Call2.

(1) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\cos(x^2) = \boxed{a} + \boxed{b}x + \boxed{c}x^2 + \boxed{d}x^3 + \frac{\boxed{e}}{\boxed{f}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{a}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{e}$:

NUMERICAL | 5 points

-1 ✓	

$\boxed{f}$:

NUMERICAL | 1 point

2 ✓	

$$\frac{2+x^2}{1-x^2} = \boxed{g} + \boxed{h}x + \boxed{i}x^2 + \boxed{j}x^3 + \boxed{k}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{g}$:

NUMERICAL | 2 points

2 ✓	

h:

NUMERICAL 1 point

0 ✓	

i:

NUMERICAL 3 points

3 ✓	

j:

NUMERICAL 1 point

0 ✓	

k:

NUMERICAL 3 points

3 ✓	

$$x \sin\left(\frac{x^3}{3}\right) = \boxed{\text{l}} + \boxed{\text{m}}x + \boxed{\text{n}}x^2 + \boxed{\text{o}}x^3 + \frac{\boxed{\text{p}}}{\boxed{\text{q}}}x^4 + o(x^4) \text{ as } x \to 0.$$

l:

NUMERICAL 1 point

0 ✓	

m:

NUMERICAL 1 point

0 ✓	

n:

NUMERICAL 1 point

0 ✓	

o:

NUMERICAL 1 point

0 ✓	

p:

NUMERICAL 3 points

1 ✓	

q:

NUMERICAL 3 points

3 ✓	

For various $\alpha, \beta, \gamma \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{\frac{2+x^2}{1-x^2} + x\sin(\frac{x^3}{3}) - \alpha - \beta x^2}{\cos(x^2) - \gamma}.$$

Let $\alpha = 0, \beta = 0$. In this case, choose all the values of γ for which this limit **diverges**.

MULTI | 10 points | Single | Shuffle

- -3 (-100%)
- $-\frac{5}{2}$ (-100%)
- -2 (-100%)
- $-\frac{3}{2}$ (-100%)
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 ✓
- $\frac{3}{2}$ (-100%)
- 2 (-100%)
- $\frac{5}{2}$ (-100%)
- 3 (-100%)

Let $\gamma = 1$. In this case, the limit **converges** only for the values $\alpha = \boxed{\text{r}}, \beta = \boxed{\text{s}}$.

$\boxed{\text{r}}$:

NUMERICAL | 5 points

2 ✓	

$\boxed{\text{s}}$:

NUMERICAL | 5 points

3 ✓	

In that case, the limit is $\frac{\boxed{\text{t}}}{\boxed{\text{u}}}$.

$\boxed{\text{t}}$:

NUMERICAL | 5 points

-20 ✓	

$\boxed{\text{u}}$:

NUMERICAL | 5 points

3 ✓	

> Use the Taylor formula $f(x) = f(0)+f'(0)x+\frac{1}{2!}f''(0)x^2+\frac{1}{3!}f^{(3)}(x)x^3+\frac{1}{4!}^{(4)}(0)x^4+o(x^4)$ as $x \to 0$. If the expansion of $f(x)$ and $g(x)$ are known, the product $f(x)g(x)$ can be obtained by taking the product of expansions. The composition (for example $f(x^2)$) can be obtained by substituting the expansion of $f(y)$ by $y = x^2$.
> If $\alpha = \beta = 0$, then the numerator has the expansion $2+o(x^2)$, while the denominator is $\cos(x^2)-\gamma = 1-\gamma-\frac{x^4}{2}$, so unless $\gamma \neq 1$ this diverges.
> To determine α, β, one only has to compare the numereator and the denominator and choose α, β, γ in such a way that they have the same degree of infinitesimal (in this case, both of them should be of order x^4).

(2) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\cos(3x^2) = \boxed{a} + \boxed{b}x + \boxed{c}x^2 + \boxed{d}x^3 + \frac{\boxed{e}}{\boxed{f}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{a}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

0 ✓	

e:

NUMERICAL | 5 points

-9 ✓	

f:

NUMERICAL | 1 point

2 ✓	

$$\frac{3-x^2}{1-x^2} = \boxed{\text{g}} + \boxed{\text{h}}x + \boxed{\text{i}}x^2 + \boxed{\text{j}}x^3 + \boxed{\text{k}}x^4 + o(x^4) \text{ as } x \to 0.$$

g:

NUMERICAL | 2 points

3 ✓	

h:

NUMERICAL | 1 point

0 ✓	

i:

