Call2.

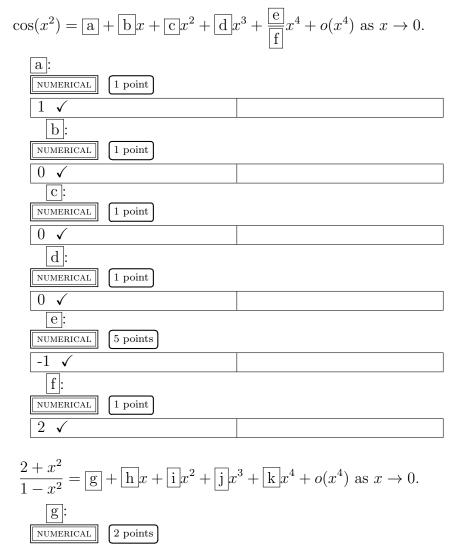
(1) **Q1**

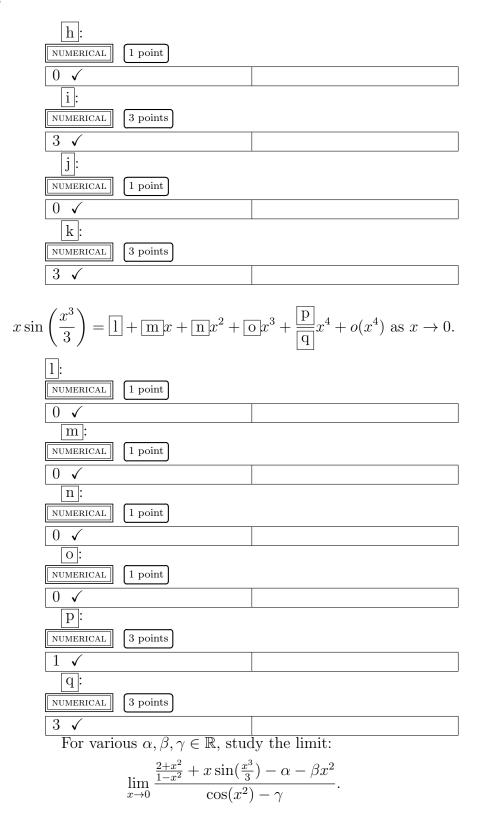
2 √

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

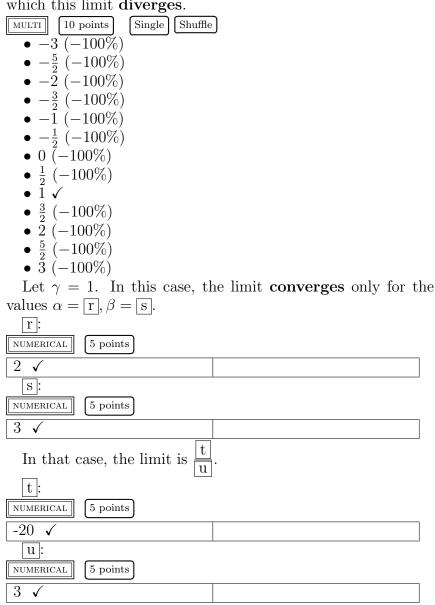
Complete the formulae.





 $\mathbf{2}$

Let $\alpha = 0, \beta = 0$. In this case, choose all the values of γ for which this limit **diverges**.



Use the Taylor formula $f(x) = f(0) + f'(0)x + \frac{1}{2!}f''(0)x^2 + \frac{1}{3!}f^{(3)}(x)x^3 + \frac{1}{4!}^{(4)}(0)x^4 + o(x^4)$ as $x \to 0$. If the expansion of f(x) and g(x) are known, the product f(x)g(x) can be obtained by taking the product of expansions. The composition (for example $f(x^2)$) can be obtained by substituting the expansion of f(y) by $y = x^2$. If $\alpha = \beta = 0$, then the numerator has the expansion $2+o(x^2)$, while the denominator is $\cos(x^2) - \gamma = 1 - \gamma - \frac{x^4}{2}$, so unless $\gamma \neq 1$ this diverges. To determine α, β , one only has to compare the numereator and the denominator and choose α, β, γ in such a way that they have the same degree of infinitesimal (in this case, both of them should be of order x^4).

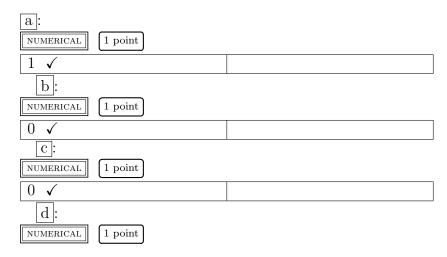
(2) **Q1**

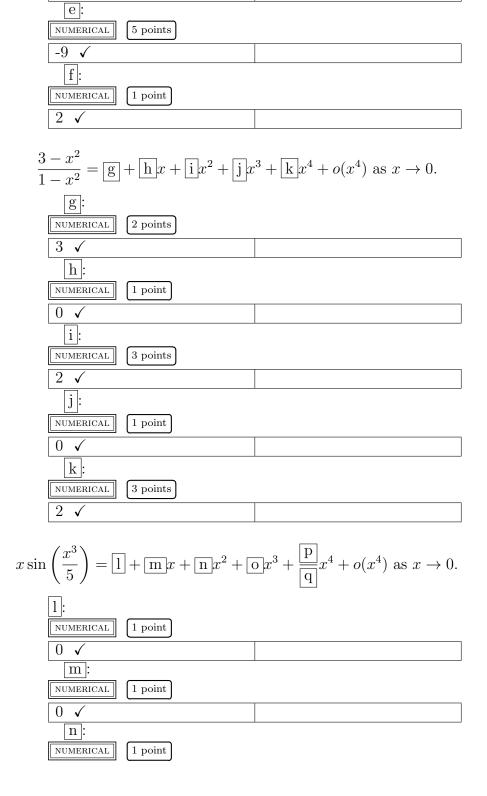
CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (pos-sibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

