Call5.

(1) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$x^3 = \boxed{\text{a}} + \boxed{\text{b}}(x+1) + \boxed{\text{c}}(x+1)^2 + \boxed{\text{d}}(x+1)^3.$$

$\boxed{\text{a}}$:

NUMERICAL | 2 points

-1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

-3 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

1 ✓	

$$(x+1)\log(x^2) = \boxed{\text{e}} + \boxed{\text{f}}(x+1) + \boxed{\text{g}}(x+1)^2 + \boxed{\text{h}}(x+1)^3 + o((x+1)^3) \text{ as } x \to -1.$$

$\boxed{\text{e}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{f}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 2 points

-2 ✓	

$\boxed{\text{h}}$:

NUMERICAL | 2 points

-1 ✓	

$\log(-x)\cdot(\exp(x+1)-1)\cdot(x+1) = \boxed{\text{i}}+\boxed{\text{j}}(x+1)+\boxed{\text{k}}(x+1)^2+\boxed{\text{l}}(x+1)^3+o((x+1)^3)$ as $x \to -1$.

$\boxed{\text{i}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{j}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{k}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{l}}$:

NUMERICAL | 3 points

-1 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to-1} \frac{x^3+\alpha(x+1)\log(x^2)+1+\beta(x+1)}{\log(-x)\cdot(\exp(x+1)-1)\cdot(x+1)}.$$

This limit converges for $\alpha = \frac{\boxed{\text{m}}}{\boxed{\text{n}}}, \beta = \boxed{\text{o}}$.

$\boxed{\text{m}}$:

NUMERICAL | 3 points

-3 ✓	

$\boxed{\text{n}}$:

NUMERICAL | 3 points

2 ✓	

$\boxed{\text{o}}$:

NUMERICAL | 6 points

-3 ✓	

In that case, the limit is $\frac{\boxed{\text{p}}}{\boxed{\text{q}}}$.

$\boxed{\text{p}}$:

NUMERICAL | 4 points

-5 ✓	

$\boxed{\text{q}}$:

NUMERICAL | 2 points

2 ✓	

Use the Taylor formula $f(x) = f(-1) + f'(-1)(x+1) + \frac{1}{2!}f''(-1)(x+1)^2 + \frac{1}{3!}f^{(3)}(-1)(x+1)^3 + o((x-3)^3)$ as $x \to -1$. As a polynomial, $x^3 = (x+1)^3 - 3(x+1)^2 + 3(x+1) - 1$ umambiguously. $\log(x^2)$ can be written as $\log(x^2) = 2\log(-x) = 2\log(-x-1+1) = 2\log(-(x+1)+1)$ and this can simplify the computation.
As for the product $\log(-x)\cdot(\exp(x+1)-1)\cdot(x+1)$, each of them has $x+1+o(x+1)$ (or $-(x+1)+o(x+1)$) and one can multiply term by term and get $\log(-x)\cdot(\exp(x+1)-1)\cdot(x+1) = -(x+1)^3 + o((x+1)^3)$.
To determine α, β, one only has to compare the numereator and the denominator and choose α, β in such a way that they have the same degree of infinitesimal (in this case, both of them should be of order $(x+1)^3$).

(2) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$x^3 = \boxed{\text{a}} + \boxed{\text{b}}(x-1) + \boxed{\text{c}}(x-1)^2 + \boxed{\text{d}}(x-1)^3.$$

$\boxed{\text{a}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

3 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

1 ✓	

$(x-1)\log(x^2) = \boxed{\mathrm{e}} + \boxed{\mathrm{f}}(x-1) + \boxed{\mathrm{g}}(x-1)^2 + \boxed{\mathrm{h}}(x-1)^3 + o((x-1)^3)$ as $x \to 1$.

$\boxed{\mathrm{e}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\mathrm{f}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\mathrm{g}}$:

NUMERICAL | 2 points

2 ✓	

$\boxed{\mathrm{h}}$:

NUMERICAL | 2 points

-1 ✓	

$\log(x)\cdot(\exp(x-1)-1)\cdot(x-1) = \boxed{\mathrm{i}} + \boxed{\mathrm{j}}(x-1) + \boxed{\mathrm{k}}(x-1)^2 + \boxed{\mathrm{l}}(x-1)^3 + o((x-1)^3)$ as $x \to 1$.

