\text{n}}$: $\boxed{0\ \checkmark}$ $\boxed{\text{o}}$: $\boxed{0\ \checkmark}$ $\boxed{\text{p}}$: $\boxed{5\ \checkmark}$

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{e^x - x(1+x)^{\frac{1}{4}} + \alpha + \beta x^2}{x\sin(5x^2)}.$$

This limit converges for $\alpha = \boxed{\text{q}}$, $\beta = \frac{\boxed{\text{r}}}{\boxed{\text{s}}}$.

$\boxed{\text{q}}$: $\boxed{-1\ \checkmark}$ $\boxed{\text{r}}$: $\boxed{-1\ \checkmark}$ $\boxed{\text{s}}$: $\boxed{4\ \checkmark}$

In that case, the limit is $\frac{\boxed{\text{t}}}{\boxed{\text{u}}}$.

$\boxed{\text{y}}$: $\boxed{5\ \checkmark}$ $\boxed{\text{u}}$: $\boxed{96\ \checkmark}$

(3) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{4^n - 2^n}{n^3+2}(x+1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = -1 + i$, calculate the partial sum $\sum_{n=0}^{2} \frac{4^n - 2^n}{n^3+2}(x+1)^n = \frac{\boxed{\text{a}}}{\boxed{\text{b}}} + \frac{\boxed{\text{c}}}{\boxed{\text{d}}} i$.

$\boxed{\text{a}}$: $\boxed{-6 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{5 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{d}}$: $\boxed{3 \ \checkmark}$

In order to discuss the convergence using the root test for $x \in \mathbb{R}$, we put $a_n = \frac{4^n - 2^n}{n^3+2}|x+1|^n$. Complete the formula.

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \boxed{\text{e}}|x + \boxed{\text{g}}|^{\boxed{\text{h}}}$$

$\boxed{\text{e}}$: $\boxed{4 \ \checkmark}$ $\boxed{\text{f}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{g}}$: $\boxed{1 \ \checkmark}$

Therefore, by the root test, the series converges absolutely for

- all x.
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-\frac{5}{4} < x < -\frac{3}{4}$. ✓
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $\frac{1}{2} < x < \frac{3}{2}$.
- $\frac{3}{4} < x < \frac{5}{4}$.
- $-1 < x < 1$.
- $-1 < x < 3$.
- $x = 0$.
- $1 < x < 3$.

For the case $x = -\frac{3}{2}$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = 1$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(4) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{4^n - 2^n}{n^3+2}(x-1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = 1-i$, calculate the partial sum $\sum_{n=0}^{2} \frac{4^n-2^n}{n^3+2}(x-1)^n = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}}i$.

$\boxed{a}$: $\boxed{-6 \quad \checkmark}$ $\boxed{b}$: $\boxed{5 \quad \checkmark}$ $\boxed{c}$: $\boxed{-2 \quad \checkmark}$ $\boxed{d}$: $\boxed{3 \quad \checkmark}$

In order to discuss the convergence using the root test for $x \in \mathbb{R}$, we put $a_n = \frac{4^n-2^n}{n^3+2}|x-1|^n$. Complete the formula.

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \boxed{e}|x + \boxed{g}|^{\boxed{h}}$$

$\boxed{e}$: $\boxed{4 \quad \checkmark}$ $\boxed{f}$: $\boxed{-1 \quad \checkmark}$ $\boxed{g}$: $\boxed{1 \quad \checkmark}$

Therefore, by the root test, the series converges absolutely for

- all x.
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-\frac{5}{4} < x < -\frac{3}{4}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $\frac{1}{2} < x < \frac{3}{2}$.
- $\frac{3}{4} < x < \frac{5}{4}$. ✓
- $-1 < x < 1$.
- $-1 < x < 3$.
- $x = 0$.
- $1 < x < 3$.

For the case $x = \frac{5}{4}$, the series

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = 1$, the series

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(5) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \arctan\left(\frac{x}{1+\log(x)}\right).$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- $-e$ ✓
- -1 ✓
- $-\frac{1}{e}$ ✓
- 0 ✓
- $\frac{1}{e}$ ✓
- 1
- e

Choose all asymptotes of $f(x)$.