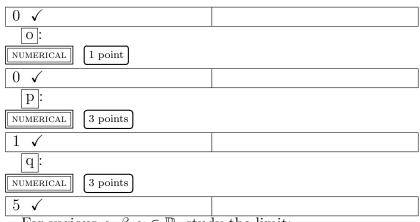
Complete the formulae.

$$\cos(3x^2) = \boxed{\mathbf{a}} + \boxed{\mathbf{b}}x + \boxed{\mathbf{c}}x^2 + \boxed{\mathbf{d}}x^3 + \frac{\boxed{\mathbf{e}}}{\boxed{\mathbf{f}}}x^4 + o(x^4) \text{ as } x \to 0.$$





0 🗸



For various $\alpha, \beta, \gamma \in \mathbb{R}$, study the limit:

$$\lim_{x \to 0} \frac{\frac{3-x^2}{1-x^2} + x\sin(\frac{x^3}{5}) - \alpha - \beta x^2}{2\cos(3x^2) - \gamma}.$$

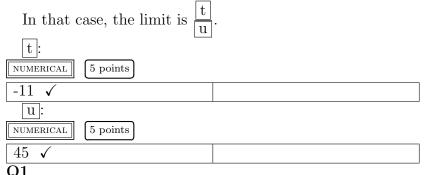
Let $\alpha = 0, \beta = 0$. In this case, choose all the values of γ for which this limit **diverges**.

MULTI 10 points Single Shuffle
\bullet -3 (-100%)
• $-\frac{5}{2}(-100\%)$
• $-2(-100\%)$
• $-\frac{3}{2}(-100\%)$
• $-1(-100\%)$
• $-\frac{1}{2}(-100\%)$
• 0 (-100%)
• $\frac{1}{2}(-100\%)$
• 1 (-100%)
• $\frac{3}{2}(-100\%)$
• 2 v
• $\frac{5}{2}$ (-100%)
• 3 (-100%)
Let $\gamma = 2$. In this case, the limit converges only for the
values $\alpha = [\mathbf{r}], \beta = [\mathbf{s}].$
r:
NUMERICAL 5 points
3 √

 S:

 NUMERICAL

 2 ✓



 $(3) \overline{\mathbf{Q1}}$

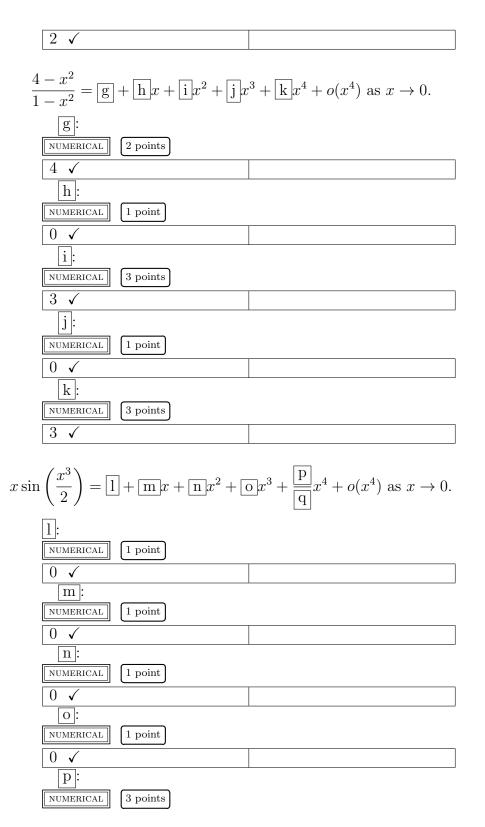
CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (pos-sibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\cos(5x^2) = \boxed{\mathbf{a}} + \boxed{\mathbf{b}}x + \boxed{\mathbf{c}}x^2 + \boxed{\mathbf{d}}x^3 + \frac{\boxed{\mathbf{e}}}{\boxed{\mathbf{f}}}x^4 + o(x^4) \text{ as } x \to 0.$$

a :	
NUMERICAL 1 point	
1 🗸	
b:	
NUMERICAL 1 point	
0 🗸	
С:	
NUMERICAL 1 point	
0 🗸	
d	
NUMERICAL 1 point	
0 🗸	
e:	
NUMERICAL 5 points	
-25 🗸	
f:	
NUMERICAL 1 point	



1 🗸				
q:				
NUMERICAL	3 points			
2 🗸				
Г	•	1 1 1	• • •	

For various $\alpha, \beta, \gamma \in \mathbb{R}$, study the limit:

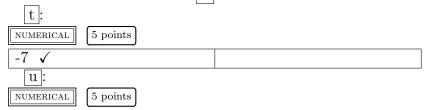
$$\lim_{x \to 0} \frac{\frac{4-x^2}{1-x^2} + x\sin(\frac{x^3}{2}) - \alpha - \beta x^2}{3\cos(5x^2) - \gamma}.$$

Let $\alpha = 0, \beta = 0$. In this case, choose all the values of γ for which this limit **diverges**.

