Call4.

(1) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$x \log x = \boxed{\text{a}} + \boxed{\text{b}}(x-1) + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}(x-1)^2 + \frac{\boxed{\text{e}}}{\boxed{\text{f}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{a}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{d}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{e}}$: $\boxed{-1 \ \checkmark}$
$\boxed{\text{f}}$: $\boxed{6 \ \checkmark}$

$$(x-1)\sqrt{x+3} = \boxed{\text{g}} + \boxed{\text{h}}(x-1) + \frac{\boxed{\text{i}}}{\boxed{\text{j}}}(x-1)^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{g}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{h}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{i}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{j}}$: $\boxed{4 \ \checkmark}$ $\boxed{\text{k}}$: $\boxed{-1 \ \checkmark}$
$\boxed{\text{l}}$: $\boxed{64 \ \checkmark}$

$$\exp((x-1)^3) = \boxed{\text{m}} + \boxed{\text{n}}(x-1) + \boxed{\text{o}}(x-1)^2 + \boxed{\text{p}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{m}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{m}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{o}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{p}}$: $\boxed{1 \ \checkmark}$

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 1} \frac{x \log x + \alpha(x-1)\sqrt{x+3} + \beta(x-1)}{\exp((x-1)^3) - 1}.$$

This limit converges for $\alpha = \boxed{\text{q}}$, $\beta = \boxed{\text{r}}$.

$\boxed{\text{q}}$: $\boxed{-2 \ \checkmark}$ $\boxed{\text{r}}$: $\boxed{3 \ \checkmark}$

In that case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

$\boxed{\text{v}}$: $\boxed{-13 \ \checkmark}$ $\boxed{\text{w}}$: $\boxed{96 \ \checkmark}$

Use the Taylor formula $f(x) = f(1) + f'(1)(x-1) + \frac{1}{2!}f''(1)(x-1)^2 + \frac{1}{3!}f^{(3)}(1)(x-1)^3 + o((x-3)^3)$ as $x \to 1$. To determine α, β, one only has to compare the numereator and the denominator and choose α, β in such a way that they have the same degree of infinitesimal (in this case, both of them should be of order $(x-1)^3$).

(2) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$x \log x = \boxed{a} + \boxed{b}(x-1) + \frac{\boxed{c}}{\boxed{d}}(x-1)^2 + \frac{\boxed{e}}{\boxed{f}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{a}$: $\boxed{0\ \checkmark}$ $\boxed{b}$: $\boxed{1\ \checkmark}$ $\boxed{c}$: $\boxed{1\ \checkmark}$ $\boxed{d}$: $\boxed{2\ \checkmark}$ $\boxed{e}$: $\boxed{-1\ \checkmark}$
$\boxed{f}$: $\boxed{6\ \checkmark}$

$$(x-1)\sqrt{x+3} = \boxed{g} + \boxed{h}(x-1) + \frac{\boxed{i}}{\boxed{j}}(x-1)^2 + \frac{\boxed{k}}{\boxed{l}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{g}$: $\boxed{0\ \checkmark}$ $\boxed{h}$: $\boxed{2\ \checkmark}$ $\boxed{i}$: $\boxed{1\ \checkmark}$ $\boxed{j}$: $\boxed{4\ \checkmark}$ $\boxed{k}$: $\boxed{-1\ \checkmark}$
$\boxed{l}$: $\boxed{64\ \checkmark}$

$$\exp((x-1)^3) = \boxed{m} + \boxed{n}(x-1) + \boxed{o}(x-1)^2 + \boxed{p}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{m}$: $\boxed{1\ \checkmark}$ $\boxed{m}$: $\boxed{0\ \checkmark}$ $\boxed{o}$: $\boxed{0\ \checkmark}$ $\boxed{p}$: $\boxed{1\ \checkmark}$

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 1} \frac{2x \log x + \alpha(x-1)\sqrt{x+3} + \beta(x-1)}{\exp((x-1)^3) - 1}.$$

This limit converges for $\alpha = \boxed{q}, \beta = \boxed{r}$.

$\boxed{q}$: $\boxed{-4\ \checkmark}$ $\boxed{r}$: $\boxed{6\ \checkmark}$

In that case, the limit is $\frac{\boxed{v}}{\boxed{w}}$.

