Call1.

(1) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\sin(3(x-1)) = \boxed{a} + \boxed{b}(x-1) + \boxed{c}(x-1)^2 + \frac{\boxed{d}}{\boxed{e}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{a}$: 0 ✓ $\boxed{b}$: 3 ✓ $\boxed{c}$: 0 ✓ $\boxed{d}$: −9 ✓ $\boxed{e}$: 2 ✓

$$(x-1)\log(x+1) = \boxed{f} + \log\boxed{g}(x-1) + \frac{\boxed{h}}{\boxed{i}}(x-1)^2 + \frac{\boxed{j}}{\boxed{k}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{f}$: 0 ✓ $\boxed{g}$: 2 ✓ $\boxed{h}$: 1 ✓ $\boxed{i}$: 2 ✓ $\boxed{j}$: −1 ✓
$\boxed{k}$: 8 ✓

$$\sin(2(x-1)^2)\cdot(x-1) = \boxed{l} + \boxed{m}(x-1) + \boxed{n}(x-1)^2 + \boxed{o}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{l}$: 0 ✓ $\boxed{m}$: 0 ✓ $\boxed{n}$: 0 ✓ $\boxed{o}$: 2 ✓

For various $\alpha, \beta, \gamma \in \mathbb{R}$, study the limit:

$$\lim_{x\to 1} \frac{\sin(3(x-1)) + (x-1)\log(x+1) - (\alpha + \log\beta)(x-1) - \gamma(x-1)^2}{\sin((x-1)^2)\cdot(x-1)}.$$

This limit converges for $\alpha = \boxed{p}$, $\beta = \boxed{q}$, $\gamma = \frac{\boxed{r}}{\boxed{s}}$.

$\boxed{p}$: 3 ✓ $\boxed{q}$: 2 ✓ $\boxed{r}$: 1 ✓ $\boxed{s}$: 2 ✓

In that case, the limit is $\frac{\boxed{t}}{\boxed{u}}$.

$\boxed{t}$: −37 ✓ $\boxed{u}$: 16 ✓

> Use the Taylor formula $f(x) = f(1) + f'(1)(x-1) + \frac{1}{2!}f''(1)(x-1)^2 + \frac{1}{3!}f^{(3)}(1)(x-1)^3 + o((x-1)^3)$ as $x \to 1$. To determine α, β, γ, one only has to compare the numereator and the denominator and choose α, β, γ in such a way that they have the same degree of infinitesimal (in this case, both of them should be of order $(x-1)^3$).

(2) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\sin(-3(x-1)) = \boxed{a} + \boxed{b}(x-1) + \boxed{c}(x-1)^2 + \frac{\boxed{d}}{\boxed{e}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{a}$: $\boxed{0\ \checkmark}$ $\boxed{b}$: $\boxed{-3\ \checkmark}$ $\boxed{c}$: $\boxed{0\ \checkmark}$ $\boxed{d}$: $\boxed{9\ \checkmark}$ $\boxed{e}$: $\boxed{2\ \checkmark}$

$$(x-1)\log(x+2) = \boxed{f} + \log\boxed{g}(x-1) + \frac{\boxed{h}}{\boxed{i}}(x-1)^2 + \frac{\boxed{j}}{\boxed{k}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{f}$: $\boxed{0\ \checkmark}$ $\boxed{g}$: $\boxed{3\ \checkmark}$ $\boxed{h}$: $\boxed{1\ \checkmark}$ $\boxed{i}$: $\boxed{3\ \checkmark}$ $\boxed{j}$: $\boxed{-1\ \checkmark}$ $\boxed{k}$: $\boxed{18\ \checkmark}$

$$\sin((x-1)^2)\cdot(x-1) = \boxed{l} + \boxed{m}(x-1) + \boxed{n}(x-1)^2 + \boxed{o}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{l}$: $\boxed{0\ \checkmark}$ $\boxed{m}$: $\boxed{0\ \checkmark}$ $\boxed{n}$: $\boxed{0\ \checkmark}$ $\boxed{o}$: $\boxed{1\ \checkmark}$

For various $\alpha, \beta, \gamma \in \mathbb{R}$, study the limit:

$$\lim_{x\to 1} \frac{\sin(-3(x-1)) + (x-1)\log(x+2) - (\alpha + \log\beta)(x-1) - \gamma(x-1)^2}{\sin((x-1)^2)\cdot(x-1)}.$$

This limit converges for $\alpha = \boxed{p}, \beta = \boxed{q}, \gamma = \frac{\boxed{r}}{\boxed{s}}$.

