Call3.

(1) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$\cos x = \boxed{\text{a}} + \boxed{\text{b}}x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}x^2 + \boxed{\text{e}}x^3 + \frac{\boxed{\text{f}}}{\boxed{\text{g}}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{\text{a}}$: $\boxed{1 \quad \checkmark}$ $\boxed{\text{b}}$: $\boxed{0 \quad \checkmark}$ $\boxed{\text{c}}$: $\boxed{-1 \quad \checkmark}$ $\boxed{\text{d}}$: $\boxed{2 \quad \checkmark}$ $\boxed{\text{e}}$: $\boxed{0 \quad \checkmark}$ $\boxed{\text{f}}$: $\boxed{1 \quad \checkmark}$ $\boxed{\text{g}}$: $\boxed{24 \quad \checkmark}$

$$x^2\sqrt{1+3x} = \boxed{\text{h}} + \boxed{\text{i}}x + \boxed{\text{j}}x^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}x^3 + \frac{\boxed{\text{m}}}{\boxed{\text{n}}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{\text{h}}$: $\boxed{0 \quad \checkmark}$ $\boxed{\text{i}}$: $\boxed{0 \quad \checkmark}$ $\boxed{\text{j}}$: $\boxed{1 \quad \checkmark}$ $\boxed{\text{k}}$: $\boxed{3 \quad \checkmark}$ $\boxed{\text{l}}$: $\boxed{2 \quad \checkmark}$ $\boxed{\text{m}}$: $\boxed{-9 \quad \checkmark}$ $\boxed{\text{n}}$: $\boxed{8 \quad \checkmark}$

$$\log(1+\sin^4 x) = \boxed{\text{o}} + \boxed{\text{p}}x + \boxed{\text{q}}x^2 + \boxed{\text{r}}x^3 + \boxed{\text{s}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{\text{o}}$: $\boxed{0 \quad \checkmark}$ $\boxed{\text{p}}$: $\boxed{0 \quad \checkmark}$ $\boxed{\text{q}}$: $\boxed{0 \quad \checkmark}$ $\boxed{\text{r}}$: $\boxed{0 \quad \checkmark}$ $\boxed{\text{s}}$: $\boxed{1 \quad \checkmark}$

For various $\alpha \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{2\cos x - 2 + x^2\sqrt{1+3x} + \alpha x^3}{\log(1+\sin^4 x)}.$$

This limit converges for $\alpha = \frac{\boxed{\text{t}}}{\boxed{\text{u}}}$.

$\boxed{\text{t}}$: $\boxed{-3 \quad \checkmark}$ $\boxed{\text{u}}$: $\boxed{2 \quad \checkmark}$

In that case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

$\boxed{\text{v}}$: $\boxed{-25 \quad \checkmark}$ $\boxed{\text{w}}$: $\boxed{24 \quad \checkmark}$

> Use the Taylor formula $f(x) = f(0)+f'(0)x+\frac{1}{2!}f''(0)x^2+\frac{1}{3!}f^{(3)}(0)x^4 + \frac{1}{4!}f^{(4)}(0)x^4 + o(x^4)$ as $x \to 0$. To determine α, one only has to compare the numereator and the denominator and choose α in such a way that they have the same degree of infinitesimal (in this case, both of them should be of order x^4).

(2) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$\cos x = \boxed{a} + \boxed{b}x + \frac{\boxed{c}}{\boxed{d}}x^2 + \boxed{e}x^3 + \frac{\boxed{f}}{\boxed{g}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{a}$: $\boxed{1 \checkmark}$ $\boxed{b}$: $\boxed{0 \checkmark}$ $\boxed{c}$: $\boxed{-1 \checkmark}$ $\boxed{d}$: $\boxed{2 \checkmark}$ $\boxed{e}$: $\boxed{0 \checkmark}$ $\boxed{f}$: $\boxed{1 \checkmark}$ $\boxed{g}$: $\boxed{24 \checkmark}$

$$x^2\sqrt{1-x} = \boxed{h} + \boxed{i}x + \boxed{j}x^2 + \frac{\boxed{k}}{\boxed{l}}x^3 + \frac{\boxed{m}}{\boxed{n}}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{h}$: $\boxed{0 \checkmark}$ $\boxed{i}$: $\boxed{0 \checkmark}$ $\boxed{j}$: $\boxed{1 \checkmark}$ $\boxed{k}$: $\boxed{-1 \checkmark}$ $\boxed{l}$: $\boxed{2 \checkmark}$ $\boxed{m}$: $\boxed{-1 \checkmark}$ $\boxed{n}$: $\boxed{8 \checkmark}$

$$\log(1+\sin^4 x) = \boxed{o} + \boxed{p}x + \boxed{q}x^2 + \boxed{r}x^3 + \boxed{s}x^4 + o(x^4) \text{ as } x \to 0.$$