$\boxed{\text{v}}$: $\boxed{-13 \;\checkmark}$ $\boxed{\text{w}}$: $\boxed{48 \;\checkmark}$

(3) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$x \log x = \boxed{\text{a}} + \boxed{\text{b}}(x-1) + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}(x-1)^2 + \frac{\boxed{\text{e}}}{\boxed{\text{f}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{a}}$: $\boxed{0 \;\checkmark}$ $\boxed{\text{b}}$: $\boxed{1 \;\checkmark}$ $\boxed{\text{c}}$: $\boxed{1 \;\checkmark}$ $\boxed{\text{d}}$: $\boxed{2 \;\checkmark}$ $\boxed{\text{e}}$: $\boxed{-1 \;\checkmark}$
$\boxed{\text{f}}$: $\boxed{6 \;\checkmark}$

$$(x-1)\sqrt{x+3} = \boxed{\text{g}} + \boxed{\text{h}}(x-1) + \frac{\boxed{\text{i}}}{\boxed{\text{j}}}(x-1)^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{g}}$: $\boxed{0 \;\checkmark}$ $\boxed{\text{h}}$: $\boxed{2 \;\checkmark}$ $\boxed{\text{i}}$: $\boxed{1 \;\checkmark}$ $\boxed{\text{j}}$: $\boxed{4 \;\checkmark}$ $\boxed{\text{k}}$: $\boxed{-1 \;\checkmark}$
$\boxed{\text{l}}$: $\boxed{64 \;\checkmark}$

$$\sin((x-1)^3) = \boxed{\text{m}} + \boxed{\text{n}}(x-1) + \boxed{\text{o}}(x-1)^2 + \boxed{\text{p}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{m}}$: $\boxed{0 \;\checkmark}$ $\boxed{\text{m}}$: $\boxed{0 \;\checkmark}$ $\boxed{\text{o}}$: $\boxed{0 \;\checkmark}$ $\boxed{\text{p}}$: $\boxed{1 \;\checkmark}$

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 1} \frac{3x \log x + \alpha(x-1)\sqrt{x+3} + \beta(x-1)}{\sin((x-1)^3)}.$$

This limit converges for $\alpha = \boxed{\text{q}}, \beta = \boxed{\text{r}}$.

$\boxed{\text{q}}$: $\boxed{-6 \;\checkmark}$ $\boxed{\text{r}}$: $\boxed{9 \;\checkmark}$

In that case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

$\boxed{\text{v}}$: $\boxed{-13 \;\checkmark}$ $\boxed{\text{w}}$: $\boxed{32 \;\checkmark}$

(4) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$x \log x = \boxed{\text{a}} + \boxed{\text{b}}(x-1) + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}(x-1)^2 + \frac{\boxed{\text{e}}}{\boxed{\text{f}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{a}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{d}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{e}}$: $\boxed{-1 \ \checkmark}$ $\boxed{\text{f}}$: $\boxed{6 \ \checkmark}$

$$(x-1)\sqrt{x+3} = \boxed{\text{g}} + \boxed{\text{h}}(x-1) + \frac{\boxed{\text{i}}}{\boxed{\text{j}}}(x-1)^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{g}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{h}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{i}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{j}}$: $\boxed{4 \ \checkmark}$ $\boxed{\text{k}}$: $\boxed{-1 \ \checkmark}$ $\boxed{\text{l}}$: $\boxed{64 \ \checkmark}$

$$\sin((x-1)^3) = \boxed{\text{m}} + \boxed{\text{n}}(x-1) + \boxed{\text{o}}(x-1)^2 + \boxed{\text{p}}(x-1)^3 + o((x-1)^3) \text{ as } x \to 1.$$

$\boxed{\text{m}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{m}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{o}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{p}}$: $\boxed{1 \ \checkmark}$

For various $\alpha, \beta \in \mathbb{R}$, study the limit:

$$\lim_{x \to 1} \frac{-x \log x + \alpha(x-1)\sqrt{x+3} + \beta(x-1)}{\sin((x-1)^3)}.$$

This limit converges for $\alpha = \boxed{\text{q}}$, $\beta = \boxed{\text{r}}$.

$\boxed{\text{q}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{r}}$: $\boxed{-3 \ \checkmark}$

In that case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

$\boxed{\text{v}}$: $\boxed{13 \ \checkmark}$ $\boxed{\text{w}}$: $\boxed{96 \ \checkmark}$

(5) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{4^n - 2^n}{n^3+2}(x+1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{4^n - 2^n}{n^3+2}(x+1)^{2n} = \frac{\boxed{\text{a}}}{\boxed{\text{b}}} + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}i$.

$\boxed{a}$: $\boxed{-24 \quad \checkmark}$ $\boxed{b}$: $\boxed{5 \quad \checkmark}$ $\boxed{c}$: $\boxed{4 \quad \checkmark}$ $\boxed{d}$: $\boxed{3 \quad \checkmark}$

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{4^n - 2^n}{n^3+2}(x+1)^{2n}$. Complete the formula.

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \boxed{e}|x + \boxed{g}|^{\boxed{h}}$$

$\boxed{e}$: $\boxed{4 \quad \checkmark}$ $\boxed{f}$: $\boxed{1 \quad \checkmark}$ $\boxed{g}$: $\boxed{2 \quad \checkmark}$

Therefore, by the root test, the series converges absolutely for

- all x.
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-2 < x < 2$.
- $-\frac{3}{2} < x < \frac{1}{2}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$. ✓
- $-1 < x < 1$.
- $-1 < x < 3$.
- $-\frac{1}{2} < x < \frac{1}{2}$.
- $x = 0$.
- $1 < x < 3$.