$\boxed{p}$: $\boxed{-3\ \checkmark}$ $\boxed{q}$: $\boxed{3\ \checkmark}$ $\boxed{r}$: $\boxed{1\ \checkmark}$ $\boxed{s}$: $\boxed{3\ \checkmark}$

In that case, the limit is $\frac{\boxed{t}}{\boxed{u}}$.

$\boxed{t}$: $\boxed{40\ \checkmark}$ $\boxed{u}$: $\boxed{9\ \checkmark}$

(3) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$\sin(3(x-1)) = \boxed{a} + \boxed{b}(x-1) + \boxed{c}(x-1)^2 + \frac{\boxed{d}}{\boxed{e}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{a}$: $\boxed{0\ \checkmark}$ $\boxed{b}$: $\boxed{3\ \checkmark}$ $\boxed{c}$: $\boxed{0\ \checkmark}$ $\boxed{d}$: $\boxed{-9\ \checkmark}$ $\boxed{e}$: $\boxed{2\ \checkmark}$

$$(x-1)\log(x+4) = \boxed{f} + \log\boxed{g}(x-1) + \frac{\boxed{h}}{\boxed{i}}(x-1)^2 + \frac{\boxed{j}}{\boxed{k}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{f}$: $\boxed{0\ \checkmark}$ $\boxed{g}$: $\boxed{5\ \checkmark}$ $\boxed{h}$: $\boxed{1\ \checkmark}$ $\boxed{i}$: $\boxed{5\ \checkmark}$ $\boxed{j}$: $\boxed{-1\ \checkmark}$
$\boxed{k}$: $\boxed{50\ \checkmark}$

$$\sin(3(x-1)^2)\cdot(x-1) = \boxed{l} + \boxed{m}(x-1) + \boxed{n}(x-1)^2 + \boxed{o}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{l}$: $\boxed{0\ \checkmark}$ $\boxed{m}$: $\boxed{0\ \checkmark}$ $\boxed{n}$: $\boxed{0\ \checkmark}$ $\boxed{o}$: $\boxed{3\ \checkmark}$

For various $\alpha, \beta, \gamma \in \mathbb{R}$, study the limit:

$$\lim_{x\to 1} \frac{\sin(3(x-1)) + (x-1)\log(x+4) - (\alpha + \log\beta)(x-1) - \gamma(x-1)^2}{\sin(3(x-1)^2)\cdot(x-1)}.$$

This limit converges for $\alpha = \boxed{p}, \beta = \boxed{q}, \gamma = \frac{\boxed{r}}{\boxed{s}}$.

$\boxed{p}$: $\boxed{3\ \checkmark}$ $\boxed{q}$: $\boxed{5\ \checkmark}$ $\boxed{r}$: $\boxed{1\ \checkmark}$ $\boxed{s}$: $\boxed{5\ \checkmark}$

In that case, the limit is $\frac{\boxed{t}}{\boxed{u}}$.

$\boxed{t}$: $\boxed{-113\ \checkmark}$ $\boxed{u}$: $\boxed{75\ \checkmark}$

(4) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(2^n-1)(2n)!}{(n!)^2}(x+1)^{3n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{(2^n-1)(2n)!}{(n!)^2}(x+1)^{3n} = \boxed{a} + \boxed{b} i$.

$\boxed{a}$: $\boxed{-4 \checkmark}$ $\boxed{b}$: $\boxed{-140 \checkmark}$

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{(2^n-1)(2n)!}{(n!)^2}|x+1|^{3n}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{c} |\boxed{d} x + \boxed{e}|^{\boxed{f}}$$

$\boxed{c}$: $\boxed{8 \checkmark}$ $\boxed{d}$: $\boxed{1 \checkmark}$ $\boxed{e}$: $\boxed{1 \checkmark}$ $\boxed{f}$: $\boxed{3 \checkmark}$

Therefore, by the ratio test, the series converges absolutely for $\frac{\boxed{j}}{\boxed{k}} < x < \frac{\boxed{l}}{\boxed{m}}$.