$\boxed{o}$: $\boxed{0 \checkmark}$ $\boxed{p}$: $\boxed{0 \checkmark}$ $\boxed{q}$: $\boxed{0 \checkmark}$ $\boxed{r}$: $\boxed{0 \checkmark}$ $\boxed{s}$: $\boxed{1 \checkmark}$

For various $\alpha \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{2\cos x - 2 + x^2\sqrt{1-x} + \alpha x^3}{\log(1+\sin^4 x)}.$$

This limit converges for $\alpha = \frac{\boxed{t}}{\boxed{u}}$.

$\boxed{t}$: $\boxed{1 \checkmark}$ $\boxed{u}$: $\boxed{2 \checkmark}$

In that case, the limit is $\frac{\boxed{\text{v}}}{\boxed{\text{w}}}$.

$\boxed{\text{v}}$: $\boxed{-1 \quad \checkmark}$ $\boxed{\text{w}}$: $\boxed{24 \quad \checkmark}$

(3) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{n(n+1)}{2(n^2+2^n)}(x+1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{n(n+1)}{2(n^2+2^n)}(x+1)^n = \frac{\boxed{\text{a}}}{\boxed{\text{b}}} + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}i$.

$\boxed{\text{a}}$: $\boxed{1 \quad \checkmark}$ $\boxed{\text{b}}$: $\boxed{3 \quad \checkmark}$ $\boxed{\text{c}}$: $\boxed{13 \quad \checkmark}$ $\boxed{\text{d}}$: $\boxed{12 \quad \checkmark}$

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{n(n+1)}{2(n^2+2^n)}|x+1|^n$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{\text{e}}}{\boxed{\text{f}}}|x + \boxed{\text{g}}|^{\boxed{\text{h}}}$$

$\boxed{\text{e}}$: $\boxed{1 \quad \checkmark}$ $\boxed{\text{f}}$: $\boxed{2 \quad \checkmark}$ $\boxed{\text{g}}$: $\boxed{1 \quad \checkmark}$ $\boxed{\text{h}}$: $\boxed{1 \quad \checkmark}$

Therefore, by the ratio test, the series converges absolutely for

- all x.
- $-\frac{1}{2} < x < \frac{1}{2}$.
- $-1 < x < 1$.
- $-2 < x < 2$.
- $x = 0$.
- $-\frac{3}{2} < x < \frac{1}{2}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $-3 < x < -1$.
- $-3 < x < 1$. ✓
- $-1 < x < 3$.
- $1 < x < 3$.

For the case $x = \frac{3}{2}$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = -3$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

> The partial sum means the following finite sum: $\sum_{n=0}^{2} a_n = a_0 + a_1 + a_2$, so one just has to apply $n = 0, 1, 2$ in the concrete series and sum the numbers up. Notice that $i^2 = -1$.
> To apply the ratio test, one considers $R = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$. Note that for any $p \in \mathbb{N}$ it holds that $\lim_{n\to\infty} \frac{n^p}{2^n} = 0$ etc. If this limit $R < 1$, then the series converges absolutely (for such x), while if $R > 1$ the series diverges.
> If $R = 1$, one needs to study the convergence with other criteria. In this case, if $x = -3$, then $a_n = \frac{n(n+1)2^n}{2(n^2+2^n)}$, and this is diverging, in particular not converging to 0. Therefore, the series is divergent.

(4) **Q2**
If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\mathrm{a}}}{\boxed{\mathrm{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{n(n+1)}{2(n^2+2^n)}(x-1)^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = -i$, calculate the partial sum $\sum_{n=0}^{2} \frac{n(n+1)}{2(n^2+2^n)}(x-1)^n = \frac{\boxed{\mathrm{a}}}{\boxed{\mathrm{b}}} + \frac{\boxed{\mathrm{c}}}{\boxed{\mathrm{d}}} i$.