NUMERICAL | 3 points

2 ✓	

j:

NUMERICAL | 1 point

0 ✓	

k:

NUMERICAL | 3 points

2 ✓	

$$x \sin\left(\frac{x^3}{5}\right) = \boxed{\text{l}} + \boxed{\text{m}}x + \boxed{\text{n}}x^2 + \boxed{\text{o}}x^3 + \frac{\boxed{\text{p}}}{\boxed{\text{q}}}x^4 + o(x^4) \text{ as } x \to 0.$$

l:

NUMERICAL | 1 point

0 ✓	

m:

NUMERICAL | 1 point

0 ✓	

n:

NUMERICAL | 1 point

0 ✓	

o:

NUMERICAL 1 point

0 ✓	

p:

NUMERICAL 3 points

1 ✓	

q:

NUMERICAL 3 points

5 ✓	

For various $\alpha, \beta, \gamma \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{\frac{3-x^2}{1-x^2} + x\sin(\frac{x^3}{5}) - \alpha - \beta x^2}{2\cos(3x^2) - \gamma}.$$

Let $\alpha = 0, \beta = 0$. In this case, choose all the values of γ for which this limit **diverges**.

MULTI 10 points Single Shuffle

- -3 (-100%)
- $-\frac{5}{2}$ (-100%)
- -2 (-100%)
- $-\frac{3}{2}$ (-100%)
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 (-100%)
- $\frac{3}{2}$ (-100%)
- 2 ✓
- $\frac{5}{2}$ (-100%)
- 3 (-100%)

Let $\gamma = 2$. In this case, the limit **converges** only for the values $\alpha =$ r, $\beta =$ s.

r:

NUMERICAL 5 points

3 ✓	

s:

NUMERICAL 5 points

2 ✓	

In that case, the limit is $\frac{\boxed{\text{t}}}{\boxed{\text{u}}}$.

$\boxed{\text{t}}$:

NUMERICAL | 5 points

-11 ✓	

$\boxed{\text{u}}$:

NUMERICAL | 5 points

45 ✓	

(3) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\cos(5x^2) = \boxed{\text{a}} + \boxed{\text{b}}x + \boxed{\text{c}}x^2 + \boxed{\text{d}}x^3 + \frac{\boxed{\text{e}}}{\boxed{\text{f}}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 5 points

-25 ✓	

$\boxed{\text{f}}$:

NUMERICAL | 1 point

2 ✓	

$$\frac{4-x^2}{1-x^2} = \boxed{\text{g}} + \boxed{\text{h}}x + \boxed{\text{i}}x^2 + \boxed{\text{j}}x^3 + \boxed{\text{k}}x^4 + o(x^4) \text{ as } x \to 0.$$

g:

NUMERICAL 2 points

4 ✓	

h:

NUMERICAL 1 point

0 ✓	

i:

NUMERICAL 3 points

3 ✓	

j:

NUMERICAL 1 point

0 ✓	

k:

NUMERICAL 3 points

3 ✓	

$$x \sin\left(\frac{x^3}{2}\right) = \boxed{\text{l}} + \boxed{\text{m}}x + \boxed{\text{n}}x^2 + \boxed{\text{o}}x^3 + \frac{\boxed{\text{p}}}{\boxed{\text{q}}}x^4 + o(x^4) \text{ as } x \to 0.$$

l:

NUMERICAL 1 point

0 ✓	

m:

NUMERICAL 1 point

0 ✓	

n:

NUMERICAL 1 point

0 ✓	

o:

NUMERICAL 1 point

0 ✓	

p:

NUMERICAL 3 points

1 ✓	

q:

NUMERICAL | 3 points

2 ✓	

For various $\alpha, \beta, \gamma \in \mathbb{R}$, study the limit:

$$\lim_{x \to 0} \frac{\frac{4-x^2}{1-x^2} + x\sin(\frac{x^3}{2}) - \alpha - \beta x^2}{3\cos(5x^2) - \gamma}.$$

Let $\alpha = 0, \beta = 0$. In this case, choose all the values of γ for which this limit **diverges**.