MULTI 10 points Single Shuffle
• $-3(-100\%)$
• $-\frac{5}{2}(-100\%)$
• $-\tilde{2} \ (-100\%)$
• $-\frac{3}{2}(-100\%)$
• $-\tilde{1}$ (-100%)
• $-\frac{1}{2}(-100\%)$
• 0 (-100%)
• $\frac{1}{2}$ (-100%)
• 1 (-100%)
• $\frac{3}{2}$ (-100%)
• 2 (-100%)
• $\frac{5}{2}$ (-100%)
• $\overline{3}$ \checkmark
Let $\gamma = 3$. In this case, the limit converges only for the
values $\alpha = [\mathbf{r}], \beta = [\mathbf{s}].$
r :
NUMERICAL 5 points
4 🗸
S:
NUMERICAL 5 points

3 √

In that case, the limit is $\begin{bmatrix} t \\ u \end{bmatrix}$



(4) $\overline{\mathbf{Q2}}$

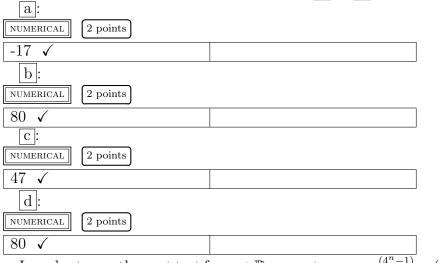
CLOZE 0.10 penalty

 \checkmark

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

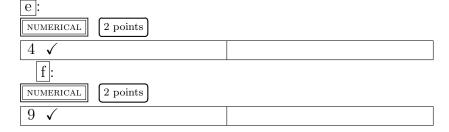
but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$. Let us study the following series $\sum_{n=0}^{\infty} \frac{(-1)^n (4^n - 1)}{(n+1)(3^n+1)^2} (x-1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i\sqrt{3}$, calculate the partial sum $\sum_{n=0}^{2} \frac{(-1)^n (4^n - 1)}{(n+1)(3^n+1)^2} (x-1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} \sqrt{3}i$.



In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{(4^n - 1)^n}{(n+1)(3^n - 1)^2}(x - 1)^{2n}$. Complete the formula.

$$\lim_{n \to \infty} (a_n)^{\frac{1}{n}} = \frac{\underline{e}}{\underline{f}} |\underline{g}x + \underline{h}|^{\underline{i}}$$



g:
NUMERICAL 1 point
$\boxed{1 \checkmark}$
h:
NUMERICAL 2 points
$\overline{-1} \checkmark$
i:
NUMERICAL 1 point
$2 \checkmark$
Therefore, by the root test, the series converges absolutely
for $\boxed{j} < x < \boxed{1}$.
$\left \mathbf{k} \right < x < \mathbf{m}.$
j:
NUMERICAL 6 points
$\overline{-1 \checkmark}$
k:
NUMERICAL 2 points
2
NUMERICAL 6 points
$5 \checkmark$
m:
NUMERICAL 2 points
$2 \checkmark$
For the case $x = -1$, the series
MULTI 8 points Single Shuffle
• converges absolutely.
• converges but not absolutely.
• diverges. ✓
For the case $x = -\frac{1}{2}$, the series
MULTI 8 points Single Shuffle
• converges absolutely.
 converges but not absolutely. ✓ diverges.

The partial sum means the following finite sum: $\sum_{n=0}^{2} a_n = a_0 + a_1 + a_2$, so one just has to apply n = 0, 1, 2in the concrete series and sum the numbers up. Notice that $i^2 = -1$. One can compute $(1 + i\sqrt{3})^4$ by using the fact that $1 + i\sqrt{3} = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}).$ To apply the root test for a positive series $\sum a_n$, one considers $L = \lim_{n \to \infty} (a_n)^{\frac{1}{n}}$. Note that $(4^n - 1)^{\frac{1}{n}} \to 4$, etc. If this limit L < 1, then the series converges absolutely (for such x), while if L > 1 the series diverges. a_n depends on x, and this gives us a condition for which the series converges. That is $\frac{4}{9}|x+1|^2 < 1$, or $-\frac{3}{2} < x+1 < \frac{3}{2}$ If L = 1, one needs to study the convergence with other criteria. In this case, if $x = -\frac{5}{2}$, then a_n is behaves asymptotically as $\frac{(-1)^n}{n+1}$, which converges by the Leibniz criterion, so it also converges.

