Call6.

(1) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$x^{\frac{1}{2}} = \boxed{\text{a}} + \frac{\boxed{\text{b}}}{\boxed{\text{c}}}(x-1) + \frac{\boxed{\text{d}}}{\boxed{\text{e}}}(x-1)^2 + \frac{\boxed{\text{f}}}{\boxed{\text{g}}}(x-1)^3 \text{ as } x \to 1.$$

$\boxed{\text{a}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{d}}$: $\boxed{-1 \ \checkmark}$ $\boxed{\text{e}}$: $\boxed{8 \ \checkmark}$ $\boxed{\text{f}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{g}}$: $\boxed{16 \ \checkmark}$

$$x\exp(x-1) = \boxed{\text{h}} + \boxed{\text{i}}(x-1) + \frac{\boxed{\text{j}}}{\boxed{\text{k}}}(x-1)^2 + \frac{\boxed{\text{l}}}{\boxed{\text{m}}}\boxed{\text{k}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{h}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{i}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{j}}$: $\boxed{3 \ \checkmark}$ $\boxed{\text{k}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{l}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{m}}$: $\boxed{3 \ \checkmark}$

$$\sin((x-1)^2)\cdot(x-1) = \boxed{\text{o}} + \boxed{\text{p}}(x-1) + \boxed{\text{q}}(x-1)^2 + \boxed{\text{r}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{o}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{p}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{q}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{r}}$: $\boxed{1 \ \checkmark}$

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to -1} \frac{x^{\frac{1}{2}} - x\exp(x-1) + \alpha(x-1) + \beta(x-1)^2}{\sin((x-1)^2)\cdot(x-1)}.$$

This limit converges for $\alpha = \frac{\boxed{\text{s}}}{\boxed{\text{t}}}, \beta = \frac{\boxed{\text{u}}}{\boxed{\text{v}}}$.

$\boxed{\text{s}}$: $\boxed{3 \ \checkmark}$ $\boxed{\text{t}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{u}}$: $\boxed{13 \ \checkmark}$ $\boxed{\text{v}}$: $\boxed{8 \ \checkmark}$

In that case, the limit is $\frac{\boxed{\text{w}}}{\boxed{\text{x}}}$.

$\boxed{\text{w}}$: $\boxed{-29 \ \checkmark}$ $\boxed{\text{x}}$: $\boxed{48 \ \checkmark}$

Use the Taylor formula $f(x) = f(1) + f'(1)(x-1) + \frac{1}{2!}f''(1)(x-1)^2 + \frac{1}{3!}f^{(3)}(1)(x-1)^3 + o((x-1)^3)$ as $x \to 1$. To determine α, β, one only has to compare the numereator and the denominator and choose α, β in such a way that they have the same degree of infinitesimal (in this case, both of them should be of order $(x-1)^3$).

(2) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$x^{\frac{1}{3}} = \boxed{a} + \frac{\boxed{b}}{\boxed{c}}(x-1) + \frac{\boxed{d}}{\boxed{e}}(x-1)^2 + \frac{\boxed{f}}{\boxed{g}}(x-1)^3 \text{ as } x \to 1.$$

$\boxed{a}$: $\boxed{1 \checkmark}$ $\boxed{b}$: $\boxed{1 \checkmark}$ $\boxed{c}$: $\boxed{3 \checkmark}$ $\boxed{d}$: $\boxed{-1 \checkmark}$ $\boxed{e}$: $\boxed{9 \checkmark}$ $\boxed{f}$: $\boxed{5 \checkmark}$ $\boxed{g}$: $\boxed{81 \checkmark}$

$$x\exp(x-1) = \boxed{h} + \boxed{i}(x-1) + \frac{\boxed{j}}{\boxed{k}}(x-1)^2 + \frac{\boxed{l}}{\boxed{m}}\boxed{k}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{h}$: $\boxed{1 \checkmark}$ $\boxed{i}$: $\boxed{2 \checkmark}$ $\boxed{j}$: $\boxed{3 \checkmark}$ $\boxed{k}$: $\boxed{2 \checkmark}$ $\boxed{l}$: $\boxed{2 \checkmark}$ $\boxed{m}$: $\boxed{3 \checkmark}$

$$\sin((x-1)^2)\cdot(x-1) = \boxed{o} + \boxed{p}(x-1) + \boxed{q}(x-1)^2 + \boxed{r}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{o}$: $\boxed{0 \checkmark}$ $\boxed{p}$: $\boxed{0 \checkmark}$ $\boxed{q}$: $\boxed{0 \checkmark}$ $\boxed{r}$: $\boxed{1 \checkmark}$