MULTI | 10 points | Single | Shuffle

- -3 (-100%)
- $-\frac{5}{2}$ (-100%)
- -2 (-100%)
- $-\frac{3}{2}$ (-100%)
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 (-100%)
- $\frac{3}{2}$ (-100%)
- 2 (-100%)
- $\frac{5}{2}$ (-100%)
- 3 ✓

Let $\gamma = 3$. In this case, the limit **converges** only for the values $\alpha =$ r, $\beta =$ s.

r:

NUMERICAL | 5 points

4 ✓	

s:

NUMERICAL | 5 points

3 ✓	

In that case, the limit is $\frac{t}{u}$.

t:

NUMERICAL | 5 points

-7 ✓	

u:

NUMERICAL | 5 points

75 ✓	

(4) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(-1)^n(4^n-1)}{(n+1)(3^n+1)^2}(x-1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i\sqrt{3}$, calculate the partial sum $\sum_{n=0}^{2} \frac{(-1)^n(4^n-1)}{(n+1)(3^n+1)^2}(x-1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}}\sqrt{3}i$.

$\boxed{a}$:

NUMERICAL | 2 points

-17 ✓	

$\boxed{b}$:

NUMERICAL | 2 points

80 ✓	

$\boxed{c}$:

NUMERICAL | 2 points

47 ✓	

$\boxed{d}$:

NUMERICAL | 2 points

80 ✓	

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{(4^n-1)}{(n+1)(3^n-1)^2}(x-1)^{2n}$. Complete the formula.

$$\lim_{n\to\infty}(a_n)^{\frac{1}{n}} = \frac{\boxed{e}}{\boxed{f}}|\boxed{g}x + \boxed{h}|^{\boxed{i}}$$

$\boxed{e}$:

NUMERICAL | 2 points

4 ✓	

$\boxed{f}$:

NUMERICAL | 2 points

9 ✓	

g:

NUMERICAL 1 point

1 ✓	

h:

NUMERICAL 2 points

-1 ✓	

i:

NUMERICAL 1 point

2 ✓	

Therefore, by the root test, the series converges absolutely for $\frac{\boxed{j}}{\boxed{k}} < x < \frac{\boxed{l}}{\boxed{m}}$.

j:

NUMERICAL 6 points

-1 ✓	

k:

NUMERICAL 2 points

2 ✓	

l:

NUMERICAL 6 points

5 ✓	

m:

NUMERICAL 2 points

2 ✓	

For the case $x = -1$, the series

MULTI 8 points Single Shuffle

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = -\frac{1}{2}$, the series

MULTI 8 points Single Shuffle

- converges absolutely.
- converges but not absolutely. ✓
- diverges.

The partial sum means the following finite sum: $\sum_{n=0}^{2} a_n = a_0+a_1+a_2$, so one just has to apply $n = 0, 1, 2$ in the concrete series and sum the numbers up. Notice that $i^2 = -1$. One can compute $(1 + i\sqrt{3})^4$ by using the fact that $1 + i\sqrt{3} = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$.
To apply the root test for a positive series $\sum a_n$, one considers $L = \lim_{n\to\infty}(a_n)^{\frac{1}{n}}$. Note that $(4^n - 1)^{\frac{1}{n}} \to 4$, etc.
If this limit $L < 1$, then the series converges absolutely (for such x), while if $L > 1$ the series diverges. a_n depends on x, and this gives us a condition for which the series converges. That is $\frac{4}{9}|x + 1|^2 < 1$, or $-\frac{3}{2} < x + 1 < \frac{3}{2}$
If $L = 1$, one needs to study the convergence with other criteria. In this case, if $x = -\frac{5}{2}$, then a_n is behaves asymptotically as $\frac{(-1)^n}{n+1}$, which converges by the Leibniz criterion, so it also converges.

(5) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(-1)^n(4^n-1)}{(n+1)(3^n+1)^2}(x + 1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i\sqrt{3}$, calculate the partial sum $\sum_{n=0}^{2} \frac{(-1)^n(4^n-1)}{(n+1)(3^n+1)^2}(x + 1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}}\sqrt{3}i$.

$\boxed{a}$:

NUMERICAL | 2 points

-17 ✓	

$\boxed{b}$:

NUMERICAL | 2 points

80 ✓	

$\boxed{c}$:

NUMERICAL | 2 points

-47 ✓	

d:

NUMERICAL | 2 points

80 ✓	

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{(4^n-1)}{(n+1)(3^n+1)^2}(x+1)^{2n}$. Complete the formula.

$$\lim_{n\to\infty}(a_n)^{\frac{1}{n}} = \frac{\boxed{\text{e}}}{\boxed{\text{f}}}|\boxed{\text{g}}x + \boxed{\text{h}}|^{\boxed{\text{i}}}$$

e:

NUMERICAL | 2 points

4 ✓	

f:

NUMERICAL | 2 points

9 ✓	

g:

NUMERICAL | 1 point

1 ✓	

h:

NUMERICAL | 2 points

1 ✓	

i:

NUMERICAL | 1 point

2 ✓	

Therefore, by the root test, the series converges absolutely for $\frac{\boxed{\text{j}}}{\boxed{\text{k}}} < x < \frac{\boxed{\text{l}}}{\boxed{\text{m}}}$.

j:

NUMERICAL | 6 points

-5 ✓	

k:

NUMERICAL | 2 points

2 ✓	

l:

NUMERICAL | 6 points

1 ✓	

m:

NUMERICAL | 2 points

2 ✓	

For the case $x = -1$, the series

MULTI | 8 points | Single | Shuffle

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = -\frac{5}{2}$, the series

MULTI | 8 points | Single | Shuffle

- converges absolutely.
- converges but not absolutely. ✓
- diverges.

(6) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(3^n-3)^2}{(n+1)(4^n+1)}(x+1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i\sqrt{3}$, calculate the partial sum $\sum_{n=0}^{2} \frac{(3^n-3)^2}{(n+1)(4^n+1)}(x+1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}}\sqrt{3}i$.

a:

NUMERICAL | 2 points

-62 ✓	

b:

NUMERICAL | 2 points

17 ✓	

c:

NUMERICAL | 2 points

-96 ✓	

d:

NUMERICAL | 2 points

17 ✓	

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{(3^n-3)^2}{(n+1)(4^n+1)}(x+1)^{2n}$. Complete the formula.

$$\lim_{n\to\infty}(a_n)^{\frac{1}{n}} = \frac{\boxed{\text{e}}}{\boxed{\text{f}}}|\boxed{\text{g}}x+\boxed{\text{h}}|^{\boxed{\text{i}}}$$

e:

NUMERICAL | 2 points

9 ✓	

f:

NUMERICAL | 2 points

4 ✓	

g:

NUMERICAL | 1 point

1 ✓	

h:

NUMERICAL | 2 points

1 ✓	

i:

NUMERICAL | 1 point

2 ✓	

Therefore, by the root test, the series converges absolutely for $\frac{\boxed{\text{j}}}{\boxed{\text{k}}} < x < \frac{\boxed{\text{l}}}{\boxed{\text{m}}}$.

j:

NUMERICAL | 6 points

-5 ✓	

k:

NUMERICAL | 2 points

3 ✓	

l:

NUMERICAL | 6 points

-1 ✓	

m:

NUMERICAL | 2 points

3 ✓	

For the case $x = -1$, the series

MULTI | 8 points | Single | Shuffle

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = -\frac{5}{3}$, the series

MULTI | 8 points | Single | Shuffle

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(7) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(3^n-3)^2}{(n+1)(4^n+1)}(x-1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i\sqrt{3}$, calculate the partial sum $\sum_{n=0}^{2} \frac{(3^n-3)^2}{(n+1)(4^n+1)}(x-1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}}\sqrt{3}i$.

a:

NUMERICAL | 2 points

-62 ✓	

b:

NUMERICAL | 2 points

17 ✓	

c:

NUMERICAL | 2 points

96 ✓	

d:

NUMERICAL | 2 points

17 ✓	

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{(3^n-3)^2}{(n+1)(4^n+1)^2}(x-1)^{2n}$. Complete the formula.

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \frac{\boxed{e}}{\boxed{f}} |\boxed{g}x + \boxed{h}|^{\boxed{i}}$$

e:

NUMERICAL 2 points

9 ✓	

f:

NUMERICAL 2 points

4 ✓	

g:

NUMERICAL 1 point

1 ✓	

h:

NUMERICAL 2 points

-1 ✓	

i:

NUMERICAL 1 point

2 ✓	

Therefore, by the root test, the series converges absolutely for $\frac{\boxed{j}}{\boxed{k}} < x < \frac{\boxed{l}}{\boxed{m}}$.

j:

NUMERICAL 6 points

1 ✓	

k:

NUMERICAL 2 points

3 ✓	

l:

NUMERICAL 6 points

5 ✓	

m:

NUMERICAL 2 points

3 ✓	

For the case $x = -1$, the series

MULTI 8 points Single Shuffle

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = \frac{1}{3}$, the series

MULTI 8 points Single Shuffle

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(8) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log \frac{x^2 + 2}{x^2 - 3x + 2}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single | Shuffle