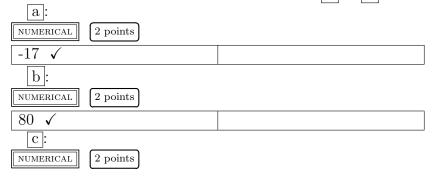
(5) **Q2**

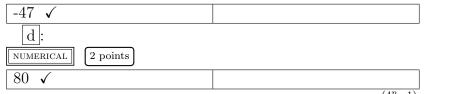
CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). log $x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(-1)^n (4^n-1)}{(n+1)(3^n+1)^2} (x+1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i\sqrt{3}$, calculate the partial sum $\sum_{n=0}^{2} \frac{(-1)^n (4^n - 1)}{(n+1)(3^n+1)^2} (x+1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} \sqrt{3}i.$



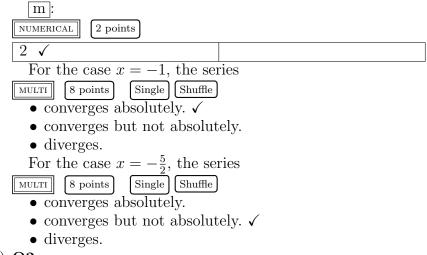


In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{(4^n - 1)}{(n+1)(3^n+1)^2}(x+1)^{2n}$. Complete the formula.

$$\lim_{n \to \infty} (a_n)^{\frac{1}{n}} = \frac{\left[\mathbf{e}\right]}{\left[\mathbf{g}\right]} \mathbf{x} + \mathbf{h} \left[\mathbf{i}\right]$$

$$\mathbf{e}:$$

$$\underbrace{\mathbf{NUMERICAL} 2 \text{ points}} \\ 4 \checkmark \\ \mathbf{f}: \\ \underbrace{\mathbf{NUMERICAL} 2 \text{ points}} \\ 9 \checkmark \\ \underbrace{\mathbf{g}:} \\ \underbrace{\mathbf{NUMERICAL} 1 \text{ point}} \\ 1 \checkmark \\ \underbrace{\mathbf{h}:} \\ \underbrace{\mathbf{numerical} 2 \text{ points}} \\ 1 \checkmark \\ \underbrace{\mathbf{i}:} \\ \underbrace{\mathbf{numerical} 1 \text{ point}} \\ 2 \checkmark \\ \underbrace{\mathbf{numerical} 1 \text{ point}} \\ 2 \checkmark \\ \underbrace{\mathbf{numerical} 1 \text{ point}} \\ \underbrace{\mathbf{g}:} \\ \underbrace{\mathbf{numerical} 1 \text{ point}} \\ \underbrace{\mathbf{numerical} 2 \text{ points}} \\ \underbrace{\mathbf{numerical} 2 \text{ points} \\ \underbrace{\mathbf{numerical} 2 \text{ points}} \\ \underbrace{\mathbf{numerical} 2 \text{ points}} \\ \underbrace{\mathbf{numerical} 2 \text{ points} \\ \underbrace{\mathbf{numerical} 2 \text{ points}} \\ \underbrace{\mathbf{numerical} 2 \text{ po$$

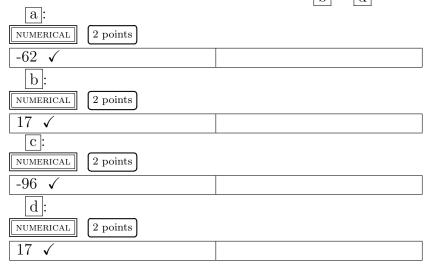


$$(6)$$
 Q2

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(3^n-3)^2}{(n+1)(4^n+1)} (x+1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i\sqrt{3}$, calculate the partial sum $\sum_{n=0}^{2} \frac{(3^n - 3)^2}{(n+1)(4^n + 1)} (x+1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} \sqrt{3}i$.



In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{(3^n - 3)^2}{(n+1)(4^n + 1)}(x+1)^{2n}$. Complete the formula.

$$\lim_{n \to \infty} (a_n)^{\frac{1}{n}} = \frac{e}{[\mathbf{f}]} [\mathbf{g} x + \mathbf{h}]^{\frac{1}{2}}$$
e:
NUMERICAL 2 points
9 \checkmark
f:
NUMERICAL 2 points
4 \checkmark
g:
NUMERICAL 1 point
1 \checkmark
h:
NUMERICAL 2 points
1 \checkmark
NUMERICAL 2 points
1 \checkmark
NUMERICAL 1 point
2 \checkmark
Therefore, by the root test, the series converges absolutely
for $\frac{\mathbf{j}}{\mathbf{k}} < x < \frac{1}{\mathbf{m}}$.
j:
NUMERICAL 6 points
-5 \checkmark
k:
NUMERICAL 2 points
3 \checkmark
I.
NUMERICAL 6 points
-1 \checkmark
Threefores
3 \checkmark
For the case $x = -1$, the series
MUETI 8 points 5 single Shuffle

- converges absolutely. \checkmark
- converges but not absolutely.
- diverges.

For the case $x = -\frac{5}{3}$, the series <u>MULTI</u> 8 points Single Shuffle

- converges absolutely.
- converges but not absolutely.
- diverges. \checkmark
- (7) **Q2**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{|\mathbf{a}|}{|\mathbf{b}|}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(3^n-3)^2}{(n+1)(4^n+1)} (x-1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i\sqrt{3}$, calculate the partial sum $\sum_{n=0}^{2} \frac{(3^n-3)^2}{(n+1)(4^n+1)} (x-1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} \sqrt{3}i$.

a :	
NUMERICAL 2 points	
-62 🗸	
b:	
NUMERICAL 2 points	
17 🗸	
<u> </u>	
NUMERICAL 2 points	
96 🗸	
d:	
NUMERICAL 2 points	
17 🗸	
T 1	$(3^n - 3)^2$

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{(3-3)}{(n+1)(4^n+1)^2}(x-3)$ $1)^{2n}$. Complete the formula.