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to -1} \frac{x^{\frac{1}{3}} - x\exp(x-1) + \alpha(x-1) + \beta(x-1)^2}{\sin((x-1)^2)\cdot(x-1)}.$$

This limit converges for $\alpha = \frac{\boxed{s}}{\boxed{t}}, \beta = \frac{\boxed{u}}{\boxed{v}}$.

$\boxed{s}$: $\boxed{5 \checkmark}$ $\boxed{t}$: $\boxed{3 \checkmark}$ $\boxed{u}$: $\boxed{29 \checkmark}$ $\boxed{v}$: $\boxed{18 \checkmark}$

In that case, the limit is $\dfrac{\boxed{\text{w}}}{\boxed{\text{x}}}$.

$\boxed{\text{w}}$: $\boxed{-49 \ \checkmark}$ $\boxed{\text{x}}$: $\boxed{81 \ \checkmark}$

(3) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\dfrac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$x^{\frac{3}{2}} = \boxed{\text{a}} + \frac{\boxed{\text{b}}}{\boxed{\text{c}}}(x-1) + \frac{\boxed{\text{d}}}{\boxed{\text{e}}}(x-1)^2 + \frac{\boxed{\text{f}}}{\boxed{\text{g}}}(x-1)^3 \text{ as } x \to 1.$$

$\boxed{\text{a}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{3 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{d}}$: $\boxed{3 \ \checkmark}$ $\boxed{\text{e}}$: $\boxed{8 \ \checkmark}$ $\boxed{\text{f}}$: $\boxed{-1 \ \checkmark}$ $\boxed{\text{g}}$: $\boxed{16 \ \checkmark}$

$$x\exp(x-1) = \boxed{\text{h}} + \boxed{\text{i}}(x-1) + \frac{\boxed{\text{j}}}{\boxed{\text{k}}}(x-1)^2 + \frac{\boxed{\text{l}}}{\boxed{\text{m}}}\boxed{\text{k}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{h}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{i}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{j}}$: $\boxed{3 \ \checkmark}$ $\boxed{\text{k}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{l}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{m}}$: $\boxed{3 \ \checkmark}$

$$\sin((x-1)^2)\cdot(x-1) = \boxed{\text{o}} + \boxed{\text{p}}(x-1) + \boxed{\text{q}}(x-1)^2 + \boxed{\text{r}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{o}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{p}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{q}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{r}}$: $\boxed{1 \ \checkmark}$

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to -1} \frac{x^{\frac{3}{2}} - x\exp(x-1) + \alpha(x-1) + \beta(x-1)^2}{\sin((x-1)^2)\cdot(x-1)}.$$

This limit converges for $\alpha = \dfrac{\boxed{\text{s}}}{\boxed{\text{t}}}$, $\beta = \dfrac{\boxed{\text{u}}}{\boxed{\text{v}}}$.

$\boxed{\text{s}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{t}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{u}}$: $\boxed{9 \ \checkmark}$ $\boxed{\text{v}}$: $\boxed{8 \ \checkmark}$

In that case, the limit is $\dfrac{\boxed{\text{w}}}{\boxed{\text{x}}}$.