- -3 (-100%)
- $-\frac{5}{2}$ (-100%)
- -2 (-100%)
- $-\frac{3}{2}$ (-100%)
- -1 (-100%)
- $-\frac{1}{2}$ (-100%)
- 0 (-100%)
- $\frac{1}{2}$ (-100%)
- 1 ✓
- $\frac{3}{2}$ ✓
- 2 ✓
- $\frac{5}{2}$ (-100%)
- 3 (-100%)

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single | Shuffle

- $y = -1$ (-100%)
- $y = 0$ ✓
- $y = 1$ (-100%)
- $x = -2$ (-100%)
- $x = -1$ (-100%)
- $x = 0$ (-100%)

- $x = 1$ ✓
- $x = 2$ ✓
- $y = x/2$ (−100%)
- $y = x/2 + 1$ (−100%)
- $y = x$ (−100%)
- $y = x + 1$ (−100%)
- $y = 2x$ (−100%)
- $y = 2x + 1$ (−100%)
- $y = -x/2$ (−100%)
- $y = -x/2 + 1$ (−100%)
- $y = -x$ (−100%)
- $y = -x + 1$ (−100%)
- $y = -2x$ (−100%)
- $y = -2x + 1$ (−100%)

One has

$$f'(-1) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 4 points

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 4 points

6 ✓	

The function $f(x)$ has $\boxed{\text{c}}$ stationary point(s) in the domain

$\boxed{\text{c}}$:

NUMERICAL | 4 points

1 ✓	

Choose the behaviour of $f(x)$ in the interval $(3, 5)$.

MULTI | 4 points | Single | Shuffle

- monotonically decreasing ✓
- monotonically increasing
- neither decreasing nor increasing

To determine the natural domain of a function, it is enough to observe the components. For example, $\log y$ is defined for $y > 0$, $\frac{1}{y-a}$ is defined only for $y \neq a$, etc. It is enought to exclude all such points where the composed function is not defined. In this case, $\frac{x^2+2}{x^2-3x+2} > 0$, while $x^2 + 2 > 0$, so this imposes that $0 < x^2 - 3x + 2 = (x-1)(x-2)$, that is $x < 1$ or $x > 2$.
There can be asymptotes for $x \to \pm\infty$, and for $x \to a$, where a is a boundary of the domain. In this case, one should check $x \to 1, 2, \pm\infty$. $x \to 1, 2$ give infinity, so there are vertical asymptote there. As for $x \to \pm\infty$, $\frac{x^2+2}{x^2-3x+2} \to 1$, so $f(x)$ tends to 0, and $y = 0$ is a horizontal asymptote.
For the derivative, the chain rule $(f(g(x)))' = g'(x)f'(g(x))$ is useful. In this case, $f(x) = \log \frac{x^2+2}{x^2-3x+2}$, $f'(x) = \frac{x^2-3x+2}{x^2+2} \cdot \frac{2x(x^2-3x+2)-(x^2+1)(2x^2-3)}{(x^2-3x+2)^2} = \frac{-3(x^2-2))}{(x^2+2)(x^2-3x+2)^2}$.
If $f'(x_0) = 0$, x_0 is called a stationary point. In this case, $x_0 = -\sqrt{2}$ because $\sqrt{2}$ is not in the domain. From the formula above for $f'(x)$, we see that $f'(x) < 0$ for $x \in (3, 5)$.

(9) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log \frac{(x+1)^2 + 2}{x(x-1)}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single | Shuffle

- -3 (-100%)

- $-\frac{5}{2}$ (−100%)
- -2 (−100%)
- $-\frac{3}{2}$ (−100%)
- -1 (−100%)
- $-\frac{1}{2}$ (−100%)
- 0 ✓
- $\frac{1}{2}$ ✓
- 1 ✓
- $\frac{3}{2}$ (−100%)
- 2 (−100%)
- $\frac{5}{2}$ (−100%)
- 3 (−100%)

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single | Shuffle

- $y = -1$ (−100%)
- $y = 0$ ✓
- $y = 1$ (−100%)
- $x = -2$ (−100%)
- $x = -1$ (−100%)
- $x = 0$ ✓
- $x = 1$ ✓
- $x = 2$ (−100%)
- $y = x/2$ (−100%)
- $y = x/2 + 1$ (−100%)
- $y = x$ (−100%)
- $y = x + 1$ (−100%)
- $y = 2x$ (−100%)
- $y = 2x + 1$ (−100%)
- $y = -x/2$ (−100%)
- $y = -x/2 + 1$ (−100%)
- $y = -x$ (−100%)
- $y = -x + 1$ (−100%)
- $y = -2x$ (−100%)
- $y = -2x + 1$ (−100%)

One has

$$f'(-1) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 4 points

3 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 4 points

2 ✓	

The function $f(x)$ has [c] stationary point(s) in the domain [c]:

NUMERICAL | 4 points

1 ✓	

Choose the behaviour of $f(x)$ in the interval $(-3, -2)$.