For the case $x = -\frac{3}{2}$, the series

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = 1$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

> The partial sum means the following finite sum: $\sum_{n=0}^{2} a_n = a_0 + a_1 + a_2$, so one just has to apply $n = 0, 1, 2$ in the concrete series and sum the numbers up. Notice that $i^2 = -1$.
> To apply the root test for a positive series $\sum a_n$, one considers $L = \lim_{n\to\infty}(a_{n+1})^{\frac{1}{n}}$. Note that for any $p \in \mathbb{N}$ it holds that $\lim_{n\to\infty}(n^p)^{\frac{1}{n}} = 1$ etc.
> If this limit $L < 1$, then the series converges absolutely (for such x), while if $R > 1$ the series diverges. a_n depends on x, and this gives us a condition for which the series converges
> If $R = 1$, one needs to study the convergence with other criteria. In this case, if $x = -\frac{3}{2}$, then $a_n = \frac{4^n - 2^n}{n^3+2}(-\frac{1}{2})^{2n} = \frac{4^n - 2^n}{4^n(n^3+2)}$, and this series is convergent.

(6) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{4^n - 2^n}{n^3+2}(x-1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{4^n - 2^n}{n^3+2}(x-1)^{2n} = \frac{\boxed{\text{a}}}{\boxed{\text{b}}} + \frac{\boxed{\text{c}}}{\boxed{\text{d}}} i$.

$\boxed{\text{a}}$: $\boxed{-24\ \checkmark}$ $\boxed{\text{b}}$: $\boxed{5\ \checkmark}$ $\boxed{\text{c}}$: $\boxed{-4\ \checkmark}$ $\boxed{\text{d}}$: $\boxed{3\ \checkmark}$

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{4^n - 2^n}{n^3+2}(x-1)^{2n}$. Complete the formula.

$$\lim_{n\to\infty}(a_n)^{\frac{1}{n}} = \boxed{\text{e}}|x + \boxed{\text{g}}|^{\boxed{\text{h}}}$$

$\boxed{\text{e}}$: $\boxed{4\ \checkmark}$ $\boxed{\text{f}}$: $\boxed{-1\ \checkmark}$ $\boxed{\text{g}}$: $\boxed{2\ \checkmark}$

Therefore, by the root test, the series converges absolutely for

- all x.
- $-3 < x < -1$.
- $-3 < x < 1$.

- $-2 < x < 2$.
- $-\frac{1}{2} < x < \frac{1}{2}$.
- $-1 < x < 1$.
- $-1 < x < 3$.
- $-\frac{3}{2} < x < \frac{1}{2}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $x = 0$.
- $\frac{1}{2} < x < \frac{3}{2}$. ✓
- $1 < x < 3$.

For the case $x = -\frac{3}{2}$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = 1$, the series
- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(7) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{9^n-2^n}{n^3+2}(x+1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{9^n-2^n}{n^3+2}(x+1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} i$.

$\boxed{a}$: $\boxed{-154 \ ✓}$ $\boxed{b}$: $\boxed{5 \ ✓}$ $\boxed{c}$: $\boxed{14 \ ✓}$ $\boxed{d}$: $\boxed{3 \ ✓}$

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{9^n-2^n}{n^3+2}(x+1)^{2n}$. Complete the formula.

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \boxed{e}|x + \boxed{g}|^{\boxed{h}}$$

$\boxed{e}$: $\boxed{9 \ ✓}$ $\boxed{f}$: $\boxed{1 \ ✓}$ $\boxed{g}$: $\boxed{2 \ ✓}$

Therefore, by the root test, the series converges absolutely for
- all x.
- $-\frac{4}{3} < x < \frac{4}{3}$.

- $-1 < x < 1$.
- $-\frac{1}{3} < x < \frac{4}{3}$.
- $-\frac{1}{3} < x < \frac{1}{3}$.
- $-3 < x < 3$.
- $x = 0$.
- $-\frac{2}{3} < x < \frac{1}{3}$.
- $-\frac{4}{3} < x < -\frac{2}{3}$. ✓
- $-\frac{2}{3} < x < \frac{2}{3}$.
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-1 < x < 3$.
- $1 < x < 3$.

For the case $x = 1$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = -\frac{4}{3}$, the series
- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(8) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{9^n - 2^n}{n^3+2}(x-1)^{2n}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{9^n - 2^n}{n^3+2}(x-1)^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} i$.

$\boxed{a}$: $\boxed{-154 \ ✓}$ $\boxed{b}$: $\boxed{5 \ ✓}$ $\boxed{c}$: $\boxed{-14 \ ✓}$ $\boxed{d}$: $\boxed{3 \ ✓}$

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{9^n - 2^n}{n^3+2}(x-1)^{2n}$. Complete the formula.