$\boxed{\text{w}}$: $\boxed{-35 \ \checkmark}$ $\boxed{\text{x}}$: $\boxed{48 \ \checkmark}$

(4) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{4^n-1}{9^n+7}(x-1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i\sqrt{3}$, calculate the partial sum $\sum_{n=0}^{2} \frac{4^n-1}{9^n+7}(x-1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}\sqrt{\boxed{d}}}{\boxed{e}} i.$

$\boxed{a}$: $\boxed{-153 \checkmark}$ $\boxed{b}$: $\boxed{88 \checkmark}$ $\boxed{c}$: $\boxed{87 \checkmark}$ $\boxed{d}$: $\boxed{3 \checkmark}$ $\boxed{e}$: $\boxed{88 \checkmark}$

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{4^n-1}{9^n+7}(x-1)^{2n}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{f}|x+\boxed{g}|^{\boxed{h}}}{\boxed{i}}$$

$\boxed{f}$: $\boxed{4 \checkmark}$ $\boxed{g}$: $\boxed{-1 \checkmark}$ $\boxed{h}$: $\boxed{2 \checkmark}$ $\boxed{i}$: $\boxed{9 \checkmark}$

Therefore, by the root test, the series converges absolutely for $\frac{\boxed{j}}{\boxed{k}} < x < \frac{\boxed{l}}{\boxed{m}}$.

$\boxed{j}$: $\boxed{-1 \checkmark}$ $\boxed{k}$: $\boxed{2 \checkmark}$ $\boxed{l}$: $\boxed{5 \checkmark}$ $\boxed{m}$: $\boxed{2 \checkmark}$

For the case $x = -1$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = \frac{1}{2}$, the series

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

The partial sum means the following finite sum: $\sum_{n=0}^{2} a_n = a_0+a_1+a_2$, so one just has to apply $n = 0, 1, 2$ in the concrete series and sum the numbers up. Notice that $i^2 = -1$. One can compute $(-1 + i\sqrt{3})^6$ by using the fact that $-1 + i\sqrt{3} = 2(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$.
To apply the ratio test for a positive series $\sum a_n$, one considers $L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$. Note that for any $p > 1, k \in \mathbb{R}$ it holds that $\lim_{n\to\infty} \frac{k}{p^n} = 0$, and $\lim_{n\to\infty} \frac{p^n+k}{p^n} = 1$ etc. If this limit $L < 1$, then the series converges absolutely (for such x), while if $L > 1$ the series diverges. a_n depends on x, and this gives us a condition for which the series converges. That is $\frac{4|x-1|^2}{9} < 1$, or $-\frac{3}{2} < x - 1 < \frac{3}{2}$
If $L = 1$, one needs to study the convergence with other criteria. In this case, if $x = \frac{5}{2}$, then a_n tends to 1, and the series diverges.

(5) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

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This series makes sense also for $x \in \mathbb{C}$. For $x = i\sqrt{3}$, calculate the partial sum $\sum_{n=0}^{2} \frac{4^n-1}{9^n+7}(x+1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}\sqrt{\boxed{d}}}{\boxed{e}}i$.

$\boxed{a}$: $\boxed{-153 \ \checkmark}$ $\boxed{b}$: $\boxed{88 \ \checkmark}$ $\boxed{c}$: $\boxed{-87 \ \checkmark}$ $\boxed{d}$: $\boxed{3 \ \checkmark}$ $\boxed{e}$: $\boxed{88 \ \checkmark}$

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{4^n-1}{9^n+7}(x+1)^{2n}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{f}|x + \boxed{g}|^{\boxed{h}}}{\boxed{i}}$$

$\boxed{f}$: $\boxed{4 \ \checkmark}$ $\boxed{g}$: $\boxed{1 \ \checkmark}$ $\boxed{h}$: $\boxed{2 \ \checkmark}$ $\boxed{i}$: $\boxed{9 \ \checkmark}$

Therefore, by the root test, the series converges absolutely for $\frac{\boxed{\text{j}}}{\boxed{\text{k}}} < x < \frac{\boxed{\text{l}}}{\boxed{\text{m}}}$.

$\boxed{\text{j}}$: $\boxed{-5 \ \checkmark}$ $\boxed{\text{k}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{l}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{m}}$: $\boxed{2 \ \checkmark}$

For the case $x = -1$, the series
- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = \frac{1}{2}$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(6) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{9^n-1}{4^n+2}(x+1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i\sqrt{3}$, calculate the partial sum $\sum_{n=0}^{2} \frac{9^n-1}{4^n+2}(x+1)^{2n} = \frac{\boxed{\text{a}}}{\boxed{\text{b}}} + \frac{\boxed{\text{c}}\sqrt{\boxed{\text{d}}}}{\boxed{\text{e}}} i$.