$\boxed{\mathrm{i}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\mathrm{j}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\mathrm{k}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\mathrm{l}}$:

NUMERICAL | 3 points

1 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to 1} \frac{x^3 + \alpha(x-1)\log(x^2) - 1 - \beta(x-1)}{\log(x)\cdot(\exp(x-1)-1)\cdot(x-1)}.$$

This limit converges for $\alpha = \frac{\boxed{\mathrm{m}}}{\boxed{\mathrm{n}}}, \beta = \boxed{\mathrm{o}}$.

$\boxed{\text{m}}$:

NUMERICAL | 3 points

-3 ✓	

$\boxed{\text{n}}$:

NUMERICAL | 3 points

2 ✓	

$\boxed{\text{o}}$:

NUMERICAL | 6 points

3 ✓	

In that case, the limit is $\frac{\boxed{\text{p}}}{\boxed{\text{q}}}$.

$\boxed{\text{p}}$:

NUMERICAL | 4 points

5 ✓	

$\boxed{\text{q}}$:

NUMERICAL | 2 points

2 ✓	

(3) **Q1**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Complete the formulae.

$$x^3 = \boxed{\text{a}} + \boxed{\text{b}}(x-1) + \boxed{\text{c}}(x-1)^2 + \boxed{\text{d}}(x-1)^3.$$

$\boxed{\text{a}}$:

NUMERICAL | 2 points

1 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

3 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

1 ✓	

$(x-1)\log(x^4) = \boxed{\text{e}} + \boxed{\text{f}}(x-1) + \boxed{\text{g}}(x-1)^2 + \boxed{\text{h}}(x-1)^3 + o((x-1)^3)$ as $x \to 1$.

$\boxed{\text{e}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{f}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 2 points

4 ✓	

$\boxed{\text{h}}$:

NUMERICAL | 2 points

-2 ✓	

$\log(x)\cdot(\exp(x-1)-1)\cdot(x-1) = \boxed{\text{i}} + \boxed{\text{j}}(x-1) + \boxed{\text{k}}(x-1)^2 + \boxed{\text{l}}(x-1)^3 + o((x-1)^3)$ as $x \to 1$.

$\boxed{\text{i}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{j}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{k}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{l}}$:

NUMERICAL | 3 points

1 ✓	

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x\to 1} \frac{3x^3 + \alpha(x-1)\log(x^4) - 3 - \beta(x-1)}{\log(x)\cdot(\exp(x-1)-1)\cdot(x-1)}.$$

This limit converges for $\alpha = \frac{\boxed{\text{m}}}{\boxed{\text{n}}}, \beta = \boxed{\text{o}}$.

$\boxed{\text{m}}$:

NUMERICAL | 3 points

-9 ✓	

$\boxed{\text{n}}$:

NUMERICAL | 3 points

4 ✓	

$\boxed{\text{o}}$:

NUMERICAL | 6 points

9 ✓	

In that case, the limit is $\frac{\boxed{\text{p}}}{\boxed{\text{q}}}$.

$\boxed{\text{p}}$:

NUMERICAL | 4 points

15 ✓	

$\boxed{\text{q}}$:

NUMERICAL | 2 points

2 ✓	

(4) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{n^3}{8^n+n^2}(x+1)^{3n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{n^3}{8^n+n^2}(x+1)^{3n} = \frac{\boxed{\text{a}}}{\boxed{\text{b}}} + \frac{\boxed{\text{c}}}{\boxed{\text{d}}} i$.

$\boxed{\text{a}}$:

NUMERICAL | 1 point

-2 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

9 ✓	

c:

NUMERICAL 1 point

-109 ✓	

d:

NUMERICAL 1 point

153 ✓	

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{n^3}{8^n+n^2}(x+1)^{3n}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{|x+\boxed{\text{e}}|^{\boxed{\text{f}}}}{\boxed{\text{g}}}$$

e:

NUMERICAL 1 point

1 ✓	

f:

NUMERICAL 1 point

3 ✓	

g:

NUMERICAL 2 points

8 ✓	

Therefore, by the root test, the series converges absolutely for

MULTI 8 points Single Shuffle

- all x.
- $-3 < x < -1$.
- $-3 < x < 1$. ✓
- $-2 < x < 2$.
- $-\frac{3}{2} < x < \frac{1}{2}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $-1 < x < 1$.
- $-1 < x < 3$.
- $-\frac{1}{2} < x < \frac{3}{2}$.
- $x = 0$.
- $0 < x < 2$.
- $0 < x < 4$.
- $\frac{1}{2} < x < \frac{3}{2}$.
- $1 < x < 1$.