MULTI | 4 points | Single | Shuffle

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(10) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log \frac{(x+2)^2 + 2}{x(x+1)}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single | Shuffle

- -3 (-100%)
- $-\frac{5}{2}$ (-100%)
- -2 (-100%)
- $-\frac{3}{2}$ (-100%)
- -1 ✓
- $-\frac{1}{2}$ ✓
- 0 ✓
- $\frac{1}{2}$ (-100%)
- 1 (-100%)
- $\frac{3}{2}$ (-100%)
- 2 (-100%)
- $\frac{5}{2}$ (-100%)
- 3 (-100%)

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single | Shuffle

- $y = -1$ (-100%)
- $y = 0$ ✓
- $y = 1$ (-100%)
- $x = -2$ (-100%)
- $x = -1$ ✓
- $x = 0$ ✓
- $x = 1$ (-100%)
- $x = 2$ (-100%)
- $y = x/2$ (-100%)
- $y = x/2 + 1$ (-100%)
- $y = x$ (-100%)
- $y = x + 1$ (-100%)
- $y = 2x$ (-100%)
- $y = 2x + 1$ (-100%)
- $y = -x/2$ (-100%)
- $y = -x/2 + 1$ (-100%)
- $y = -x$ (-100%)
- $y = -x + 1$ (-100%)
- $y = -2x$ (-100%)
- $y = -2x + 1$ (-100%)

One has

$$f'(1) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 4 points

-21 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 4 points

22 ✓	

The function $f(x)$ has $\boxed{\text{c}}$ stationary point(s) in the domain

$\boxed{\text{c}}$:

NUMERICAL | 4 points

1 ✓	

Choose the behaviour of $f(x)$ in the interval $(-2, -1)$.

MULTI | 4 points | Single | Shuffle

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(11) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{\log 2}^{\log 3} \frac{1 + e^{2x}}{e^x - 1} dx.$$

Let us apply the change of variables $x = \log t$. Then we have

$$\int_{\log 2}^{\log 3} \frac{1 + e^{2x}}{e^x - 1} dx = \int_{\boxed{a}}^{\boxed{b}} \frac{\boxed{c} t^2 + \boxed{d} t + \boxed{e}}{\boxed{f} t^2 + \boxed{g} t + \boxed{h}}.$$

a:

NUMERICAL | 4 points

2 ✓	

b:

NUMERICAL | 4 points

3 ✓	

c:

NUMERICAL | 2 points

1 ✓	

d:

NUMERICAL | 1 point

0 ✓	

e:

NUMERICAL | 2 points

1 ✓	

f:

NUMERICAL | 1 point

1 ✓	

g:

NUMERICAL | 1 point

-1 ✓	

h:

NUMERICAL 1 point

0 ✓	

Choose a primitive of $\frac{t}{t(t-1)}$.

MULTI 8 points Single Shuffle

- $\log|t+1|$
- $\log|t(t-1)|$
- $\log|t/(t-1)|$
- $\log|(t-1)/t|$
- $\log|t-1|$ ✓
- $\log|t^2-1|$
- $\log|t^2-1|/2$
- $1/t+1/(t-1)$
- $1/t-1/(t-1)$
- $t\arctan(t)$
- $\arctan(t-1)$
- $\arctan((t-1)/2)$

By continuing, we get

$$\int_{\log 2}^{\log 3} \frac{1+e^{2x}}{e^x-1}dx = \boxed{\text{i}} + \boxed{\text{j}}\log 2 + \boxed{\text{k}}\log 3.$$

i:

NUMERICAL 8 points

1 ✓	

j:

NUMERICAL 8 points

3 ✓	

k:

NUMERICAL 8 points

-1 ✓	

By the change of the variables $x = \log t$, $e^x = t$ and we also have $\frac{dx}{dt} = \frac{1}{t}$, so $\int_{\log 2}^{\log 3} \frac{1+e^{2x}}{e^x-1}dx = \int_2^3 \frac{1+t^2}{(t-1)t}dt$. Furthermore, $\frac{1+t^2}{(t-1)t} = 1 + \frac{t+1}{(t-1)t} = 1 + \frac{2}{t-1} - \frac{1}{t}$, and they can be integrated separately.