- $y = -\pi$
- $y = -\frac{\pi}{2}$
- $y = -\frac{\pi}{4}$
- $y = 0$
- $y = \frac{\pi}{4}$
- $y = \frac{\pi}{2}$ ✓
- $y = e$
- $x = -1$
- $x = -\frac{1}{e}$
- $x = 0$
- $x = 1$
- $x = \frac{1}{e}$
- $y = x$
- $y = -x$
- $y = ex$

One has

$$f'(e) = \frac{\boxed{\text{a}}}{e^2 + \boxed{\text{b}}}.$$

$\boxed{\text{a}}$: $\boxed{1 \ ✓}$ $\boxed{\text{b}}$: $\boxed{4 \ ✓}$

The function $f(x)$ has $\boxed{\text{c}}$ stationary point(s) in the domain

$\boxed{\text{c}}$: $\boxed{1 \ ✓}$

Choose the behaviour of $f(x)$ in the interval $[1,2]$..

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(6) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \arctan\left(-\frac{x}{1+\log(x)}\right).$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- $-e$ ✓
- -1 ✓
- $-\frac{1}{e}$ ✓
- 0 ✓
- $\frac{1}{e}$ ✓
- 1
- e

Choose all asymptotes of $f(x)$.

- $y = -\pi$
- $y = -\frac{\pi}{2}$ ✓
- $y = -\frac{\pi}{4}$
- $y = 0$
- $y = \frac{\pi}{4}$
- $y = \frac{\pi}{2}$
- $y = e$
- $x = -1$
- $x = -\frac{1}{e}$
- $x = 0$
- $x = 1$
- $x = \frac{1}{e}$
- $y = x$
- $y = -x$
- $y = ex$

One has

$$f'(e) = \frac{\boxed{\text{a}}}{e^2 + \boxed{\text{b}}}.$$

$\boxed{\text{a}}$: $\boxed{-1 \ ✓}$ $\boxed{\text{b}}$: $\boxed{4 \ ✓}$

The function $f(x)$ has $\boxed{\text{c}}$ stationary point(s) in the domain

$\boxed{\text{c}}$: $\boxed{1 \ \checkmark}$

Choose the behaviour of $f(x)$ in the interval $[1, 2]$..

- monotonically decreasing ✓
- monotonically increasing
- neither decreasing nor increasing

(7) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{\cos\theta} d\theta.$$

With the change of variables $\sin\theta = t$, we get

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{\cos\theta} d\theta = \int_X^Y \boxed{\text{e}}\, dt.$$

Fill in the blanks $X = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}, Y = \frac{\boxed{\text{c}}}{\boxed{\text{d}}}$.

$\boxed{\text{a}}$: $\boxed{-1 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{d}}$: $\boxed{2 \ \checkmark}$

Choose the function $\boxed{\text{e}}$.

- $\arccos t$
- t^2
- $t^2 - 1$
- $1/(1 - t^2)$ ✓
- $1/(t^2)$
- 1

By continuing, we get $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{\cos\theta} d\theta = \log \boxed{\text{f}}$.

$\boxed{\text{f}}$: $\boxed{3 \ \checkmark}$

(8) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_0^{\frac{\pi}{6}} \frac{1}{\cos\theta} d\theta.$$

With the change of variables $\sin\theta = t$, we get

$$\int_0^{\frac{\pi}{6}} \frac{1}{\cos\theta} d\theta = \int_X^Y \boxed{\text{d}} dt.$$

Fill in the blanks $X = \boxed{\text{a}}, Y = \dfrac{\boxed{\text{b}}}{\boxed{\text{c}}}$.

$\boxed{\text{a}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{2 \ \checkmark}$

Choose the function $\boxed{\text{d}}$.

- $\arccos t$
- t^2
- $t^2 - 1$
- $1/(1-t^2)$ ✓
- $1/(t^2)$
- 1

By continuing, we get $\int_0^{\frac{\pi}{6}} \frac{1}{\cos\theta} d\theta = \dfrac{\log \boxed{\text{e}}}{\boxed{\text{f}}}$.