- $1 < x < 3$.

For the case $x = -\frac{3}{2}$, the series

MULTI | 4 points | Single | Shuffle

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = 1$, the series

MULTI | 4 points | Single | Shuffle

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

The partial sum means the following finite sum: $\sum_{n=0}^{2} a_n = a_0 + a_1 + a_2$, so one just has to apply $n = 0, 1, 2$ in the concrete series and sum the numbers up. Notice that $i^2 = -1$. One can compute $(1+i)^6$ by using the fact that $1 + i = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$.
To apply the ratio test for a positive series $\sum a_n$, one considers $L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$. Note that for any $p \in \mathbb{N}$ it holds that $\lim_{n\to\infty} \frac{(n+1)^p}{n^p} = 1$, and $\lim_{n\to\infty} \frac{n^p}{8^n} = 0$ etc.
If this limit $L < 1$, then the series converges absolutely (for such x), while if $L > 1$ the series diverges. a_n depends on x, and this gives us a condition for which the series converges. That is $\frac{|x+1|^3}{8} < 1$, or $-2 < x + 1 < 2$
If $L = 1$, one needs to study the convergence with other criteria. In this case, if $x = 1$, then $a_n = \frac{n^3}{8^n + n^2} 2^{3n}$, and this diverges, hence so does the series.

(5) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{n^3}{8^n + n^2}(x-1)^{3n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = -i$, calculate the partial sum $\sum_{n=0}^{2} \frac{n^3}{8^n+n^2}(x-1)^{3n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} i$.

$\boxed{a}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{b}$:

NUMERICAL | 1 point

9 ✓	

$\boxed{c}$:

NUMERICAL | 1 point

-177 ✓	

$\boxed{d}$:

NUMERICAL | 1 point

153 ✓	

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{n^3}{8^n+n^2}(x-1)^{3n}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{|x + \boxed{e}|^{\boxed{f}}}{\boxed{g}}$$

$\boxed{e}$:

NUMERICAL | 1 point

-1 ✓	

$\boxed{f}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{g}$:

NUMERICAL | 2 points

8 ✓	

Therefore, by the root test, the series converges absolutely for

MULTI | 8 points | Single

- all x.
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-2 < x < 2$.
- $-\frac{3}{2} < x < \frac{1}{2}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.

- $-1 < x < 1$.
- $-1 < x < 3$. ✓
- $-\frac{1}{2} < x < \frac{3}{2}$.
- $x = 0$.
- $0 < x < 2$.
- $0 < x < 4$.
- $\frac{1}{2} < x < \frac{3}{2}$.
- $1 < x < 1$.
- $1 < x < 3$.

For the case $x = -\frac{3}{2}$, the series

MULTI | 4 points | Single | Shuffle

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = 1$, the series

MULTI | 4 points | Single | Shuffle

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(6) **Q2**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us study the following series $\sum_{n=0}^{\infty} \frac{n^3}{8^n+n^2}(x-2)^{3n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = 1-i$, calculate the partial sum $\sum_{n=0}^{2} \frac{n^3}{8^n+n^2}(x-2)^{3n} = \frac{\boxed{\text{a}}}{\boxed{\text{b}}} + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}i$.

$\boxed{\text{a}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

9 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 1 point

-177 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

153 ✓	

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{n^3}{8^n+n^2}(x-2)^{3n}$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{|x+\boxed{\text{e}}|^{\boxed{\text{f}}}}{\boxed{\text{g}}}$$

$\boxed{\text{e}}$:

NUMERICAL | 1 point

-2 ✓	

$\boxed{\text{f}}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 2 points

8 ✓	

Therefore, by the root test, the series converges absolutely for

MULTI | 8 points | Single

- all x.
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-2 < x < 2$.
- $-\frac{3}{2} < x < \frac{1}{2}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $-1 < x < 1$.
- $-1 < x < 3$.
- $-\frac{1}{2} < x < \frac{3}{2}$.
- $x = 0$.
- $0 < x < 2$.
- $0 < x < 4$. ✓
- $\frac{1}{2} < x < \frac{3}{2}$.
- $1 < x < 1$.
- $1 < x < 3$.