$$\lim_{n \to \infty} (a_n)^{\frac{1}{n}} = \frac{\underline{e}}{\underline{f}} |\underline{g}x + \underline{h}|^{\underline{i}}$$

e:
NUMERICAL 2 points
9 🗸
f:
NUMERICAL 2 points
$4 \checkmark$
g:
NUMERICAL 1 point
$1 \checkmark$
h:
NUMERICAL 2 points
<u> </u>
NUMERICAL 1 point
$2 \checkmark$
Therefore, by the root test, the series converges absolutely
i I
for $\frac{ \mathbf{J} }{ \mathbf{k} } < x < \frac{ \mathbf{I} }{ \mathbf{m} }$.
j
NUMERICAL 6 points
$1 \checkmark$
NUMERICAL 2 points
Image: Numerical 6 points
$5 \checkmark$
<u> </u>
NUMERICAL 2 points
$3 \checkmark$
For the case $x = -1$, the series
MULTI 8 points Single Shuffle
• converges absolutely. \checkmark
• converges but not absolutely.
• diverges.
For the case $x = \frac{1}{3}$, the series
MULTI 8 points Single Shuffle

- converges absolutely.
- converges but not absolutely.
- diverges. \checkmark
- (8) **Q3**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

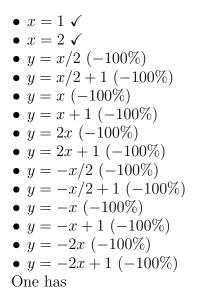
$$f(x) = \log \frac{x^2 + 2}{x^2 - 3x + 2}$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of f(x).

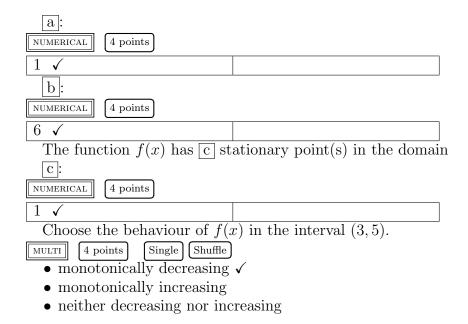
```
MULTI
        4 points
                    Single
                            Shuffle
  • -3(-100\%)
  • -\frac{5}{2} (-100%)
  • -2(-100\%)
    -\frac{3}{2}(-100\%)
    -1 (-100%)
  • -\frac{1}{2} (-100%)
  • 0 (-100\%)
  • \frac{1}{2} (-100%)
  • 1 🗸
  • \frac{3}{2} \checkmark

    2 ✓

  • \frac{5}{2} (-100%)
  • 3(-100\%)
  Choose all asymptotes of f(x).
                    Single Shuffle
MULTI 4 points
  • y = -1 \ (-100\%)
  • y = 0 \checkmark
  • y = 1 \ (-100\%)
  • x = -2 \ (-100\%)
  • x = -1 \ (-100\%)
  • x = 0 \ (-100\%)
```



$$f'(-1) = \frac{a}{b}$$



To determine the natural domain of a function, it is enough to observe the components. For example, $\log y$ is defined for y > 0, $\frac{1}{y-a}$ is defined only for $y \neq a$, etc. It is enought to exclude all such points where the composed function is not defined. In this case, $\frac{x^2+2}{x^2-3x+2} > 0$, while $x^2 + 2 > 0$, so this imposes that $0 < x^2 - 3x + 2 =$ (x-1)(x-2), that is x < 1 or x > 2. There can be asymptotes for $x \to \pm \infty$, and for $x \to a$, where a is a boundary of the domain. In this case, one should check $x \to 1, 2, \pm \infty$. $x \to 1, 2$ give infinity, so there are vertical asymptote there. As for $x \to \pm \infty$, $\frac{x^2+2}{x^2-3x+2} \to 1$, so f(x) tends to 0, and y = 0 is a horizontal asymptote. For the derivative, the chain rule (f(g(x)))'g'(x)f'(g(x)) is useful. In this case, $f(x) = \frac{x^2-3x+2}{x^2-3x+2}$, $f'(x) = \frac{x^2-3x+2}{x^2+2} \cdot \frac{2x(x^2-3x+2)-(x^2+1)(2x^2-3)}{(x^2-3x+2)^2} =$ $\tfrac{-3(x^2-2))}{(x^2+2)(x^2-3x+2)^2}.$ If $f'(x_0) = 0$, x_0 is called a stationary point. In this case, $x_0 = -\sqrt{2}$ because $\sqrt{2}$ is not in the domain. From the formula above for f'(x), we see that f'(x) < 0 for $x \in (3, 5).$

(9) **Q3**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (pos-sibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\boxed{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

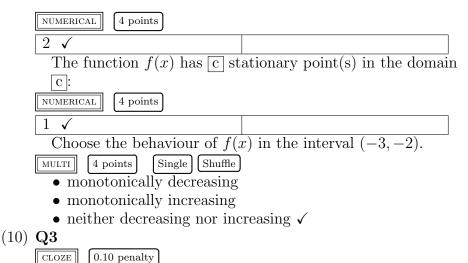
Let us consider the following function

$$f(x) = \log \frac{(x+1)^2 + 2}{x(x-1)}$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of f(x).