$\boxed{j}$: $\boxed{-3 \checkmark}$ $\boxed{k}$: $\boxed{2 \checkmark}$ $\boxed{l}$: $\boxed{-1 \checkmark}$ $\boxed{m}$: $\boxed{2 \checkmark}$

For the case $x = -1$, the series

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = \frac{1}{2}$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

The partial sum means the following finite sum: $\sum_{n=0}^{2} a_n = a_0+a_1+a_2$, so one just has to apply $n = 0, 1, 2$ in the concrete series and sum the numbers up. Notice that $i^2 = -1$. One can compute $(1+i)^{10}$ by using the fact that $1+i = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$.
To apply the ratio test for a positive series $\sum a_n$, one considers $L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$. Note that $(2(n+1))! = (2n+2)(2n+1)(2n)!$.
If this limit $L < 1$, then the series converges absolutely (for such x), while if $L > 1$ the series diverges. a_n depends on x, and this gives us a condition for which the series converges. That is $8|x+1|^3 < 1$, or $-\frac{1}{2} < x+1 < \frac{1}{2}$
If $L = 1$, one needs to study the convergence with other criteria. In this case, if $x = -\frac{1}{2}$, then a_n diverges, and the series diverges as well.

(5) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(2^n-1)(2n)!}{(n!)^2}(x-1)^{3n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{(2^n-1)(2n)!}{(n!)^2}(x-1)^{3n} = \boxed{a} + \boxed{b}i$.

$\boxed{a}$: 4 ✓ $\boxed{b}$: 148 ✓

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{(2^n-1)(2n)!}{(n!)^2}|x-1|^{3n}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{c}|\boxed{d}x + \boxed{e}|^{\boxed{f}}$$

$\boxed{c}$: 8 ✓ $\boxed{d}$: 1 ✓ $\boxed{e}$: −1 ✓ $\boxed{f}$: 3 ✓

Therefore, by the ratio test, the series converges absolutely for $\frac{\boxed{j}}{\boxed{k}} < x < \frac{\boxed{l}}{\boxed{m}}$.

$\boxed{j}$: 1 ✓ $\boxed{k}$: 2 ✓ $\boxed{l}$: 3 ✓ $\boxed{m}$: 2 ✓

For the case $x = -\frac{1}{2}$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = \frac{2}{3}$, the series
- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(6) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(n^2-1)(3n)!}{(n!)^3}(x-1)^{3n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{(n^2-1)(3n)!}{(n!)^3}(x-1)^{3n} = \boxed{\text{a}} + \boxed{\text{b}} i$.

$\boxed{\text{a}}$: $\boxed{-1\ ✓}$ $\boxed{\text{b}}$: $\boxed{2160\ ✓}$

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{(n^2-1)(3n)!}{(n!)^3}(x-1)^{3n}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{\text{c}} \left| \boxed{\text{d}} x + \boxed{\text{e}} \right|^{\boxed{\text{f}}}$$

$\boxed{\text{c}}$: $\boxed{27\ ✓}$ $\boxed{\text{d}}$: $\boxed{1\ ✓}$ $\boxed{\text{e}}$: $\boxed{-1\ ✓}$ $\boxed{\text{f}}$: $\boxed{3\ ✓}$

Therefore, by the ratio test, the series converges absolutely for $\frac{\boxed{\text{j}}}{\boxed{\text{k}}} < x < \frac{\boxed{\text{l}}}{\boxed{\text{m}}}$.

$\boxed{\text{j}}$: $\boxed{2\ ✓}$ $\boxed{\text{k}}$: $\boxed{3\ ✓}$ $\boxed{\text{l}}$: $\boxed{4\ ✓}$ $\boxed{\text{m}}$: $\boxed{3\ ✓}$

For the case $x = \frac{4}{3}$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = \frac{4}{5}$, the series
- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(7) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{(n^2-1)(3n)!}{(n!)^3}(x+1)^{3n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{(n^2-1)(3n)!}{(n!)^3}(x+1)^{3n} = \boxed{a} + \boxed{b}i$.