For the case $x = -\frac{3}{2}$, the series

MULTI 4 points Single

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = -1$, the series

MULTI 4 points Single

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(7) **Q3**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log \frac{x^3+1}{x^2-4}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI 4 points Single

- -3 ✓
- $-\frac{5}{2}$ ✓
- -2 ✓
- $-\frac{3}{2}$ (-100%)
- -1 ✓
- $-\frac{1}{2}$ ✓
- 0 ✓
- $\frac{1}{2}$ ✓
- 1 ✓
- $\frac{3}{2}$ ✓
- 2 ✓
- $\frac{5}{2}$ (-100%)
- 3 (-100%)

Choose all asymptotes of $f(x)$.

MULTI 4 points Single

- $y = -e$ (−100%)
- $y = -1$ (−100%)
- $y = 0$ (−100%)
- $y = 1$ (−100%)
- $y = e$ (−100%)
- $x = -2$ ✓
- $x = -\sqrt{3}$ (−100%)
- $x = -\sqrt{2}$ (−100%)
- $x = -1$ ✓
- $x = 0$ (−100%)
- $x = 1$ (−100%)
- $x = \sqrt{2}$ (−100%)
- $x = \sqrt{3}$ (−100%)
- $x = 2$ ✓
- $y = x/2$ (−100%)
- $y = x$ (−100%)
- $y = 2x$ (−100%)
- $y = -x/2$ (−100%)
- $y = -x$ (−100%)
- $y = -2x$ (−100%)

One has

$$f'(3) = \frac{\boxed{a}}{\boxed{b}}.$$

$\boxed{a}$:

NUMERICAL | 4 points

-33 ✓	

$\boxed{b}$:

NUMERICAL | 4 points

140 ✓	

The function $f(x)$ has $\boxed{a}$ stationary point(s) in the domain

$\boxed{a}$:

NUMERICAL | 4 points

1 ✓	

Choose the behaviour of $f(x)$ in the interval $(4, 5)$.

MULTI | 4 points | Single

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

To determine the natural domain of a function, it is enough to observe the components. For example, $\log y$ is defined for $y > 0$, $\frac{1}{y-a}$ is defined only for $y \neq a$, etc. It is enought to exclude all such points where the composed function is not defined. In this case, $\frac{x^3+1}{x^2-4} > 0$, so $x^3 + 1 > 0$ and $x^2 - 4 > 0$ or $x^3 + 1 < 0$ and $x^2 - 4 < 0$. That is, $x > 2$ or $-2 < x < -1$.
There can be asymptotes for $x \to \pm\infty$, and for $x \to a$, where a is a boundary of the domain. In this case, one should check $x \to -2, -1, 1, \infty$. $x \to -2, -1, 1$ give infinity, so there are vertical asymptote there. As for $x \to \infty$, $f(x)$ diverges, so there is no horizontal asymptote. To see whethere there are oblique asymptotes, we note that $f(x) = \log \frac{x^3+1}{x^2-4} < \log(2x^3)$, and $\frac{\log 2 + \log x^3}{x} \to 0$ as $x \to \infty$. Hence there is no oblique asymptote.
For the derivative, the chain rule $(f(g(x)))' = g'(x)f'(g(x))$ is useful. In this case, $f(x) = \log \frac{x^3+1}{x^2-4}$, $f'(x) = \frac{x^2-4}{x^3+1} \cdot \frac{3x^2(x^2-4)-(x^3+1)2x}{(x^2-4)^2} = \frac{x(x^3-12x-2)}{(x^3+1)(x^2-4)}$.
If $f'(x_0) = 0$, x_0 is called a stationary point. In this case, $x_0 = 0$ is the only possibility. Note that, $f'(0) = 0$ but $x = 0$ is not in the domain. As for $g(x) = x^3 - 12x - 2$, it has one solution in $[3, 4]$ because $g(3) < 0, g(4) > 0$. Other solutions are again outside the domain (they are in $x < -2$ and in $[-1, 0]$, by the same reasoning as above).
If $f'(x) \geq 0$ (≤ 0) in one interval, then $f(x)$ is monotonically increasing (decreasing) there. If $x \in [4, 5]$, $f'(x)$ is positive, because $g(4) = 64 - 48 - 2 > 0$, and $g'(x) = 3x^3 - 12 > 0$ in $x \in [4, 5]$.