$\boxed{\text{e}}$: $\boxed{3 \ \checkmark}$ $\boxed{\text{f}}$: $\boxed{2 \ \checkmark}$

(9) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\dfrac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = (y(x)+1)^{\frac{1}{2}} x \cos(x^2)$$

.

- $y(x) = (\sin(x^2)/4)^2 + C$
- $y(x) = (\sin(x^2)/4 + C/2)^2 - 1$ ✓
- $y(x) = (\sin(x^2)/2)^2 + C$
- $y(x) = (\sin(x^2)/2 + C/2)^2 - 2$
- $y(x) = (-\cos(x^2)/4)^2 + C$
- $y(x) = (-\cos(x^2)/4 + C/2)^2 - 1$

- $y(x) = (-\cos(x^2)/2)^2 + C$
- $y(x) = (-\cos(x^2)/2 + C/2)^2 - 2$

Determine $C = \boxed{\text{a}} > 0$ with the initial condition $y(0) = 0$

$\boxed{\text{a}}$: $\boxed{2\ \checkmark}$

Choose the general solution of the following differential equation.

$$y''(x) + 2y'(x) - 3y(x) = 0.$$

- $y(x) = C_1 \sin(-3x) + C_2 \cos(x)$
- $y(x) = C_1 \sin(x) + C_2 \cos(x)$
- $y(x) = C_1 \cos(-3x) + C_2 \sin(-x)$
- $y(x) = C_1 \cos(-x) + C_2 \sin(3x)$
- $y(x) = C_1 \exp(-3x) + C_2 \exp(x)$ ✓
- $y(x) = C_1 \exp(-3x) + C_2 \exp(-x)$
- $y(x) = C_1 \exp(3x) + C_2 \exp(-x)$
- $y(x) = C_1 \exp(3x) + C_2 \exp(x)$
- $y(x) = C_1 \exp(3x) + C_2 \cos(-x)$

Choose $C_1 = \boxed{\text{b}}, C_2 = \boxed{\text{c}}$ in such a way that $y(0) = 5$ and $\lim_{x\to\infty} y(x) = 0$.

$\boxed{\text{b}}$: $\boxed{5\ \checkmark}$ $\boxed{\text{c}}$: $\boxed{0\ \checkmark}$

(10) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = (y(x) + 1)^{\frac{1}{2}} x \sin(x^2)$$

.

- $y(x) = (\sin(x^2)/4)^2 + C$
- $y(x) = (\sin(x^2)/4 + C/2)^2 - 1$
- $y(x) = (\sin(x^2)/2)^2 + C$
- $y(x) = (\sin(x^2)/2 + C/2)^2 - 2$
- $y(x) = (-\cos(x^2)/4)^2 + C$
- $y(x) = (-\cos(x^2)/4 + C/2)^2 - 1$ ✓
- $y(x) = (-\cos(x^2)/2)^2 + C$
- $y(x) = (-\cos(x^2)/2 + C/2)^2 - 2$

Determine $C = \dfrac{\boxed{\text{a}}}{\boxed{\text{b}}} > 0$ with the initial condition $y(0) = 0$

$\boxed{\text{a}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{a}}$: $\boxed{2 \ \checkmark}$

Choose the general solution of the following differential equation.

$$y''(x) - 2y'(x) - 3y(x) = 0$$

.

- $y(x) = C_1 \sin(-3x) + C_2 \cos(x)$
- $y(x) = C_1 \sin(x) + C_2 \cos(x)$
- $y(x) = C_1 \cos(-3x) + C_2 \sin(-x)$
- $y(x) = C_1 \cos(-x) + C_2 \sin(3x)$
- $y(x) = C_1 \exp(-3x) + C_2 \cos(x)$
- $y(x) = C_1 \exp(-3x) + C_2 \exp(x)$
- $y(x) = C_1 \exp(3x) + C_2 \exp(-x)$ ✓
- $y(x) = C_1 \exp(3x) + C_2 \exp(x)$
- $y(x) = C_1 \exp(3x) + C_2 \cos(-x)$

Choose $C_1 = \boxed{\text{c}}, C_2 = \boxed{\text{d}}$ in such a way that $y(0) = 5$ and $\lim_{x\to\infty} y(x) = 0$.

$\boxed{\text{c}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{d}}$: $\boxed{5 \ \checkmark}$