(12) **Q4**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{\log 3}^{\log 4} \frac{1+e^{2x}}{e^x-1}dx.$$

Let us apply the change of variables $x = \log t$. Then we have

$$\int_{\log 3}^{\log 4} \frac{1+e^{2x}}{e^x-1}dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} \frac{\boxed{\text{c}}t^2+\boxed{\text{d}}t+\boxed{\text{e}}}{\boxed{\text{f}}t^2+\boxed{\text{g}}t+\boxed{\text{h}}}.$$

a:

NUMERICAL | 4 points

3 ✓	

b:

NUMERICAL | 4 points

4 ✓	

c:

NUMERICAL | 2 points

1 ✓	

d:

NUMERICAL | 1 point

0 ✓	

e:

NUMERICAL | 2 points

1 ✓	

f:

NUMERICAL | 1 point

1 ✓	

g:

NUMERICAL | 1 point

-1 ✓	

h:

NUMERICAL | 1 point

0 ✓	

Choose a primitive of $\frac{1}{t(t-1)}$.

MULTI | 8 points | Single | Shuffle

- $\log|t+1|$
- $\log|t(t-1)|$
- $\log|t/(t-1)|$
- $\log|(t-1)/t|$ ✓
- $\log|t-1|$
- $\log|t^2-1|$
- $\log|t^2-1|/2$
- $1/t+1/(t-1)$
- $1/t-1/(t-1)$
- $t\arctan(t)$
- $\arctan(t-1)$
- $\arctan((t-1)/2)$

By continuing, we get

$$\int_{\log 3}^{\log 4} \frac{1+e^{2x}}{e^x-1}dx = \boxed{\text{i}} + \boxed{\text{j}}\log 2 + \boxed{\text{k}}\log 3.$$

$\boxed{\text{i}}$:

NUMERICAL | 8 points

1 ✓	

$\boxed{\text{j}}$:

NUMERICAL | 8 points

-4 ✓	

$\boxed{\text{k}}$:

NUMERICAL | 8 points

3 ✓	

(13) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{\log 2}^{\log 4} \frac{1+e^{2x}}{e^x-1} dx.$$

Let us apply the change of variables $x = \log t$. Then we have

$$\int_{\log 2}^{\log 4} \frac{1+e^{2x}}{e^x-1} dx = \int_{\boxed{a}}^{\boxed{b}} \frac{\boxed{c}t^2 + \boxed{d}t + \boxed{e}}{\boxed{f}t^2 + \boxed{g}t + \boxed{h}}.$$

a:

NUMERICAL | 4 points

2 ✓	

b:

NUMERICAL | 4 points

4 ✓	

c:

NUMERICAL | 2 points

1 ✓	

d:

NUMERICAL | 1 point

0 ✓	

e:

NUMERICAL | 2 points

1 ✓	

f:

NUMERICAL | 1 point

1 ✓	

g:

NUMERICAL | 1 point

-1 ✓	

h:

NUMERICAL | 1 point

0 ✓	

Choose a primitive of $\frac{t^2}{t(t-1)}$.

MULTI | 8 points | Single | Shuffle

- $t + \log|t+1|$
- $\log|t(t-1)|$
- $t + \log|t/(t-1)|$

- $\log|(t-1)/t|$
- $t+\log|t-1|$ ✓
- $\log|t^2-1|$
- $\log|t^2-1|/2$
- $1/t+1/(t-1)$
- $1/t-1/(t-1)$
- $t\arctan(t)$
- $\arctan(t-1)$
- $\arctan((t-1)/2)$

By continuing, we get

$$\int_{\log 2}^{\log 4} \frac{1+e^{2x}}{e^x-1}dx = \boxed{\text{i}} + \boxed{\text{j}}\log 2 + \boxed{\text{k}}\log 3.$$

$\boxed{\text{i}}$:

NUMERICAL | 8 points

2 ✓	

$\boxed{\text{j}}$:

NUMERICAL | 8 points

-1 ✓	

$\boxed{\text{k}}$:

NUMERICAL | 8 points

2 ✓	

(14) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Consider the following improper integral for various $\alpha \in \mathbb{R}$.

$$\int_0^\infty \frac{\sin x}{x^\alpha(x+1)}dx$$

.