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \boxed{e}|x + \boxed{g}|^{\boxed{h}}$$

$\boxed{e}$: $\boxed{9 \ ✓}$ $\boxed{f}$: $\boxed{-1 \ ✓}$ $\boxed{g}$: $\boxed{2 \ ✓}$

Therefore, by the root test, the series converges absolutely for

- all x.
- $-3 < x < -1$.
- $-3 < x < 3$.
- $-3 < x < 1$.
- $-\frac{4}{3} < x < -\frac{2}{3}$.
- $-\frac{4}{3} < x < \frac{4}{3}$.
- $-1 < x < 1$.
- $-1 < x < 3$.
- $-\frac{2}{3} < x < \frac{1}{3}$.
- $-\frac{1}{3} < x < \frac{4}{3}$.
- $-\frac{1}{3} < x < \frac{1}{3}$.
- $x = 0$.
- $\frac{1}{3} < x < \frac{2}{3}$.
- $\frac{2}{3} < x < \frac{4}{3}$. ✓
- $1 < x < 3$.

For the case $x = 1$, the series

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = \frac{4}{3}$, the series

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

(9) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us consider the following function

$$f(x) = \sqrt{\frac{x^4 - 5x^2 + 4}{x^2 + 1}}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- -3

- $-\frac{5}{2}$
- -2
- $-\frac{3}{2}$ ✓
- -1
- $-\frac{1}{2}$
- 0
- $\frac{1}{2}$
- 1
- $\frac{3}{2}$ ✓
- 2
- $\frac{5}{2}$
- 3

Choose all asymptotes of $f(x)$.

- $y = -e$
- $y = -1$
- $y = 0$
- $y = 1$
- $y = e$
- $x = -2$
- $x = -\sqrt{3}$
- $x = -\sqrt{2}$
- $x = -1$
- $x = 0$
- $x = 1$
- $x = \sqrt{2}$
- $x = \sqrt{3}$
- $x = 2$
- $y = x/2$
- $y = x$ ✓
- $y = 2x$
- $y = -x/2$
- $y = -x$ ✓
- $y = -2x$

One has

$$f'(3) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$: $\boxed{27 \ ✓}$ $\boxed{\text{b}}$: $\boxed{20 \ ✓}$

The function $f(x)$ has $\boxed{\text{a}}$ stationary point(s) in the domain

$\boxed{\text{a}}$: $\boxed{1 \ ✓}$

Choose the behaviour of $f(x)$ in the interval $(3, 5)$.

- monotonically decreasing

- monotonically increasing ✓
- neither decreasing nor increasing

To determine the natural domain of a function, it is enough to observe the components. For example, $\sqrt{y}$ is defined for $y \geq 0$, $\frac{1}{y-a}$ is defined only for $y \neq a$, etc. It is enought to exclude all such points where the composed function is not defined. In this case, $\frac{x^4-5x^2+4}{x^2+1} \geq 0$, so $x^4-5x^2+4 \geq 0$ This is equivalent to $(x^2-1)(x^2-4) \geq 0$, that is, $x \leq -2, -1 \leq x \leq x$ or $2 \leq x$.
There can be asymptotes for $x \to \pm\infty$, and for $x \to a$, where a is a boundary of the domain. In this case, one should check $x \to \pm 1, \pm 4, \pm\infty$. $x \to \pm 1, \pm 4$ give finite result, so there is no asymptote there. As for $\pm\infty$, $f(x)$ tends to ∞, so there is no horizontal asymptote. To see whethere there are oblique asymptotes, we first compute $\lim_{x \pm\infty} \frac{f(x)}{x} = \pm 1$. After this, we compute $\lim_{x \pm \infty} \frac{f(x)}{x} - \pm x = 0$. Therefore, the oblique asymptotes are $y = \pm x$.
For the derivative, the chain rule $(f(g(x)))' = g'(x) f'(g(x))$ is useful. In this case, $f(x) = \sqrt{\frac{x^4-5x^2+4}{x^2+1}}$, $f'(x) = \frac{1}{2}\sqrt{\frac{x^2+1}{x^4-5x^2+4}} \frac{(4x^3-10x)(x^2+1)-2x(x^4-5x+4)}{(x^2+1)^2} = \sqrt{\frac{x^2+1}{x^4-5x^2+4}} \frac{x(x^4+2x^2-9)}{(x^2+1)^2}$.
If $f'(x_0) = 0$, x_0 is called a stationary point. In this case, $x_0 = 0$ is the only possibility. Note that, $g(x) = x^4+2x^2-9 = 0$ have other solutions, but they are outside the domain, because $g(2) > 0$, and
If $f'(x) \geq 0$ (≤ 0) in one interval, then $f(x)$ is monotonically increasing (decreasing) there. If $x \in [3, 5]$, $f'(x)$ is positive. Note that $\sqrt{\frac{x^2+1}{x^4-5x^2+4}}$ is positive in the domain.