(8) **Q3**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us consider the following function

$$f(x) = \log \frac{1 - x^3}{x^2 - 4}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

MULTI | 4 points | Single

- -3 (-100%)
- $-\frac{5}{2}$ (-100%)
- -2 ✓
- $-\frac{3}{2}$ ✓
- -1 ✓
- $-\frac{1}{2}$ ✓
- 0 ✓
- $\frac{1}{2}$ ✓
- 1 ✓
- $\frac{3}{2}$ (-100%)
- 2 ✓
- $\frac{5}{2}$ ✓
- 3 ✓

Choose all asymptotes of $f(x)$.

MULTI | 4 points | Single

- $y = -e$ (-100%)
- $y = -1$ (-100%)
- $y = 0$ (-100%)
- $y = 1$ (-100%)
- $y = e$ (-100%)
- $x = -2$ ✓
- $x = -\sqrt{3}$ (-100%)
- $x = -\sqrt{2}$ (-100%)
- $x = -1$ (-100%)
- $x = 0$ (-100%)
- $x = 1$ ✓
- $x = \sqrt{2}$ (-100%)
- $x = \sqrt{3}$ (-100%)
- $x = 2$ ✓
- $y = x/2$ (-100%)
- $y = x$ (-100%)
- $y = 2x$ (-100%)
- $y = -x/2$ (-100%)
- $y = -x$ (-100%)

- $y = -2x$ (-100%)

One has

$$f'(-3) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$:

NUMERICAL | 4 points

33 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 4 points

140 ✓	

The function $f(x)$ has $\boxed{\text{a}}$ stationary point(s) in the domain

$\boxed{\text{a}}$:

NUMERICAL | 4 points

1 ✓	

Choose the behaviour of $f(x)$ in the interval $(-5, -4)$.

MULTI | 4 points | Single

- monotonically decreasing ✓
- monotonically increasing
- neither decreasing nor increasing

(9) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_1^2 \frac{1}{2^x + 4 + 3(2^{-x})} dx.$$

Let us change the variables $2^x = t$. Complete the formula

$$\int_1^2 \frac{1}{2^x + 4 + 3(2^{-x})} dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} \frac{1}{\log \boxed{\text{c}} (t^2 + \boxed{\text{d}} t + \boxed{\text{e}})} dt$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

4 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

2 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

4 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

3 ✓	

By continuing, we get

$$\int_1^2 \frac{1}{2^x + 4 + 3(2^{-x})}dx = \frac{\log \frac{\boxed{\text{f}}}{\boxed{\text{g}}}}{\boxed{\text{h}} \log \boxed{\text{i}}}.$$

$\boxed{\text{f}}$:

NUMERICAL | 2 points

25 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 2 points

21 ✓	

$\boxed{\text{h}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{i}}$:

NUMERICAL | 1 point

2 ✓	

By substitution $2^x = t$, we need to consider $\frac{dt}{dx} = \log 2 2^x$, or formally $dx = \frac{1}{\log 2t}dt$. Accordingly we need to change the limits of the integral.
After the change of variables, the integral becomes $\int_2^4 \frac{1}{\log 2(t^2+4t+3)}dt$. The part $\frac{1}{t^2+4t+3} = \frac{1}{2}(\frac{1}{t+1} - \frac{1}{t+3})$ can be integrated by the formula $\int \frac{1}{x+a}dx = \log(x+a) + C$.

(10) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_2^3 \frac{1}{2^x + 4 + 3(2^{-x})} dx.$$

Let us change the variables $2^x = t$. Complete the formula

$$\int_2^3 \frac{1}{2^x + 4 + 3(2^{-x})} dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} \frac{1}{\log \boxed{\text{c}} (t^2 + \boxed{\text{d}}\, t + \boxed{\text{e}})} dt$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

4 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

8 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

2 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

4 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

3 ✓	

By continuing, we get

$$\int_2^3 \frac{1}{2^x + 4 + 3(2^{-x})} dx = \frac{\log \frac{\boxed{\text{f}}}{\boxed{\text{g}}}}{\boxed{\text{h}} \log \boxed{\text{i}}}.$$

$\boxed{\text{f}}$:

NUMERICAL | 2 points

63 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 2 points

55 ✓	

$\boxed{\text{h}}$:

NUMERICAL | 1 point

2 ✓	

$\boxed{\text{i}}$:

NUMERICAL | 1 point

2 ✓	

(11) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_2^4 \frac{1}{2^x - 2 - 3(2^{-x})} dx.$$