$\boxed{\text{a}}$: $\boxed{-344 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{9 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{-296 \ \checkmark}$ $\boxed{\text{d}}$: $\boxed{3 \ \checkmark}$ $\boxed{\text{e}}$: $\boxed{9 \ \checkmark}$

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{9^n-1}{4^n+2}(x+1)^{2n}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{\text{f}}|x+\boxed{\text{g}}|^{\boxed{\text{h}}}}{\boxed{\text{i}}}$$

$\boxed{\text{f}}$: $\boxed{9 \ \checkmark}$ $\boxed{\text{g}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{h}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{i}}$: $\boxed{4 \ \checkmark}$

Therefore, by the root test, the series converges absolutely for $\frac{\boxed{\text{j}}}{\boxed{\text{k}}} < x < \frac{\boxed{\text{l}}}{\boxed{\text{m}}}$.

$\boxed{\text{j}}$: $\boxed{-5 \ \checkmark}$ $\boxed{\text{k}}$: $\boxed{3 \ \checkmark}$ $\boxed{\text{l}}$: $\boxed{-1 \ \checkmark}$ $\boxed{\text{m}}$: $\boxed{3 \ \checkmark}$

For the case $x = -1$, the series

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = -\frac{1}{2}$, the series
- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(7) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{9^n-1}{4^n+2}(x-1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i\sqrt{3}$, calculate the partial sum $\sum_{n=0}^{2} \frac{9^n-1}{4^n+2}(x-1)^{2n} = \frac{\boxed{\text{a}}}{\boxed{\text{b}}} + \frac{\boxed{\text{c}}\sqrt{\boxed{\text{d}}}}{\boxed{\text{e}}} i.$

$\boxed{\text{a}}$: $\boxed{-344 \ ✓}$ $\boxed{\text{b}}$: $\boxed{9 \ ✓}$ $\boxed{\text{c}}$: $\boxed{296 \ ✓}$ $\boxed{\text{d}}$: $\boxed{3 \ ✓}$ $\boxed{\text{e}}$: $\boxed{9 \ ✓}$

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{9^n-1}{4^n+2}(x-1)^{2n}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{\text{f}}|x + \boxed{\text{g}}|^{\boxed{\text{h}}}}{\boxed{\text{i}}}$$

$\boxed{\text{f}}$: $\boxed{9 \ ✓}$ $\boxed{\text{g}}$: $\boxed{-1 \ ✓}$ $\boxed{\text{h}}$: $\boxed{2 \ ✓}$ $\boxed{\text{i}}$: $\boxed{4 \ ✓}$

Therefore, by the root test, the series converges absolutely for $\frac{\boxed{\text{j}}}{\boxed{\text{k}}} < x < \frac{\boxed{\text{l}}}{\boxed{\text{m}}}$.

$\boxed{\text{j}}$: $\boxed{1 \ ✓}$ $\boxed{\text{k}}$: $\boxed{3 \ ✓}$ $\boxed{\text{l}}$: $\boxed{5 \ ✓}$ $\boxed{\text{m}}$: $\boxed{3 \ ✓}$

For the case $x = -1$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = \frac{1}{6}$, the series
- converges absolutely.

- converges but not absolutely.
- diverges. ✓

(8) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \sqrt{\frac{x^4}{x^2 + 4x - 5}}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- -6
- -5 ✓
- -4 ✓
- -3 ✓
- -2 ✓
- -1 ✓
- 1 ✓
- 2
- 3
- 4
- 5
- 6

Choose all asymptotes of $f(x)$.