(10) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us consider the following function

$$f(x) = \sqrt{\frac{4x^4 - 20x^2 + 16}{x^2 + 1}}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- -3
- $-\frac{5}{2}$
- -2
- $-\frac{3}{2}$ ✓
- -1
- $-\frac{1}{2}$
- 0
- $\frac{1}{2}$
- 1
- $\frac{3}{2}$ ✓
- 2
- $\frac{5}{2}$
- 3

Choose all asymptotes of $f(x)$.

- $y = -e$
- $y = -1$
- $y = 0$
- $y = 1$
- $y = e$
- $x = -2$
- $x = -\sqrt{3}$
- $x = -\sqrt{2}$
- $x = -1$
- $x = 0$
- $x = 1$
- $x = \sqrt{2}$
- $x = \sqrt{3}$
- $x = 2$
- $y = x/2$
- $y = x$
- $y = 2x$ ✓
- $y = -x/2$
- $y = -x$
- $y = -2x$ ✓

One has

$$f'(3) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$: $\boxed{27 \quad \checkmark}$ $\boxed{\text{b}}$: $\boxed{10 \quad \checkmark}$

The function $f(x)$ has $\boxed{\text{a}}$ stationary point(s) in the domain

$\boxed{\text{a}}$: $\boxed{1 \quad \checkmark}$

Choose the behaviour of $f(x)$ in the interval $(-5, -3)$.

- monotonically decreasing ✓
- monotonically increasing
- neither decreasing nor increasing

(11) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us consider the following function

$$f(x) = \sqrt{\frac{x^4 - 5x^2 + 4}{4x^2 + 4}}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- -3
- $-\frac{5}{2}$
- -2
- $-\frac{3}{2}$ ✓
- -1
- $-\frac{1}{2}$
- 0
- $\frac{1}{2}$
- 1
- $\frac{3}{2}$ ✓
- 2
- $\frac{5}{2}$
- 3

Choose all asymptotes of $f(x)$.

- $y = -e$
- $y = -1$

- $y = 0$
- $y = 1$
- $y = e$
- $x = -2$
- $x = -\sqrt{3}$
- $x = -\sqrt{2}$
- $x = -1$
- $x = 0$
- $x = 1$
- $x = \sqrt{2}$
- $x = \sqrt{3}$
- $x = 2$
- $y = x/2$ ✓
- $y = x$
- $y = 2x$
- $y = -x/2$ ✓
- $y = -x$
- $y = -2x$

One has

$$f'(3) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$: $\boxed{27\ ✓}$ $\boxed{\text{b}}$: $\boxed{40\ ✓}$

The function $f(x)$ has $\boxed{\text{a}}$ stationary point(s) in the domain

$\boxed{\text{a}}$: $\boxed{1\ ✓}$

Choose the behaviour of $f(x)$ in the interval $(-1, 1)$.

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

(12) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us calculate the following integral.

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2(x) \sin\left(x + \frac{\pi}{6}\right) dx.$$

Complete the formula

$$\sin\left(x+\frac{\pi}{6}\right) = \frac{\sqrt{\boxed{\text{a}}}}{\boxed{\text{b}}}\sin x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}\cos x.$$

$\boxed{\text{a}}$: $\boxed{3\ \checkmark}$ $\boxed{\text{b}}$: $\boxed{2\ \checkmark}$ $\boxed{\text{c}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{d}}$: $\boxed{2\ \checkmark}$

Choose a primitive of $\cos^2(x)\sin(x)$.

- $\frac{1}{3}\cos^3(x)\sin(x)$
- $-\frac{1}{3}\cos^3(x)\sin(x)$
- $\frac{1}{3}\cos^3(x)$
- $-\frac{1}{3}\cos^3(x)$ ✓
- $\frac{1}{2}\cos^3(\sin(x))$
- $-\frac{1}{2}\cos^3(\sin(x))$
- $\frac{1}{2}\sin^3(\cos(x))$
- $-\frac{1}{2}\sin^3(\cos(x))$

Choose a primitive of $\cos^3(x)$.

- $-\frac{1}{4}\cos^4(x)$
- $\frac{1}{4}\sin^4(x)$
- $\sin(x)-\frac{1}{3}\sin^3(x)$ ✓
- $\cos(x)-\frac{1}{3}\cos^3(x)$
- $x-\frac{1}{3}\sin^3(x)$
- $x-\frac{1}{3}\cos^3(x)$
- $x+\frac{1}{4}\sin^4(x)$
- $x-\frac{1}{4}\cos^4(x)$

By continuing, we get

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} \cos^2(x)\sin\left(x+\frac{\pi}{6}\right)dx = \frac{\boxed{\text{e}}}{\boxed{\text{f}}} + \frac{\boxed{\text{g}}\sqrt{\boxed{\text{h}}}}{\boxed{\text{i}}} + \frac{\sqrt{\boxed{\text{j}}}}{\boxed{\text{k}}}$$

.