Complete the formulae.

$$\frac{\sin 3x}{x+1} = \boxed{\text{a}} + \boxed{\text{b}}x + \boxed{\text{c}}x^2 + o(x^2) \text{ as } x \to 0.$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

0 ✓	

b:

NUMERICAL | 1 point

3 ✓	

c:

NUMERICAL | 2 points

-3 ✓	

Choose all values of α such that $\int_0^1 \frac{\sin 3x}{x^\alpha (x+1)} dx$ is convergent.

MULTI | 8 points | Single | Shuffle

- -3 ✓
- $-\frac{5}{2}$ ✓
- -2 ✓
- $-\frac{3}{2}$ ✓
- -1 ✓
- $-\frac{1}{2}$ ✓
- 0 ✓
- $\frac{1}{2}$ ✓
- 1 ✓
- $\frac{3}{2}$ ✓
- 2 (-100%)
- $\frac{5}{2}$ (-100%)
- 3 (-100%)

Choose all values of α such that $\int_1^\infty \frac{\sin 3x}{x^\alpha (x+1)} dx$ is convergent.

MULTI | 8 points | Single | Shuffle

- -3 (-100%)
- $-\frac{5}{2}$ (-100%)
- -2 (-100%)
- $-\frac{3}{2}$ (-100%)
- -1 (-100%)
- $-\frac{1}{2}$ ✓
- 0 ✓
- $\frac{1}{2}$ ✓
- 1 ✓
- $\frac{3}{2}$ ✓
- 2 ✓
- $\frac{5}{2}$ ✓
- 3 ✓

Calculate the following improper integral.

$$\int_0^\infty \frac{1}{x^2+1}dx = \frac{\boxed{\text{d}}\pi}{\boxed{\text{e}}}.$$

$\boxed{\text{d}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 2 points

2 ✓	

> $\int_0^1 \frac{\sin x}{x^\alpha(x+1)}dx$ converges if $\frac{\sin x}{x^\alpha(x+1)}$ behaves asymptotically as $x^\gamma, \gamma > -1$ as $x \to 0$. This can be studied by Taylor's formula. On the other hand, $\int_1^\infty \frac{\sin x}{x^\alpha(x+1)}dx$ converges if $\frac{\sin x}{x^\alpha(x+1)}$ behaves asymptotically as $x^\gamma, \gamma < -1$ as $x \to \infty$. As $\sin x$ is bounded, this holds if $\alpha > 0$. The borderline case $\alpha = 0$ is also convergent because as a whole it is asymptotically equal to $\frac{\sin x}{x+1}$, which is convergent (not absolutely).
> Note that $\int \frac{1}{x^2+1}dx = \arctan x$.

(15) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Consider the following improper integral for various $\alpha \in \mathbb{R}$.

$$\int_0^\infty \frac{\sin x}{x^\alpha(x+1)}dx$$

.

Complete the formulae.

$$\frac{\sin x}{(x+1)^2} = \boxed{\text{a}} + \boxed{\text{b}}x + \boxed{\text{c}}x^2 + o(x^2) \text{ as } x \to 0.$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

0 ✓	

b:

NUMERICAL 1 point

1 ✓	

c:

NUMERICAL 2 points

-2 ✓	

Choose all values of α such that $\int_1^\infty \frac{\sin x}{x^\alpha (x+1)^2} dx$ is convergent.

MULTI 8 points Single Shuffle

- -3 (-100%)
- $-\frac{5}{2}$ (-100%)
- -2 (-100%)
- $-\frac{3}{2}$ ✓
- -1 ✓
- $-\frac{1}{2}$ ✓
- 0 ✓
- $\frac{1}{2}$ ✓
- 1 ✓
- $\frac{3}{2}$ ✓
- 2 ✓
- $\frac{5}{2}$ ✓
- 3 ✓

Choose all values of α such that $\int_0^1 \frac{\sin x}{x^\alpha (x+1)^2} dx$ is convergent.

MULTI 8 points Single Shuffle

- -3 ✓
- $-\frac{5}{2}$ ✓
- -2 ✓
- $-\frac{3}{2}$ ✓
- -1 ✓
- $-\frac{1}{2}$ ✓
- 0 ✓
- $\frac{1}{2}$ ✓
- 1 ✓
- $\frac{3}{2}$ ✓
- 2 (-100%)
- $\frac{5}{2}$ (-100%)
- 3 (-100%)

Calculate the following improper integral.

$$\int_1^\infty \frac{1}{x^2+1}dx = \frac{\boxed{\text{d}}\pi}{\boxed{\text{e}}}.$$

$\boxed{\text{d}}$:

NUMERICAL | 1 point

1 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 3 points

4 ✓	

Total of marks: 636