- $y = -5$
- $y = -1$
- $y = 0$
- $y = 1$
- $y = 5$
- $x = -5$ ✓
- $x = -\sqrt{5}$
- $x = -\sqrt{2}$
- $x = -1$
- $x = 0$
- $x = 1$ ✓

- $x = \sqrt{2}$
- $x = \sqrt{5}$
- $x = 5$
- $y = x/2$
- $y = x$ ✓
- $y = 2x$
- $y = -x/2$
- $y = -x$ ✓
- $y = -2x$

One has

$$f'(2) = \frac{\boxed{\text{a}}\sqrt{\boxed{\text{b}}}}{\boxed{\text{c}}}.$$

$\boxed{\text{a}}$: $\boxed{12 \ ✓}$ $\boxed{\text{b}}$: $\boxed{7 \ ✓}$ $\boxed{\text{c}}$: $\boxed{49 \ ✓}$

The function $f(x)$ has $\boxed{\text{d}}$ stationary point(s) in the domain

$\boxed{\text{d}}$: $\boxed{2 \ ✓}$

Choose the behaviour of $f(x)$ in the interval $(-8, -7)$.

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

To determine the natural domain of a function, it is enough to observe the components. For example, $\sqrt{y}$ is defined for $y \geq 0$, $\frac{1}{y-a}$ is defined only for $y \neq a$, etc. It is enought to exclude all such points where the composed function is not defined. In this case, $\frac{x^4}{x^2+4x-5} \geq 0$, so $x = 0$, or $x^2 + 4x - 5 > 0$ (because $x^4 \geq 0$). As $x^2 + 4x - 5 = (x + 5)(x - 1) > 0$, we have $x < -5$ or $x > 1$ or $x = 0$.
There can be asymptotes for $x \to \pm\infty$, and for $x \to a$, where a is a boundary of the domain. In this case, one should check $x \to -5, 1, \infty$. $x \to -5, 1$ give infinity, so there are vertical asymptote there. As for $x \to \infty$, $f(x)$ diverges, so there is no horizontal asymptote. To see whethere there are oblique asymptotes, we note that $\frac{f(x)}{x} = \pm\sqrt{\frac{x^2}{x^2+4x-5}}$ depending on $x > 0$ or $x < 0$ This tends to ± 1 as $x \to \pm\infty$. One can also calculate $\lim_{x\to\pm\infty} f(x) - \pm x = 0$, therefore, the oblique asymptotes are $y = \pm x$.
For the derivative, the chain rule $(f(g(x)))' = g'(x)f'(g(x))$ is useful. In this case, $f(x) = \sqrt{\frac{x^4}{x^2+4x-5}}$, $f'(x) = \frac{1}{2}\sqrt{\frac{x^2+4x-5}{x^4}} \cdot \frac{4x^3(x^2+4x-5)-x^4(2x+4)}{(x^2+4x-5)^2} = \frac{1}{2}\sqrt{\frac{x^2+4x-5}{x^4}}\frac{2x^3(x^2-6x-10)}{(x^2+4x-5)^2}$.
If $f'(x_0) = 0$, x_0 is called a stationary point. In this case, $x_0 = 0, -3 \pm \sqrt{19}$. Note that, $x = 0$ is an isolated point in the domain, so $x = 0$ is not a stationary point. As $f'(-8) > 0, f'(-7) < 0$, f is not monotonic in the interval $(-8, -7)$.

(9) **Q3**
If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \sqrt{\frac{x^4}{x^2 - 4x - 5}}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- -6
- -5
- -4
- -3
- -2
- -1 ✓
- 1 ✓
- 2 ✓
- 3 ✓
- 4 ✓
- 5 ✓
- 6

Choose all asymptotes of $f(x)$.

- $y = -5$
- $y = -1$
- $y = 0$
- $y = 1$
- $y = 5$
- $x = -5$
- $x = -\sqrt{5}$
- $x = -\sqrt{2}$
- $x = -1$ ✓
- $x = 0$
- $x = 1$
- $x = \sqrt{2}$
- $x = \sqrt{5}$
- $x = 5$ ✓
- $y = x/2$
- $y = x$ ✓
- $y = 2x$
- $y = -x/2$
- $y = -x$ ✓
- $y = -2x$

One has

$$f'(-2) = \frac{\boxed{\text{a}}\sqrt{\boxed{\text{b}}}}{\boxed{\text{c}}}.$$

$\boxed{\text{a}}$: $\boxed{-12 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{7 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{49 \ \checkmark}$

The function $f(x)$ has $\boxed{\text{d}}$ stationary point(s) in the domain

$\boxed{\text{d}}$: $\boxed{2 \ \checkmark}$

Choose the behaviour of $f(x)$ in the interval $(7, 8)$.