$\boxed{\text{e}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{f}}$: $\boxed{3\ \checkmark}$ $\boxed{\text{g}}$: $\boxed{5\ \checkmark}$ $\boxed{\text{h}}$: $\boxed{2\ \checkmark}$ $\boxed{\text{i}}$: $\boxed{24\ \checkmark}$ $\boxed{\text{j}}$: $\boxed{6\ \checkmark}$ $\boxed{\text{k}}$: $\boxed{24\ \checkmark}$

> The formula $\sin(x+a) = \sin x\cos a + \cos x \sin a$ allows to decompose $\sin(x+a)$ as a linear combination of $\sin(x+a)$. The integral $\int \sin x \cos^2 x dx$ can be carried out by the substitution $t = \cos$, hence is reduced to $-\int t^3 dt$. On the other hand, we have $\int \sin^3 x dx = \int \sin x(1-\cos x)dx$ and this can be calculated as well.

(13) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us calculate the following integral.

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2(x)\sin\left(x+\frac{7\pi}{6}\right)dx.$$

Complete the formula

$$\sin\left(x+\frac{7\pi}{6}\right) = -\frac{\sqrt{\boxed{\text{a}}}}{\boxed{\text{b}}}\sin x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}\cos x.$$

$\boxed{\text{a}}$: $\boxed{3\ \checkmark}$ $\boxed{\text{b}}$: $\boxed{2\ \checkmark}$ $\boxed{\text{c}}$: $\boxed{-1\ \checkmark}$ $\boxed{\text{d}}$: $\boxed{2\ \checkmark}$

Choose a primitive of $\cos^2(x)\sin(x)$.

- $\frac{1}{3}\cos^3(x)\sin(x)$
- $-\frac{1}{3}\cos^3(x)\sin(x)$
- $\frac{1}{3}\cos^3(x)$
- $-\frac{1}{3}\cos^3(x)$ ✓
- $\frac{1}{2}\cos^3(\sin(x))$
- $-\frac{1}{2}\cos^3(\sin(x))$
- $\frac{1}{2}\sin^3(\cos(x))$
- $-\frac{1}{2}\sin^3(\cos(x))$

Choose a primitive of $\cos^3(x)$.

- $-\frac{1}{4}\cos^4(x)$
- $\frac{1}{4}\sin^4(x)$
- $\sin(x)-\frac{1}{3}\sin^3(x)$ ✓
- $\cos(x)-\frac{1}{3}\cos^3(x)$
- $x-\frac{1}{3}\sin^3(x)$
- $x-\frac{1}{3}\cos^3(x)$
- $x+\frac{1}{4}\sin^4(x)$
- $x-\frac{1}{4}\cos^4(x)$

By continuing, we get

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2(x)\sin\left(x+\frac{7\pi}{6}\right)dx = \frac{\boxed{\text{e}}}{\boxed{\text{f}}} + \frac{\boxed{\text{g}}\sqrt{\boxed{\text{h}}}}{\boxed{\text{i}}} - \frac{\sqrt{\boxed{\text{j}}}}{\boxed{\text{k}}}$$

.

$\boxed{\text{e}}$: $\boxed{-1 \quad \checkmark}$ $\boxed{\text{f}}$: $\boxed{3 \quad \checkmark}$ $\boxed{\text{g}}$: $\boxed{-5 \quad \checkmark}$ $\boxed{\text{h}}$: $\boxed{2 \quad \checkmark}$ $\boxed{\text{i}}$: $\boxed{24 \quad \checkmark}$
$\boxed{\text{j}}$: $\boxed{6 \quad \checkmark}$ $\boxed{\text{k}}$: $\boxed{24 \quad \checkmark}$

(14) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us calculate the following integral.

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2(x) \sin\left(x + \frac{\pi}{3}\right) dx.$$

Complete the formula

$$\sin\left(x + \frac{\pi}{3}\right) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}} \sin x + \frac{\sqrt{\boxed{\text{c}}}}{\boxed{\text{d}}} \cos x.$$

$\boxed{\text{a}}$: $\boxed{1 \quad \checkmark}$ $\boxed{\text{b}}$: $\boxed{2 \quad \checkmark}$ $\boxed{\text{c}}$: $\boxed{3 \quad \checkmark}$ $\boxed{\text{d}}$: $\boxed{2 \quad \checkmark}$

Choose a primitive of $\cos^2(x)\sin(x)$.