$\boxed{a}$: $\boxed{-1 \ \checkmark}$ $\boxed{b}$: $\boxed{-2160 \ \checkmark}$

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{(n^2-1)(3n)!}{(n!)^3}(x+1)^{3n}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \boxed{c}\left|\boxed{d}x + \boxed{e}\right|^{\boxed{f}}$$

$\boxed{c}$: $\boxed{27 \ \checkmark}$ $\boxed{d}$: $\boxed{1 \ \checkmark}$ $\boxed{e}$: $\boxed{1 \ \checkmark}$ $\boxed{f}$: $\boxed{3 \ \checkmark}$

Therefore, by the ratio test, the series converges absolutely for $\frac{\boxed{j}}{\boxed{k}} < x < \frac{\boxed{l}}{\boxed{m}}$.

$\boxed{j}$: $\boxed{-4 \ \checkmark}$ $\boxed{k}$: $\boxed{3 \ \checkmark}$ $\boxed{l}$: $\boxed{-2 \ \checkmark}$ $\boxed{m}$: $\boxed{3 \ \checkmark}$

For the case $x = \frac{4}{5}$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = -\frac{4}{3}$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(8) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \sqrt{\frac{x^3 - 1}{x}}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- -3
- $-\frac{5}{2}$
- -2
- $-\frac{3}{2}$
- -1
- $-\frac{1}{2}$
- 0 ✓
- $\frac{1}{2}$ ✓
- 1
- $\frac{3}{2}$
- 2
- $\frac{5}{2}$
- 3

Choose all asymptotes of $f(x)$.

- $y = -1$
- $y = 0$
- $y = 1$
- $x = -2$
- $x = -1$
- $x = 0$ ✓
- $x = 1$
- $x = 2$
- $y = x/2$
- $y = x/2 + 1$
- $y = x$ ✓
- $y = x + 1$
- $y = 2x$
- $y = 2x + 1$
- $y = -x/2$
- $y = -x/2 + 1$
- $y = -x$ ✓
- $y = -x + 1$
- $y = -2x$
- $y = -2x + 1$

One has

$$f'(-1) = \frac{\boxed{\text{a}}\sqrt{\boxed{\text{b}}}}{\boxed{\text{c}}}.$$

$\boxed{\text{a}}$: $\boxed{-1 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{4 \ \checkmark}$

The function $f(x)$ has $\boxed{\text{d}}$ stationary point(s) in the domain

$\boxed{\text{d}}$: $\boxed{1 \ \checkmark}$

Choose the behaviour of $f(x)$ in the interval $(3, 5)$.

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

> To determine the natural domain of a function, it is enough to observe the components. For example, $\sqrt{y}$ is defined for $y \geq 0$, $\frac{1}{y-a}$ is defined only for $y \neq a$, etc. It is enought to exclude all such points where the composed function is not defined. In this case, $\frac{x^3-1}{x} \geq 0$, so both of x and $x^3 - 1$ are ≥ 0, or both of them are ≤ 0 From this we get that $x < 0$ or $x \geq 0$.
> There can be asymptotes for $x \to \pm\infty$, and for $x \to a$, where a is a boundary of the domain. In this case, one should check $x \to 0, 1, \infty$. $x \to 0$ give infinity, so there are vertical asymptote there. As for $x \to \infty$, $f(x)$ diverges, so there is no horizontal asymptote. To see whether there are oblique asymptotes, we note that $\frac{f(x)}{x} = \pm\sqrt{\frac{x^3-x}{x^2}}$ depending on $x > 0$ or $x < 0$ This tends to ± 1 as $x \to \pm\infty$. One can also calculate $\lim_{x\to\pm\infty} f(x) - \pm x = 0$, therefore, the oblique asymptotes are $y = \pm x$.
> For the derivative, the chain rule $(f(g(x)))' = g'(x)f'(g(x))$ is useful. In this case, $f(x) = \sqrt{\frac{x^3-1}{x}}$, $f'(x) = \frac{1}{2}\sqrt{\frac{x}{x^3-1}} \cdot \frac{3x^3-(x^3-1)}{x^2} = \frac{1}{2}\sqrt{\frac{x}{x^3-1}} \cdot \frac{2x^3+1)}{x^2}$.
> If $f'(x_0) = 0$, x_0 is called a stationary point. In this case, $x_0 =, -(\frac{1}{2})^{\frac{1}{3}}$. From the formula above for $f'(x)$, we see that $f'(x) > 0$ for $x \in (3, 5)$.