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(10) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_1^2 2x(x^2+1)\log(x^2+1)dx.$$

Let us change the variables $x^2+1 = t$. Complete the formula

$$\int_1^2 2x(x^2+1)\log(x^2+1)dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} t^{\boxed{\text{c}}}\log(t+\boxed{\text{d}})dt$$

$\boxed{\text{a}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{5 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{d}}$: $\boxed{0 \ \checkmark}$

This integral in t can be carried out by integration by parts. We get

$$\int_1^2 2x(x^2+1)\log(x^2+1)dx = \frac{\boxed{\text{e}}}{\boxed{\text{f}}} + \boxed{\text{g}}\log\boxed{\text{h}} + \frac{\boxed{\text{i}}\log\boxed{\text{j}}}{\boxed{\text{k}}}$$

where $\boxed{\text{g}} < \boxed{\text{j}}$.

$\boxed{\text{e}}$: $\boxed{-21 \ \checkmark}$ $\boxed{\text{f}}$: $\boxed{4 \ \checkmark}$ $\boxed{\text{g}}$: $\boxed{-2 \ \checkmark}$ $\boxed{\text{h}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{i}}$: $\boxed{25 \ \checkmark}$ $\boxed{\text{j}}$: $\boxed{5 \ \checkmark}$ $\boxed{\text{k}}$: $\boxed{2 \ \checkmark}$

By substitution $x^2+1=t$, we need to consider $\frac{dt}{dx}=2x$, or formally $2xdx=dt$. Accordingly we need to change the limits of the integral.
After the change of variables, the integral becomes $\int_2^5 t\log t dt$. As $t=(\frac{t^2}{2})'$, we have $\int t\log t = \frac{t^2\log t}{2} - \int \frac{t}{2}dt = \frac{t^2\log t}{2} - \frac{t^2}{4} + C$ by integration by parts.

(11) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{\sqrt{2}}^{2} 2x(x^2+1)\log(x^2+1)dx.$$

Let us change the variables $x^2+1=t$. Complete the formula

$$\int_{\sqrt{2}}^{2} 2x(x^2+1)\log(x^2+1)dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} t^{\boxed{\text{c}}}\log(t+\boxed{\text{d}})dt$$

$\boxed{\text{a}}$: $\boxed{3\ \checkmark}$ $\boxed{\text{b}}$: $\boxed{5\ \checkmark}$ $\boxed{\text{c}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{d}}$: $\boxed{0\ \checkmark}$

This integral in t can be carried out by integration by parts. We get

$$\int_{\sqrt{2}}^{2} 2x(x^2+1)\log(x^2+1)dx = \boxed{\text{e}} + \frac{\boxed{\text{f}}\log\boxed{\text{g}}}{\boxed{\text{h}}} + \frac{\boxed{\text{i}}\log\boxed{\text{j}}}{\boxed{\text{k}}}$$

where $\boxed{\text{g}} < \boxed{\text{j}}$.

$\boxed{\text{e}}$: $\boxed{-4\ \checkmark}$ $\boxed{\text{f}}$: $\boxed{-9\ \checkmark}$ $\boxed{\text{g}}$: $\boxed{3\ \checkmark}$ $\boxed{\text{h}}$: $\boxed{2\ \checkmark}$ $\boxed{\text{i}}$: $\boxed{25\ \checkmark}$ $\boxed{\text{j}}$: $\boxed{5\ \checkmark}$ $\boxed{\text{k}}$: $\boxed{2\ \checkmark}$

(12) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_{\sqrt{2}}^{\sqrt{3}} 2x(x^2+1)\log(x^2+1)dx.$$

Let us change the variables $x^2+1=t$. Complete the formula

$$\int_{\sqrt{2}}^{\sqrt{3}} 2x(x^2+1)\log(x^2+1)dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} t^{\boxed{\text{c}}}\log(t+\boxed{\text{d}})dt$$