- $\frac{1}{3}\cos^3(x)\sin(x)$
- $-\frac{1}{3}\cos^3(x)\sin(x)$
- $\frac{1}{3}\cos^3(x)$
- $-\frac{1}{3}\cos^3(x)$ ✓
- $\frac{1}{2}\cos^3(\sin(x))$
- $-\frac{1}{2}\cos^3(\sin(x))$
- $\frac{1}{2}\sin^3(\cos(x))$
- $-\frac{1}{2}\sin^3(\cos(x))$

Choose a primitive of $\cos^3(x)$.

- $-\frac{1}{4}\cos^4(x)$
- $\frac{1}{4}\sin^4(x)$
- $\sin(x) - \frac{1}{3}\sin^3(x)$ ✓
- $\cos(x) - \frac{1}{3}\cos^3(x)$
- $x - \frac{1}{3}\sin^3(x)$
- $x - \frac{1}{3}\cos^3(x)$
- $x + \frac{1}{4}\sin^4(x)$
- $x - \frac{1}{4}\cos^4(x)$

By continuing, we get

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2(x)\sin\left(x+\frac{\pi}{3}\right)dx = \frac{\sqrt{\boxed{\text{e}}}}{\boxed{\text{f}}} + \frac{\sqrt{\boxed{\text{g}}}}{\boxed{\text{h}}} - \frac{\boxed{\text{i}}\sqrt{\boxed{\text{j}}}}{\boxed{\text{k}}}$$

.

Fill them in, with $\boxed{\text{e}} < \boxed{\text{g}}$. $\boxed{\text{e}}$: $\boxed{2\ \checkmark}$ $\boxed{\text{f}}$: $\boxed{24\ \checkmark}$ $\boxed{\text{g}}$: $\boxed{3\ \checkmark}$ $\boxed{\text{h}}$: $\boxed{3\ \checkmark}$ $\boxed{\text{i}}$: $\boxed{5\ \checkmark}$ $\boxed{\text{j}}$: $\boxed{6\ \checkmark}$ $\boxed{\text{k}}$: $\boxed{24\ \checkmark}$

(15) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Determine whether the following improper integral converges, and if so, calculate the value. If it does not converge, write $\frac{1}{0}$.

$$\int_0^\infty x^{-\frac{2}{5}} e^{-x^{\frac{3}{5}}} dx = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$: $\boxed{5\ \checkmark}$ $\boxed{\text{b}}$: $\boxed{3\ \checkmark}$

Let us consider the improper integral $\int_0^\infty f(x)dx$. Choose all function(s) $f(x)$ for which this improper integral converges.

- $f(x) = \exp(x)$
- $f(x) = \exp(-x)$ ✓
- $f(x) = x\exp(x)$
- $f(x) = x\exp(-x)$ ✓
- $f(x) = \frac{1}{x}\exp(x)$
- $f(x) = \frac{1}{x}\exp(-x)$
- $f(x) = \exp(x^2)$
- $f(x) = \exp(-x^2)$ ✓
- $f(x) = x^2\exp(x^2)$
- $f(x) = x^2\exp(-x^2)$ ✓
- $f(x) = \frac{1}{x}\exp(x^2)$
- $f(x) = \frac{1}{x}\exp(-x^2)$

Among the following improper integrals, choose the smallest (and convergent) one and give its value $\frac{\boxed{\text{c}}}{\boxed{\text{d}}}e^{\boxed{\text{e}}}$.

- $\int_0^\infty x\exp(-x)dx$
- $\int_0^\infty x^2\exp(-x)dx$

- $\int_0^\infty \exp(-x)$
- $\int_0^\infty \frac{1}{x}\exp(-x)$
- $\int_0^\infty x\exp(-2x)dx$
- $\int_0^\infty x^2\exp(-2x)dx$
- $\int_2^\infty \exp(-2x)$
- $\int_2^\infty \frac{1}{x}\exp(-2x)$
- $\int_2^\infty x\exp(-2x)dx$
- $\int_2^\infty x^2\exp(-2x)dx$
- $\int_2^\infty \exp(-3x)$

$\boxed{\text{c}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{d}}$: $\boxed{3\ \checkmark}$ $\boxed{\text{e}}$: $\boxed{-6\ \checkmark}$

To consider the (improper) integral $\int_0^\infty f(x)dx$, we need to determine where $f(x)$ is unbounded, and also the behaviour of $f(x)$ as $x \to \infty$. In the examples at hand, $f(x)$ is unbounded only around $x = 0$, so we need to take $\int_a^1 f(x)dx + \int_1^b f(x)dx$, and then take the limits $a \to 0, b \to \infty$ separately.
The integral $\int_a^b x^{-\frac{2}{5}}e^{-x^{\frac{3}{5}}}dx$ can be carried out by the substitution $x^{\frac{3}{5}} = t$, because $\frac{dt}{dx} = \frac{5}{3}x^{-\frac{2}{5}}$.
We know that $\int_0^1 x^\alpha dx$ is convergent only for $\alpha > -1$, while $\int_1^\infty x^p e^{-x}dx$ is convergent for any $p \in \mathbb{R}$. e^{-x^2} decays faster than e^{-x}.
As for the last question, we have $\int_a^b f(x)dx \geq \int_a^b g(x)$ if $f(x) \geq g(x)$, and $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ and $\int_a^b f(x)dx \geq 0$ if $f(x) \geq 0$. All functions are positive, and e^{-3x} is the smalles on the interval $[2, \infty)$, so this gives the smallest integral.