(9) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \sqrt{\frac{x^3 - 1}{4x}}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- -3
- $-\frac{5}{2}$
- -2
- $-\frac{3}{2}$
- -1
- $-\frac{1}{2}$
- 0 ✓
- $\frac{1}{2}$ ✓
- 1
- $\frac{3}{2}$
- 2
- $\frac{5}{2}$
- 3

Choose all asymptotes of $f(x)$.

- $y = -1$
- $y = 0$
- $y = 1$
- $x = -2$
- $x = -1$
- $x = 0$ ✓
- $x = 1$
- $x = 2$
- $y = x/2$ ✓
- $y = x/2 + 1$
- $y = x$
- $y = x + 1$
- $y = 2x$
- $y = 2x + 1$
- $y = -x/2$ ✓
- $y = -x/2 + 1$
- $y = -x$

- $y = -x + 1$
- $y = -2x$
- $y = -2x + 1$

One has

$$f'(-1) = \frac{\boxed{\text{a}}\sqrt{\boxed{\text{b}}}}{\boxed{\text{c}}}.$$

$\boxed{\text{a}}$: $\boxed{-1 \;\checkmark}$ $\boxed{\text{b}}$: $\boxed{2 \;\checkmark}$ $\boxed{\text{c}}$: $\boxed{8 \;\checkmark}$

The function $f(x)$ has $\boxed{\text{d}}$ stationary point(s) in the domain

$\boxed{\text{d}}$: $\boxed{1 \;\checkmark}$

Choose the behaviour of $f(x)$ in the interval $(-5, -3)$.

- monotonically decreasing ✓
- monotonically increasing
- neither decreasing nor increasing

(10) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \sqrt{\frac{8x^3 - 1}{2x}}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- -3
- $-\frac{5}{2}$
- -2
- $-\frac{3}{2}$
- -1
- $-\frac{1}{2}$
- 0 ✓
- $\frac{1}{2}$
- 1
- $\frac{3}{2}$
- 2
- $\frac{5}{2}$

- 3

Choose all asymptotes of $f(x)$.

- $y=-1$
- $y=0$
- $y=1$
- $x=-2$
- $x=-1$
- $x=0$ ✓
- $x=1$
- $x=2$
- $y=x/2$
- $y=x/2+1$
- $y=x$
- $y=x+1$
- $y=2x$ ✓
- $y=2x+1$
- $y=-x/2$
- $y=-x/2+1$
- $y=-x$
- $y=-x+1$
- $y=-2x$ ✓
- $y=-2x+1$

One has

$$f'(-1)=\frac{\boxed{\text{a}}\sqrt{\boxed{\text{b}}}}{\boxed{\text{c}}}.$$

$\boxed{\text{a}}$: $\boxed{-5\ ✓}$ $\boxed{\text{b}}$: $\boxed{2\ ✓}$ $\boxed{\text{c}}$: $\boxed{4\ ✓}$

The function $f(x)$ has $\boxed{\text{d}}$ stationary point(s) in the domain

$\boxed{\text{d}}$: $\boxed{1\ ✓}$

Choose the behaviour of $f(x)$ in the interval $(-1,0)$.