Let us change the variables $2^x = t$. Complete the formula

$$\int_2^4 \frac{1}{2^x - 2 - 3(2^{-x})} dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} \frac{1}{\log \boxed{\text{c}} (t^2 + \boxed{\text{d}} t + \boxed{\text{e}})} dt$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

4 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

8 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

2 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

-2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

-3 ✓	

By continuing, we get

$$\int_2^4 \frac{1}{2^x - 2 - 3(2^{-x})} dx = \frac{\log \frac{\boxed{\text{f}}}{\boxed{\text{g}}}}{\boxed{\text{h}} \log \boxed{\text{i}}}.$$

$\boxed{\text{f}}$:

NUMERICAL | 2 points

65 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 2 points

17 ✓	

$\boxed{\text{h}}$:

NUMERICAL | 1 point

4 ✓	

$\boxed{\text{i}}$:

NUMERICAL | 1 point

2 ✓	

(12) **Q4**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Let us calculate the following integral.

$$\int_3^5 \frac{1}{2^x - 2 - 3(2^{-x})} dx.$$

Let us change the variables $2^x = t$. Complete the formula

$$\int_3^5 \frac{1}{2^x - 2 - 3(2^{-x})} dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} \frac{1}{\log \boxed{\text{c}} (t^2 + \boxed{\text{d}}\, t + \boxed{\text{e}})} dt$$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

4 ✓	

$\boxed{\text{b}}$:

NUMERICAL | 1 point

32 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 2 points

2 ✓	

$\boxed{\text{d}}$:

NUMERICAL | 1 point

-2 ✓	

$\boxed{\text{e}}$:

NUMERICAL | 1 point

-3 ✓	

By continuing, we get

$$\int_3^5 \frac{1}{2^x - 2 - 3(2^{-x})} dx = \frac{\log \frac{\boxed{\text{f}}}{\boxed{\text{g}}}}{\boxed{\text{h}} \log \boxed{\text{i}}}.$$

$\boxed{\text{f}}$:

NUMERICAL | 2 points

87 ✓	

$\boxed{\text{g}}$:

NUMERICAL | 2 points

55 ✓	

$\boxed{\text{h}}$:

NUMERICAL | 1 point

4 ✓	

$\boxed{\text{i}}$:

NUMERICAL | 1 point

2 ✓	

(13) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = 6x^2 \exp(x^3) y^{\frac{1}{2}}$$

.

MULTI | 1 point | Single

- $y(x) = C \exp(x^3)$
- $y(x) = \exp(x^3) + \frac{C}{2}$
- $y(x) = 2\exp(x^3) + \frac{C}{2}$
- $y(x) = C \exp(x^6)$
- $y(x) = \exp(x^6) + C$
- $y(x) = 2\exp(x^6) + C$
- $y(x) = (\exp(x^3) + \frac{C}{2})^2$ ✓
- $y(x) = 2(\exp(x^3) + \frac{C}{2})^2$
- $y(x) = (\exp(x^3) + \frac{C}{2})^3$
- $y(x) = 2(\exp(x^3) + \frac{C}{2})^3$

Determine $C = \boxed{\text{a}} > 0$ with the initial condition $y((\log 2)^{\frac{1}{3}}) = 9$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

2 ✓	

Choose the general solution of the following differential equation.

$$y''(x) + y'(x) - 6y(x) = 0$$

.

MULTI | 1 point | Single

- $y(x) = C_1 \exp(-3x) + C_2 \exp(2x)$ ✓
- $y(x) = C_1 \exp(-2x) + C_2 \exp(3x)$
- $y(x) = C_1 \exp(-x) + C_2 \exp(6x)$
- $y(x) = C_1 \exp(-6x) + C_2 \exp(1x)$
- $y(x) = C_1 \sin(-3x) + C_2 \cos(2x)$
- $y(x) = C_1 \sin(-2x) + C_2 \cos(3x)$
- $y(x) = C_1 \sin(-x) + C_2 \cos(6x)$
- $y(x) = C_1 \sin(-6x) + C_2 \cos(1x)$

Find a solution $y(x)$ such that $y(0) = 4$ and $\lim_{x\to\infty} y(x) = 0$.
$C_1 = \boxed{\text{a}}, C_2 = \boxed{\text{b}}$.