(16) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Determine whether the following improper integral converges, and if so, calculate the value. If it does not converge, write $\frac{1}{0}$.

$$\int_0^\infty x^{-\frac{1}{3}} e^{-x^{\frac{2}{3}}}\, dx = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$: $\boxed{3 \quad \checkmark}$ $\boxed{\text{b}}$: $\boxed{2 \quad \checkmark}$

Let us consider the improper integral $\int_{-\infty}^0 f(x)dx$. Choose all function(s) $f(x)$ for which this improper integral converges.

- $f(x) = \exp(x)$ ✓
- $f(x) = \exp(-x)$
- $f(x) = x\exp(x)$ ✓
- $f(x) = x\exp(-x)$
- $f(x) = \frac{1}{x}\exp(x)$
- $f(x) = \frac{1}{x}\exp(-x)$
- $f(x) = \exp(x^2)$
- $f(x) = \exp(-x^2)$ ✓
- $f(x) = x^2\exp(x^2)$
- $f(x) = x^2\exp(-x^2)$ ✓
- $f(x) = \frac{1}{x}\exp(x^2)$
- $f(x) = \frac{1}{x}\exp(-x^2)$

Among the following improper integrals, choose the smallest (and convergent) one and give its value $\frac{\boxed{\text{c}}}{\boxed{\text{d}}}e^{\boxed{\text{e}}}$.

- $\int_0^\infty x\exp(-x)dx$
- $\int_0^\infty x^2\exp(-x)dx$
- $\int_0^\infty \exp(-x)$
- $\int_0^\infty \frac{1}{x}\exp(-x)$
- $\int_0^\infty x\exp(-2x)dx$
- $\int_0^\infty x^2\exp(-2x)dx$
- $\int_2^\infty \exp(-2x)$
- $\int_2^\infty \frac{1}{x}\exp(-2x)$
- $\int_2^\infty x\exp(-2x)dx$
- $\int_2^\infty x^2\exp(-2x)dx$
- $\int_2^\infty \exp(-4x)$

$\boxed{\text{c}}$: $\boxed{1 \quad \checkmark}$ $\boxed{\text{d}}$: $\boxed{4 \quad \checkmark}$ $\boxed{\text{e}}$: $\boxed{-8 \quad \checkmark}$

(17) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Determine whether the following improper integral converges, and if so, calculate the value. If it does not converge, write $\frac{1}{0}$.

$$\int_0^\infty x^{-\frac{3}{7}} e^{-x^{\frac{4}{7}}}\,dx = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$: $\boxed{7 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{4 \ \checkmark}$

Let us consider the improper integral $\int_1^\infty f(x)dx$. Choose all function(s) $f(x)$ for which this improper integral converges.

- $f(x) = \exp(x)$
- $f(x) = \exp(-x)$ ✓
- $f(x) = x\exp(x)$
- $f(x) = x\exp(-x)$ ✓
- $f(x) = \frac{1}{x}\exp(x)$
- $f(x) = \frac{1}{x}\exp(-x)$ ✓
- $f(x) = \exp(x^2)$
- $f(x) = \exp(-x^2)$ ✓
- $f(x) = x^2\exp(x^2)$
- $f(x) = x^2\exp(-x^2)$ ✓
- $f(x) = \frac{1}{x}\exp(x^2)$
- $f(x) = \frac{1}{x}\exp(-x^2)$ ✓

Among the following improper integrals, choose the smallest (and convergent) one and give its value $\frac{\boxed{\text{c}}}{\boxed{\text{d}}}e^{\boxed{\text{e}}}$.

- $\int_0^\infty x\exp(-x)dx$
- $\int_0^\infty x^2\exp(-x)dx$
- $\int_0^\infty \exp(-x)$
- $\int_0^\infty \frac{1}{x}\exp(-x)$
- $\int_0^\infty x\exp(-3x)dx$
- $\int_0^\infty x^2\exp(-3x)dx$
- $\int_2^\infty \exp(-3x)$
- $\int_2^\infty \frac{1}{x}\exp(-3x)$
- $\int_2^\infty x\exp(-3x)dx$
- $\int_2^\infty x^2\exp(-3x)dx$
- $\int_2^\infty \exp(-5x)$

$\boxed{\text{c}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{d}}$: $\boxed{5 \ \checkmark}$ $\boxed{\text{e}}$: $\boxed{-10 \ \checkmark}$