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(11) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x=\log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{-1}^{0} \frac{4x^2-1}{x^3-2x^2+x-2}dx.$$

Complete the formula

$$\frac{4x^2-1}{x^3-2x^2+x-2} = \frac{\boxed{a}x+\boxed{b}}{x^2+\boxed{c}} + \frac{\boxed{d}}{x+\boxed{f}}.$$

$\boxed{a}$: $\boxed{1 \ \checkmark}$ $\boxed{b}$: $\boxed{2 \ \checkmark}$ $\boxed{c}$: $\boxed{1 \ \checkmark}$ $\boxed{d}$: $\boxed{3 \ \checkmark}$ $\boxed{e}$: $\boxed{-2 \ \checkmark}$

Choose a primitive of $\frac{x}{x^2+1}$.

- $\arctan(x+1)$
- $\frac{x}{2}\arctan(x)$
- $x\arctan(x^2+1)$
- $\frac{1}{4}\log(x^2+1)$
- $\frac{1}{2}\log(x^2+1)$ ✓
- $\log(x(x^2+1))$
- $\frac{1}{4}\arcsin(x^2+1)$
- $\frac{1}{2}\arcsin(x^2+1)$
- $\arcsin(x(x^2+1))$

By continuing, we get

$$\int_{-1}^{0} \frac{4x^2-1}{x^3-2x^2+x-2}dx = \frac{\boxed{f}}{\boxed{g}}\pi + \frac{\boxed{h}}{\boxed{i}}\log 2 + \boxed{j}\log 3.$$

$\boxed{f}$: $\boxed{1 \ \checkmark}$ $\boxed{g}$: $\boxed{2 \ \checkmark}$ $\boxed{h}$: $\boxed{5 \ \checkmark}$ $\boxed{i}$: $\boxed{2 \ \checkmark}$ $\boxed{i}$: $\boxed{-3 \ \checkmark}$

> The partial fractions of $\frac{4x^2-1}{x^3-2x^2+x-2}$ can be found first by factorizing the denominator $x^3-2x^2+x-2 = (x^2+1)(x-2)$ and then by putting $\frac{4x^2-1}{x^3-2x^2+x-2} = \frac{A}{x-2} + \frac{Bx+C}{x^2+1}$. The indefinite integral of $\frac{x}{x^2+1}$ can be found by the substitution $u = x^2+1$, while one can use $\int \frac{1}{x^2+1}dx = \arctan x$.

(12) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{-1}^{0} \frac{3x^2-2}{x^3-2x^2+x-2}dx.$$

Complete the formula

$$\frac{3x^2-2}{x^3-2x^2+x-2} = \frac{\boxed{\text{a}}x+\boxed{\text{b}}}{x^2+\boxed{\text{c}}} + \frac{\boxed{\text{d}}}{x+\boxed{\text{f}}}.$$

$\boxed{\text{a}}$: $\boxed{1 \checkmark}$ $\boxed{\text{b}}$: $\boxed{2 \checkmark}$ $\boxed{\text{c}}$: $\boxed{1 \checkmark}$ $\boxed{\text{d}}$: $\boxed{2 \checkmark}$ $\boxed{\text{e}}$: $\boxed{-2 \checkmark}$

Choose a primitive of $\frac{x}{x^2+1}$.

- $\arctan(x+1)$
- $\frac{x}{2}\arctan(x)$
- $x\arctan(x^2+1)$
- $\frac{1}{4}\log(x^2+1)$
- $\frac{1}{2}\log(x^2+1)$ ✓
- $\log(x(x^2+1))$
- $\frac{1}{4}\arcsin(x^2+1)$
- $\frac{1}{2}\arcsin(x^2+1)$
- $\arcsin(x(x^2+1))$

By continuing, we get

$$\int_{-1}^{0} \frac{3x^2-2}{x^3-2x^2+x-2}dx = \frac{\boxed{\text{f}}}{\boxed{\text{g}}}\pi + \frac{\boxed{\text{h}}}{\boxed{\text{i}}}\log 2 + \boxed{\text{j}}\log 3.$$

$\boxed{\text{f}}$: $\boxed{1 \checkmark}$ $\boxed{\text{g}}$: $\boxed{2 \checkmark}$ $\boxed{\text{h}}$: $\boxed{3 \checkmark}$ $\boxed{\text{i}}$: $\boxed{2 \checkmark}$ $\boxed{\text{j}}$: $\boxed{-2 \checkmark}$