 $\underbrace{ \begin{array}{c} \hline \text{MULTI} \\ \bullet \end{array} } \underbrace{ \begin{array}{c} 4 \text{ points} \\ \bullet \end{array} } \underbrace{ \begin{array}{c} \text{Single} \\ \text{Shuffle} \end{array} } \underbrace{ \begin{array}{c} \text{Shuffle} \\ \text{Shuffle} \end{array} } \\ \end{array}$

• $-\frac{5}{2}$ (-100%) • $-\frac{2}{2}(-100\%)$ • $-\frac{3}{2}(-100\%)$ • -1(-100%)• $-\frac{1}{2}(-100\%)$ • 0 v $\begin{array}{c} \bullet \quad \frac{1}{2} \checkmark \\ \bullet \quad 1 \checkmark \end{array}$ • $\frac{3}{2}$ (-100%) • $\tilde{2}$ (-100%) • $\frac{5}{2}$ (-100%) • 3(-100%)Choose all asymptotes of f(x). MULTI 4 points Single Shuffle • $y = -1 \ (-100\%)$ • $y = 0 \checkmark$ • $y = 1 \ (-100\%)$ • $x = -2 \ (-100\%)$ • $x = -1 \ (-100\%)$ • $x = 0 \checkmark$ • $x = 1 \checkmark$ • $x = 2 \ (-100\%)$ • $y = x/2 \ (-100\%)$ • $y = x/2 + 1 \ (-100\%)$ • y = x (-100%)• $y = x + 1 \ (-100\%)$ • $y = 2x \ (-100\%)$ • $y = 2x + 1 \ (-100\%)$ • $y = -x/2 \ (-100\%)$ • $y = -x/2 + 1 \ (-100\%)$ • $y = -x \ (-100\%)$ • $y = -x + 1 \ (-100\%)$ • $y = -2x \ (-100\%)$ • $y = -2x + 1 \ (-100\%)$ One has $f'(-1) = \frac{\boxed{a}}{\boxed{b}}.$ a: NUMERICAL 4 points 3 🗸 b:



If not specified otherwise, fill in the blanks with **integers (pos-sibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log \frac{(x+2)^2 + 2}{x(x+1)}.$$

The function f(x) is not defined on the whole real line \mathbb{R} . Choose all the points that are **not** in the natural domain of f(x).

4 points MULTI Single Shuffle • -3(-100%)• $-\frac{5}{2}(-100\%)$ • -2(-100%)• $-\frac{3}{2}(-100\%)$ • -ī √ • $-\frac{1}{2}$ \checkmark • 0 \ • $\frac{1}{2}$ (-100%) • $\bar{1}$ (-100%) • $\frac{3}{2}$ (-100%) • 2(-100%)• $\frac{5}{2}$ (-100%) • $\bar{3}$ (-100%)

Choose all asymptotes of f(x). MULTI 4 points Single Shuffle • $y = -1 \ (-100\%)$ • $y = 0 \checkmark$ • $y = 1 \ (-100\%)$ • $x = -2 \ (-100\%)$ • $x = -1 \checkmark$ • $x = 0 \checkmark$ • $x = 1 \ (-100\%)$ • $x = 2 \ (-100\%)$ • $y = x/2 \ (-100\%)$ • $y = x/2 + 1 \ (-100\%)$ • y = x (-100%)• $y = x + 1 \ (-100\%)$ • $y = 2x \ (-100\%)$ • $y = 2x + 1 \ (-100\%)$ • $y = -x/2 \ (-100\%)$ • $y = -x/2 + 1 \ (-100\%)$ • $y = -x \ (-100\%)$ • $y = -x + 1 \ (-100\%)$ • $y = -2x \ (-100\%)$ • $y = -2x + 1 \ (-100\%)$ One has $f'(1) = \frac{\boxed{a}}{\boxed{b}}.$ a: NUMERICAL 4 points -21 **√** b : NUMERICAL 4 points 22 √ The function f(x) has c stationary point(s) in the domain c: NUMERICAL 4 points 1 🗸 Choose the behaviour of f(x) in the interval (-2, -1). MULTI 4 points Single Shuffle • monotonically decreasing • monotonically increasing \checkmark

• neither decreasing nor increasing

(11) **Q4** [CLOZE] 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as \boxed{a}) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

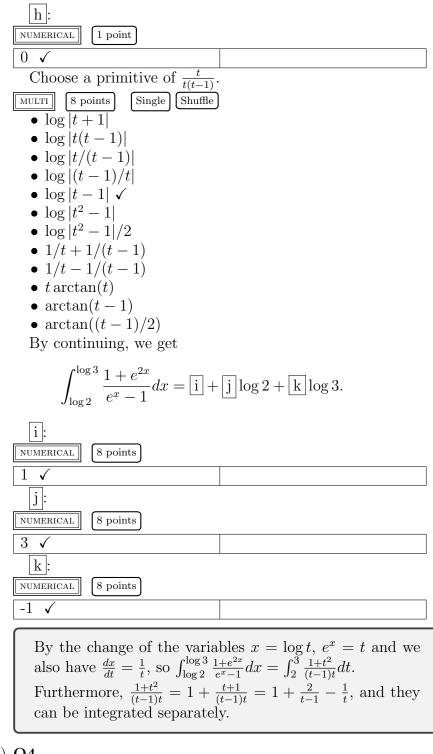
Let us calculate the following integral.