$\boxed{\text{b}}$:

NUMERICAL 1 point

4 ✓	

c:

NUMERICAL 1 point

0 ✓	

With this solution, find a value $a =$ d such that $\lim_{x\to-\infty} \frac{y(x)}{e^{ax}}$ converges to a non-zero limit.

c:

NUMERICAL 1 point

-3 ✓	

The differential equation $y'(x) = 6x^2 \exp(x^3)y^{\frac{1}{2}}$ is separable. Write it formally as $y^{-\frac{1}{2}}dy = 6x^2\exp(x^3)dx$ and we can integrate separately, to obtain $2y^{\frac{1}{2}} = 2\exp(x^3) + C$, or $y = (\exp(x^3) + \frac{C}{2})^2$. To obtain the solution with the initial condition, note that If $x = (\log 2)^{\frac{1}{3}}$, we have to have $9 = (\exp(((\log 2)^{\frac{1}{3}})^3 + \frac{C}{2})^2 = (2 + \frac{C}{2})^2$, or $C = 2$. The equation $y''(x) + y'(x) - 6y(x) = 0$ can be solved by finding the solutions of the equation $z^2 + z - 6 = 0$, that are $z = 2, -3$. With them, the general solution is $y(x) = C_1\exp(-3x) + C_2\exp(2x)$. The term $C_1\exp(-3x)$ diverges as $x \to -\infty$, while the term $C_2\exp(3x)$ diverges as $x \to \infty$. The only possible value of a such that $\frac{e^{3x}}{e^{ax}}$ tends to a non-zero limit as $x \to -\infty$ is $a = 3$ (for other values, it is either ∞ or 0).

(14) **Q5**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = 6x^2\exp(x^3)y^{\frac{1}{2}}$$

.

MULTI 1 point Single

- $y(x) = C\exp(x^3)$
- $y(x) = \exp(x^3) + \frac{C}{2}$
- $y(x) = 2\exp(x^3) + \frac{C}{2}$
- $y(x) = C\exp(x^6)$
- $y(x) = \exp(x^6) + C$
- $y(x) = 2\exp(x^6) + C$
- $y(x) = (\exp(x^3) + \frac{C}{2})^2$ ✓
- $y(x) = 2(\exp(x^3) + \frac{C}{2})^2$
- $y(x) = (\exp(x^3) + \frac{C}{2})^3$
- $y(x) = 2(\exp(x^3) + \frac{C}{2})^3$

Determine $C = \boxed{\text{a}}$ with the initial condition $y((\log 2)^{\frac{1}{3}}) = 4$

$\boxed{\text{a}}$:

NUMERICAL 1 point

0 ✓	

Choose the general solution of the following differential equation.

$$y''(x) + y'(x) - 6y(x) = 0$$

.

MULTI 1 point Single

- $y(x) = C_1\exp(-3x) + C_2\exp(2x)$ ✓
- $y(x) = C_1\exp(-2x) + C_2\exp(3x)$
- $y(x) = C_1\exp(-x) + C_2\exp(6x)$
- $y(x) = C_1\exp(-6x) + C_2\exp(1x)$
- $y(x) = C_1\sin(-3x) + C_2\cos(2x)$
- $y(x) = C_1\sin(-2x) + C_2\cos(3x)$
- $y(x) = C_1\sin(-x) + C_2\cos(6x)$
- $y(x) = C_1\sin(-6x) + C_2\cos(1x)$

Find a solution $y(x)$ such that $y(0) = 4$ and $\lim_{x\to-\infty} y(x) = 0$. $C_1 = \boxed{\text{a}}, C_2 = \boxed{\text{b}}$.

$\boxed{\text{b}}$:

NUMERICAL 1 point

0 ✓	

$\boxed{\text{c}}$:

NUMERICAL 1 point

4 ✓	

With this solution, find a value $a = \boxed{\text{d}}$ such that $\lim_{x\to\infty} \frac{y(x)}{e^{ax}}$ converges to a non-zero limit.

$\boxed{\text{c}}$:

NUMERICAL 1 point

2 ✓	

(15) **Q5**

CLOZE 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = 9x^2 \exp(x^3) y^{\frac{2}{3}}$$

.