(13) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{-1}^{0} \frac{2x^2-3}{x^3-2x^2+x-2}dx.$$

Complete the formula

$$\frac{2x^2-3}{x^3-2x^2+x-2} = \frac{\boxed{\text{a}}x+\boxed{\text{b}}}{x^2+\boxed{\text{c}}} + \frac{\boxed{\text{d}}}{x+\boxed{\text{f}}}.$$

[a]: [1 ✓] [b]: [2 ✓] [c]: [1 ✓] [d]: [1 ✓] [e]: [−2 ✓]

Choose a primitive of $\frac{x}{x^2+1}$.

- $\arctan(x+1)$
- $\frac{x}{2}\arctan(x)$
- $x\arctan(x^2+1)$
- $\frac{1}{4}\log(x^2+1)$
- $\frac{1}{2}\log(x^2+1)$ ✓
- $\log(x(x^2+1))$
- $\frac{1}{4}\arcsin(x^2+1)$
- $\frac{1}{2}\arcsin(x^2+1)$
- $\arcsin(x(x^2+1))$

By continuing, we get

$$\int_{-1}^{0}\frac{2x^2-3}{x^3-2x^2+x-2}dx = \frac{\boxed{\text{f}}}{\boxed{\text{g}}}\pi + \frac{\boxed{\text{h}}}{\boxed{\text{i}}}\log 2 + \boxed{\text{j}}\log 3.$$

[f]: [1 ✓] [g]: [2 ✓] [h]: [1 ✓] [i]: [2 ✓] [j]: [−1 ✓]

(14) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = x\exp(x)y(x)^2$$

.

- $y(x) = C/((x-1)\exp x)$
- $y(x) = -1/((x-C)\exp x)$
- $y(x) = -1/((x-1)\exp x + C)$ ✓
- $y(x) = -1/((x-1)\exp Cx)$
- $y(x) = -1/((x-1)\exp x) + C$
- $y(x) = (x-1)\exp x + C$
- $y(x) = (x-C)\exp x$
- $y(x) = C(x-1)\exp x$
- $y(x) = (x-1)\exp Cx$

Determine $C = \boxed{\text{a}}$ with the initial condition $y(0) = 0.5$

[a]: [−1 ✓]

Choose the general solution of the following differential equation.

$$y''(x) - y'(x) - 6y(x) = 0$$

.

- $y(x) = C_1 \sin(-3x) + C_2 \cos(2x)$
- $y(x) = C_1 \cos(-3x) + C_2 \sin(-2x)$
- $y(x) = C_1 \exp(3x) + C_2 \exp(-2x)$ ✓
- $y(x) = C_1 \exp(-3x) + C_2 \exp(2x)$
- $y(x) = C_1 \sin(1x) + C_2 \cos(6x)$
- $y(x) = C_1 \cos(-1x) + C_2 \sin(6x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(6x)$
- $y(x) = C_1 \exp(3x) + C_2 \exp(x)\cos(x)$
- $y(x) = C_1 \exp(-3x) + C_2 \exp(-x)\cos(x)$
- $y(x) = C_1 \exp(3x)\sin(x) + C_2 \cos(-x)$

Choose $C_1 = \boxed{\text{b}}, C_2 = \boxed{\text{c}}$ in such a way that $y(0) = 5$ and $\lim_{x\to\infty} y(x) = 0$.

$\boxed{\text{b}}$: $\boxed{0 \quad ✓}$ $\boxed{\text{c}}$: $\boxed{5 \quad ✓}$

> The equation $y'(x) = x\exp(x)y(x)^2$ is separable, hence one obtains the relation $-\frac{1}{y} = \int e^{-y}dy = \int x\exp(x)dx + C = (x-1)\exp(x) + C$, or $y(x) = \frac{-1}{(x-1)\exp(x)+C}$.
> The second-order differential equation $y'' + ay' + by = 0$ can be solved as follows: put $z^2 + az + b = 0$, and solve this equation. If this has two real solutions z_1, z_2, then the general solution is $y = C_1e^{z_1x} + C_2e^{z_2x}$. If it has two real solutions z_1, z_2, then $y = C_1e^{z_1x} + C_2e^{z_2x}$.
> The constant can be obtaind by substituting the initial condition and noticing that $\lim_{x\to\infty} C_1 \exp(3x) = 0$ only if $C_1 = 0$.