$$\int_{\log 2}^{\log 3} \frac{1+e^{2x}}{e^x-1} dx.$$

Let us apply the change of variables $x = \log t$. Then we have

$$\int_{\log 2}^{\log 3} \frac{1+e^{2x}}{e^x-1} dx = \int_{\boxed{a}}^{\boxed{b}} \frac{\boxed{c}t^2+\boxed{d}t+\boxed{e}}{\boxed{f}t^2+\boxed{g}t+\boxed{h}}.$$

a:	
NUMERICAL 4 points	
$2 \checkmark$	
b:	
NUMERICAL 4 points	
3 🗸	
C	
NUMERICAL 2 points	
1 🗸	
d	
NUMERICAL 1 point	
0 🗸	
e:	
NUMERICAL 2 points	
1 🗸	
f:	
NUMERICAL 1 point	
1 🗸	
g:	
NUMERICAL 1 point	
-1 🗸	



(12) **Q4**

CLOZE 0.10 penalty

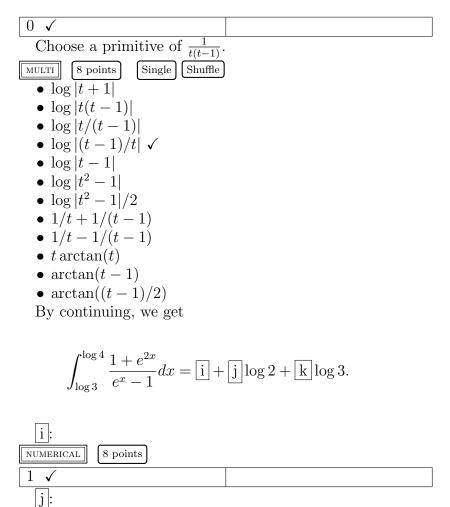
If not specified otherwise, fill in the blanks with integers (possibly 0 or negative). A fraction should be reduced (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$. Let us calculate the following integral.

$$\int_{\log 3}^{\log 4} \frac{1+e^{2x}}{e^x-1} dx.$$

Let us apply the change of variables $x = \log t$. Then we have

$$\int_{\log 3}^{\log 4} \frac{1+e^{2x}}{e^x-1} dx = \int_{\boxed{a}}^{\boxed{b}} \frac{\boxed{c}t^2+\boxed{d}t+\boxed{e}}{\boxed{f}t^2+\boxed{g}t+\boxed{h}}.$$

a:	
NUMERICAL 4 points	
3 🗸	
b:	
NUMERICAL 4 points	
4 🗸	
<u>C</u> :	
NUMERICAL 2 points	
1 🗸	
d:	
NUMERICAL 1 point	
0 🗸	
e:	
NUMERICAL 2 points	
1 🗸	
f:	
NUMERICAL 1 point	
1 🗸	
g:	
NUMERICAL 1 point	
-1 🗸	
h:	
NUMERICAL 1 point	



(13) $\bf{Q4}$

NUMERICAL -4 ✓ k: NUMERICAL

3 √

8 points

8 points

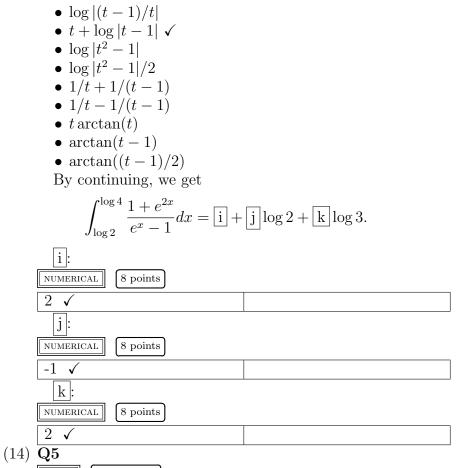
If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). log $x = \log_e x$, not $\log_{10} x$. Let us calculate the following integral.

$$\int_{\log 2}^{\log 4} \frac{1 + e^{2x}}{e^x - 1} dx.$$

Let us apply the change of variables $x = \log t$. Then we have

$\int_{\log 2}^{\log 4} \frac{1+e^{2x}}{e^x-1} dx = \int_{\boxed{a}}^{\boxed{b}} \frac{\boxed{c}t^2+\boxed{d}t+\boxed{e}}{\boxed{f}t^2+\boxed{g}t+\boxed{h}}.$
$J_{\log 2}$ $e^{z} - 1$ $J[a]$ $[t]t^{2} + [g]t + [h]$
NUMERICAL 4 points
b:
NUMERICAL 4 points
NUMERICAL 2 points
NUMERICAL 1 point
0 ✓ e:
NUMERICAL 2 points
Image:
g:
NUMERICAL 1 point
$-1 \checkmark$
h:
NUMERICAL 1 point
Choose a primitive of $\frac{t^2}{t(t-1)}$.
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$
• $t + \log t+1 $
• $\log t(t-1) $
• $t + \log t/(t-1) $

h ____



29

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). log $x = \log_e x$, not $\log_{10} x$.