$\boxed{\text{a}}$: $\boxed{3\ \checkmark}$ $\boxed{\text{b}}$: $\boxed{4\ \checkmark}$ $\boxed{\text{c}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{d}}$: $\boxed{0\ \checkmark}$

This integral in t can be carried out by integration by parts. We get

$$\int_{\sqrt{2}}^{\sqrt{3}} 2x(x^2+1)\log(x^2+1)dx = \frac{\boxed{\text{e}}}{\boxed{\text{f}}} + \boxed{\text{g}}\log\boxed{\text{h}} + \frac{\boxed{\text{i}}\log\boxed{\text{j}}}{\boxed{\text{k}}}$$

where $\boxed{\text{g}} < \boxed{\text{j}}$.

$\boxed{\text{e}}$: $\boxed{-7\ \checkmark}$ $\boxed{\text{f}}$: $\boxed{4\ \checkmark}$ $\boxed{\text{g}}$: $\boxed{16\ \checkmark}$ $\boxed{\text{h}}$: $\boxed{2\ \checkmark}$ $\boxed{\text{i}}$: $\boxed{-9\ \checkmark}$ $\boxed{\text{j}}$: $\boxed{3\ \checkmark}$ $\boxed{\text{k}}$: $\boxed{2\ \checkmark}$

(13) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Consider the following improper integral for various $\alpha \in \mathbb{R}$.

$$\int_0^{\infty} xe^{\alpha x^2}dx.$$

Choose all values of α such that the improper integral is convergent.

- -5 ✓
- $-\pi$ ✓
- $-e$ ✓
- -2 ✓
- -1 ✓
- $-\frac{1}{2}$ ✓
- 0

- $\frac{1}{2}$
- 1
- 2
- π
- e
- 5

Among the correct options above, take the smallest value of α and calculate the improper integral: $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$.

$\boxed{\text{a}}$: $\boxed{1 \quad \checkmark}$ $\boxed{\text{b}}$: $\boxed{10 \quad \checkmark}$

Choose the values of β such that the following integral converges.

$$\int_0^\infty x^\beta e^{-x^3} dx.$$

- -5
- $-\pi$
- $-e$
- -2
- -1
- $-\frac{1}{2}$ ✓
- 0 ✓
- $\frac{1}{2}$ ✓
- 1 ✓
- 2 ✓
- π ✓
- e ✓
- 5 ✓

The function $f(x) = e^{\alpha x^2}$ diverges as $x \to \infty$ if $\alpha > 0$, is equal to 1 if $\alpha = 0$ and tends to 0 very fast as $x \to \infty$ if $\alpha < 0$. Therefore, the integral $\int_0^\infty x e^{\alpha x^2} dx$ converges only if $\alpha < 0$. By the substitution $t = x^2$ this can be calculated.
As for $\int_0^\infty x^\beta e^{-x^3} dx$, the part $x \to \infty$ is convergent for any value of β as in the previous case. The problem is $x = 0$, and we know that $e^{-0^3} = 1$, hence this is convergent if and only if $\beta > -1$ by comparison with $\int_0^1 x^\beta dx$.

(14) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Consider the following improper integral for various $\alpha \in \mathbb{R}$.

$$\int_0^\infty xe^{-\alpha x^2}dx.$$

Choose all values of α such that the improper integral is convergent.

- -6
- $-\pi$
- $-e$
- -2
- -1
- $-\frac{1}{2}$
- 0
- $\frac{1}{2}$ ✓
- 1 ✓
- 2 ✓
- π ✓
- e ✓
- 6 ✓

Among the correct options above, take the largest value of α and calculate the improper integral: $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$.

$\boxed{\text{a}}$: $\boxed{1 ✓}$ $\boxed{\text{b}}$: $\boxed{12 ✓}$

Choose the values of β such that the following integral converges.

$$\int_0^\infty x^{\beta-1}e^{-x^3}dx.$$

- -5
- $-\pi$
- $-e$
- -2
- -1
- $-\frac{1}{2}$
- 0

- $\frac{1}{2}$ ✓
- 1 ✓
- 2 ✓
- π ✓
- e ✓
- 5 ✓