(15) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = x\exp(x)y(x)^2$$

.

- $y(x) = C/((x-1)\exp x)$
- $y(x) = -1/((x-C)\exp x)$
- $y(x) = -1/((x-1)\exp x + C)$ ✓
- $y(x) = -1/((x-1)\exp Cx)$
- $y(x) = -1/((x-1)\exp x) + C$
- $y(x) = (x-1)\exp x + C$
- $y(x) = (x-C)\exp x$
- $y(x) = C(x-1)\exp x$
- $y(x) = (x-1)\exp Cx$

Determine $C = \boxed{\text{a}}$ with the initial condition $y(0) = 0.25$

$\boxed{\text{a}}$: $\boxed{-3 \ ✓}$

Choose the general solution of the following differential equation.

$$y''(x) + y'(x) - 6y(x) = 0$$

.

- $y(x) = C_1\sin(-3x) + C_2\cos(2x)$
- $y(x) = C_1\cos(-3x) + C_2\sin(-2x)$
- $y(x) = C_1\exp(3x) + C_2\exp(-2x)$
- $y(x) = C_1\exp(-3x) + C_2\exp(2x)$ ✓
- $y(x) = C_1\sin(1x) + C_2\cos(6x)$
- $y(x) = C_1\cos(-1x) + C_2\sin(6x)$
- $y(x) = C_1\exp(-x) + C_2\exp(6x)$
- $y(x) = C_1\exp(3x) + C_2\exp(x)\cos(x)$
- $y(x) = C_1\exp(-3x) + C_2\exp(-x)\cos(x)$
- $y(x) = C_1\exp(3x)\sin(x) + C_2\cos(-x)$

Choose $C_1 = \boxed{\text{b}}$, $C_2 = \boxed{\text{c}}$ in such a way that $y(0) = 5$ and $\lim_{x\to\infty} y(x) = 0$.

$\boxed{\text{b}}$: $\boxed{5 \ ✓}$ $\boxed{\text{c}}$: $\boxed{0 \ ✓}$

(16) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = x\exp(x)y(x)^2$$

.

- $y(x) = C/((x-1)\exp x)$
- $y(x) = -1/((x-C)\exp x)$
- $y(x) = -1/((x-1)\exp x + C)$ ✓
- $y(x) = -1/((x-1)\exp Cx)$
- $y(x) = -1/((x-1)\exp x) + C$
- $y(x) = (x-1)\exp x + C$
- $y(x) = (x-C)\exp x$
- $y(x) = C(x-1)\exp x$
- $y(x) = (x-1)\exp Cx$

Determine $C =$ [a] with the initial condition $y(0) = 0.2$

[a]: [-4 ✓]

Choose the general solution of the following differential equation.

$$y''(x) - 5y'(x) - 6y(x) = 0$$

.

- $y(x) = C_1\sin(-3x) + C_2\cos(2x)$
- $y(x) = C_1\cos(-3x) + C_2\sin(-2x)$
- $y(x) = C_1\exp(3x) + C_2\exp(-2x)$
- $y(x) = C_1\exp(-3x) + C_2\exp(2x)$
- $y(x) = C_1\sin(1x) + C_2\cos(6x)$
- $y(x) = C_1\cos(-1x) + C_2\sin(6x)$
- $y(x) = C_1\exp(-x) + C_2\exp(6x)$ ✓
- $y(x) = C_1\exp(3x) + C_2\exp(x)\cos(x)$
- $y(x) = C_1\exp(-3x) + C_2\exp(-x)\cos(x)$
- $y(x) = C_1\exp(3x)\sin(x) + C_2\cos(-x)$

Choose $C_1 =$ [b], $C_2 =$ [c] in such a way that $y(0) = 3$ and $\lim_{x\to\infty} y(x) = 0$.

[b]: [3 ✓] [c]: [0 ✓]