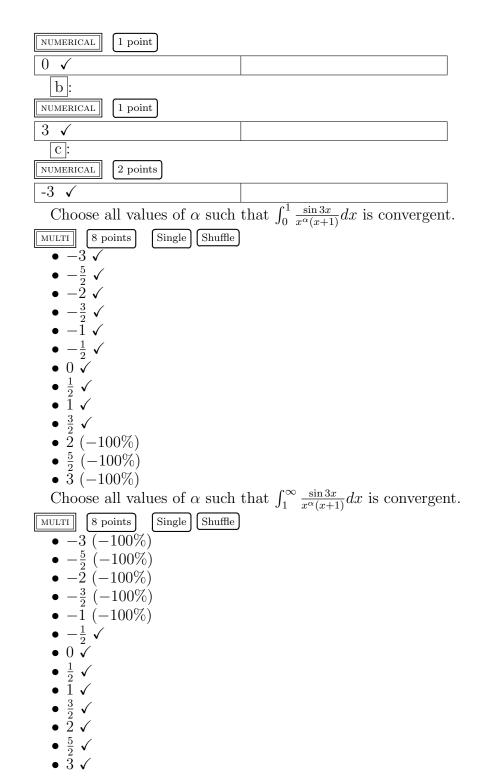
Consider the following improper integral for various $\alpha \in \mathbb{R}$.

$$\int_0^\infty \frac{\sin x}{x^\alpha (x+1)} dx$$

Complete the formulae.

$$\frac{\sin 3x}{x+1} = [a] + [b]x + [c]x^2 + o(x^2) \text{ as } x \to 0.$$

a :



Calculate the following improper integral.

$$\int_{0}^{\infty} \frac{1}{x^{2} + 1} dx = \frac{d}{e} \pi.$$

$$\boxed{d:}$$

$$\boxed{1 \sqrt{}}$$

$$\boxed{e:}$$

$$\boxed{1 \sqrt{}}$$

$$\boxed{e:}$$

$$\boxed{\text{NUMERICAL} \quad 2 \text{ points}}$$

$$\boxed{2 \sqrt{}}$$

$$\boxed{\int_{0}^{1} \frac{\sin x}{x^{\alpha}(x+1)} dx \text{ converges if } \frac{\sin x}{x^{\alpha}(x+1)} \text{ behaves asymptotically}}$$

 $\int_{0}^{\infty} \frac{\sin x}{x^{\alpha}(x+1)} dx \text{ converges if } \frac{\sin x}{x^{\alpha}(x+1)} \text{ behaves asymptotically} \\ \text{as } x^{\gamma}, \gamma > -1 \text{ as } x \to 0. \text{ This can be studied by Taylor's} \\ \text{formula. On the other hand, } \int_{1}^{\infty} \frac{\sin x}{x^{\alpha}(x+1)} dx \text{ converges if} \\ \frac{\sin x}{x^{\alpha}(x+1)} \text{ behaves asymptotically as } x^{\gamma}, \gamma < -1 \text{ as } x \to \infty. \\ \text{As } \sin x \text{ is bounded, this holds if } \alpha > 0. \text{ The border-line case } \alpha = 0 \text{ is also convergent because as a whole it} \\ \text{is asymptotically equal to } \frac{\sin x}{x+1}, \text{ which is convergent (not absolutely).} \\ \text{Note that } \int \frac{1}{x^{2}+1} dx = \arctan x. \end{cases}$

(15) **Q5**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{a}{b}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). log $x = \log_e x$, not $\log_{10} x$.

Consider the following improper integral for various $\alpha \in \mathbb{R}$.

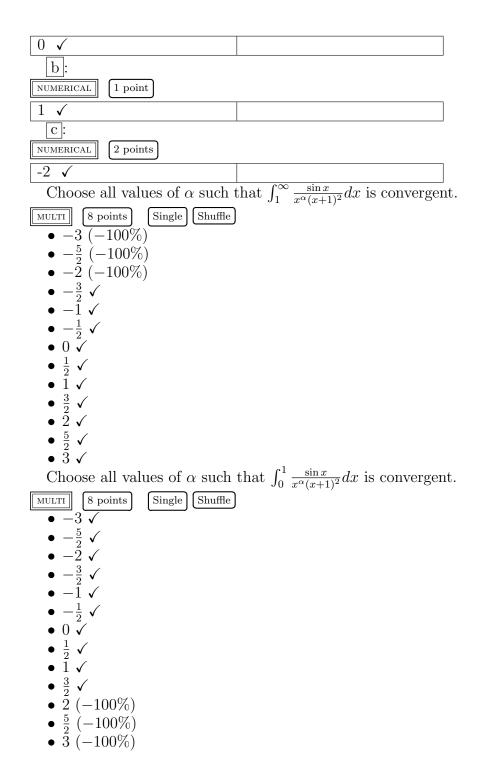
$$\int_0^\infty \frac{\sin x}{x^\alpha (x+1)} dx$$

Complete the formulae.

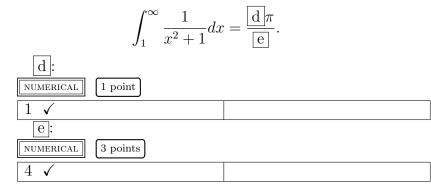
$$\frac{\sin x}{(x+1)^2} = \boxed{\mathbf{a}} + \boxed{\mathbf{b}}x + \boxed{\mathbf{c}}x^2 + o(x^2) \text{ as } x \to 0.$$

$$\boxed{\mathbf{a}}:$$

$$\boxed{\text{NUMERICAL}} \qquad \boxed{1 \text{ point}}$$



Calculate the following improper integral.



Total of marks: 636