MULTI 1 point Single

- $y(x) = C\exp(x^3)$
- $y(x) = \exp(x^3) + \frac{C}{2}$
- $y(x) = 2\exp(x^3) + \frac{C}{2}$
- $y(x) = C\exp(x^6)$
- $y(x) = \exp(x^6) + C$
- $y(x) = 2\exp(x^6) + C$
- $y(x) = (\exp(x^3) + \frac{C}{2})^2$
- $y(x) = 2(\exp(x^3) + \frac{C}{2})^2$
- $y(x) = (\exp(x^3) + \frac{C}{2})^3$ ✓
- $y(x) = 2(\exp(x^3) + \frac{C}{2})^3$

Determine $C = \boxed{a}$ with the initial condition $y((\log 2)^{\frac{1}{3}}) = 1$

$\boxed{a}$:

NUMERICAL 1 point

-3 ✓	

Choose the general solution of the following differential equation.

$$y''(x) - y'(x) - 6y(x) = 0$$

.

MULTI 1 point Single

- $y(x) = C_1\exp(-3x) + C_2\exp(2x)$
- $y(x) = C_1\exp(-2x) + C_2\exp(3x)$ ✓
- $y(x) = C_1\exp(-x) + C_2\exp(6x)$
- $y(x) = C_1\exp(-6x) + C_2\exp(1x)$

- $y(x) = C_1 \sin(-3x) + C_2 \cos(2x)$
- $y(x) = C_1 \sin(-2x) + C_2 \cos(3x)$
- $y(x) = C_1 \sin(-x) + C_2 \cos(6x)$
- $y(x) = C_1 \sin(-6x) + C_2 \cos(1x)$

Find a solution $y(x)$ such that $y(0) = 3$ and $\lim_{x\to-\infty} y(x) = 0$. $C_1 = \boxed{\text{a}}, C_2 = \boxed{\text{b}}$.

$\boxed{\text{b}}$:

NUMERICAL | 1 point

0 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 1 point

3 ✓	

With this solution, find a value $a = \boxed{\text{d}}$ such that $\lim_{x\to\infty} \frac{y(x)}{e^{ax}}$ converges to a non-zero limit.

$\boxed{\text{c}}$:

NUMERICAL | 1 point

3 ✓	

(16) **Q5**

CLOZE | 0.10 penalty

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not). $\log x = \log_e x$, not $\log_{10} x$.

Choose the general solution of the following differential equation.

$$y'(x) = 6x^2 \exp(x^3) y^{\frac{1}{2}}$$

.

MULTI | 1 point | Single

- $y(x) = C \exp(x^3)$
- $y(x) = \exp(x^3) + \frac{C}{2}$
- $y(x) = 2\exp(x^3) + \frac{C}{2}$
- $y(x) = C \exp(x^6)$
- $y(x) = \exp(x^6) + C$
- $y(x) = 2\exp(x^6) + C$
- $y(x) = (\exp(x^3) + \frac{C}{2})^2$ ✓
- $y(x) = 2(\exp(x^3) + \frac{C}{2})^2$

Determine $C = \boxed{\text{a}}$ with the initial condition $y((\log 2)^{\frac{1}{3}}) = 27$

$\boxed{\text{a}}$:

NUMERICAL | 1 point

3 ✓	

Choose the general solution of the following differential equation.

$$y''(x) - y'(x) - 6y(x) = 0$$

.

MULTI | 1 point | Single

- $y(x) = C_1 \exp(-3x) + C_2 \exp(2x)$
- $y(x) = C_1 \exp(-2x) + C_2 \exp(3x)$ ✓
- $y(x) = C_1 \exp(-x) + C_2 \exp(6x)$
- $y(x) = C_1 \exp(-6x) + C_2 \exp(1x)$
- $y(x) = C_1 \sin(-3x) + C_2 \cos(2x)$
- $y(x) = C_1 \sin(-2x) + C_2 \cos(3x)$
- $y(x) = C_1 \sin(-x) + C_2 \cos(6x)$
- $y(x) = C_1 \sin(-6x) + C_2 \cos(1x)$

Find a solution $y(x)$ such that $y(0) = 3$ and $\lim_{x\to\infty} y(x) = 0$. $C_1 = \boxed{\text{a}}, C_2 = \boxed{\text{b}}$.

$\boxed{\text{b}}$:

NUMERICAL | 1 point

3 ✓	

$\boxed{\text{c}}$:

NUMERICAL | 1 point

0 ✓	

With this solution, find a value $a = \boxed{\text{d}}$ such that $\lim_{x\to-\infty} \frac{y(x)}{e^{ax}}$ converges to a non-zero limit.

$\boxed{\text{c}}$:

NUMERICAL | 1 point

-2 ✓	

Total of marks: 300