$\boxed{\mathrm{a}}$: $\boxed{-1 \checkmark}$ $\boxed{\mathrm{b}}$: $\boxed{3 \checkmark}$ $\boxed{\mathrm{c}}$: $\boxed{5 \checkmark}$ $\boxed{\mathrm{d}}$: $\boxed{12 \checkmark}$

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{n(n+1)}{2(n^2+2^n)}|x-1|^n$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{\mathrm{e}}}{\boxed{\mathrm{f}}} |x + \boxed{\mathrm{g}}|^{\boxed{\mathrm{h}}}$$

$\boxed{\mathrm{e}}$: $\boxed{1 \checkmark}$ $\boxed{\mathrm{f}}$: $\boxed{2 \checkmark}$ $\boxed{\mathrm{g}}$: $\boxed{-1 \checkmark}$ $\boxed{\mathrm{h}}$: $\boxed{1 \checkmark}$

Therefore, by the ratio test, the series converges absolutely for

- all x.
- $-\frac{1}{2} < x < \frac{1}{2}$.
- $-1 < x < 1$.
- $-2 < x < 2$.
- $x = 0$.
- $-\frac{3}{2} < x < \frac{1}{2}$.
- $-\frac{3}{2} < x < -\frac{1}{2}$.
- $-3 < x < -1$.
- $-3 < x < 1$.
- $-1 < x < 3$. ✓
- $1 < x < 3$.

For the case $x = \frac{3}{2}$, the series
- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = -3$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(5) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us consider the following function

$$f(x) = \exp\left(\frac{x^2}{x^2 - 1}\right).$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.
- $-e$
- -2
- -1 ✓
- 0
- 1 ✓
- 2
- e

Choose all asymptotes of $f(x)$.
- $y = -e$
- $y = -1$
- $y = 0$
- $y = 1$
- $y = e$ ✓
- $x = -2$
- $x = -\sqrt{3}$
- $x = -\sqrt{2}$
- $x = -1$ ✓
- $x = 0$
- $x = 1$ ✓
- $x = \sqrt{2}$
- $x = \sqrt{3}$
- $x = 2$
- $y = x$
- $y = -x$

One has

$$f'(2) = \exp\left(\frac{\boxed{\text{a}}}{\boxed{\text{b}}}\right)\frac{\boxed{\text{c}}}{\boxed{\text{d}}}.$$

$\boxed{\text{a}}$: $\boxed{4\ \checkmark}$ $\boxed{\text{b}}$: $\boxed{3\ \checkmark}$ $\boxed{\text{c}}$: $\boxed{-4\ \checkmark}$ $\boxed{\text{d}}$: $\boxed{9\ \checkmark}$

The function $f(x)$ has one stationary point: $x = \boxed{\text{e}}$.

$\boxed{\text{e}}$: $\boxed{0\ \checkmark}$

Choose the behaviour of $f(x)$ in the interval $(3, 5)$.
- monotonically decreasing ✓
- monotonically increasing
- neither decreasing nor increasing

To determine the natural domain of a function, it is enough to observe the components. For example, $\exp y$ is defined for all $y \in \mathbb{R}$, $\frac{1}{y-a}$ is defined only for $y \neq a$, etc. It is enought to exclude all such points where the composed function is not defined. In this case, $x^2 - 1 =\neq 0$.
There can be asymptotes for $x \to \pm\infty$, and for $x \to a$, where a is a boundary of the domain. In this case, one should check $x \to \pm 1, \pm\infty$. All of them are asymptotes.
For the derivative, the chain rule $(f(g(x)))' = g'(x)f'(g(x))$ is useful. In this case, $f(x) = \exp(\frac{x^2}{x^2-1})$, $f'(x) = \exp(\frac{x}{x^2-1})\frac{2x(x^2-1)-x^2\cdot 2x}{(x^2-1)^2} = \exp(\frac{x}{x^2-1})\frac{-2x}{(x^2-1)^2}$.
If $f'(x_0) = 0$, x_0 is called a stationary point. In this case, $x_0 = 0$ is the only possibility.
If $f'(x) \geq 0$ (≤ 0) in one interval, then $f(x)$ is monotonically increasing (decreasing) there. If $x \in [3, 5]$, $f'(x)$ is negative.

(6) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us consider the following function

$$f(x) = \exp\left(\frac{(x-1)^2}{x^2-2x}\right).$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- $-e$
- -2
- -1
- 0 ✓
- 1
- 2 ✓
- e

Choose all asymptotes of $f(x)$.

- $y = -e$
- $y = -1$
- $y = 0$
- $y = 1$
- $y = e$ ✓
- $x = -2$
- $x = -\sqrt{3}$
- $x = -\sqrt{2}$
- $x = -1$
- $x = 0$ ✓
- $x = 1$
- $x = \sqrt{2}$
- $x = \sqrt{3}$
- $x = 2$ ✓
- $y = x$
- $y = -x$

One has

$$f'(3) = \exp\left(\frac{\boxed{\text{a}}}{\boxed{\text{b}}}\right)\frac{\boxed{\text{c}}}{\boxed{\text{d}}}.$$

$\boxed{\text{a}}$: $\boxed{4\ ✓}$ $\boxed{\text{b}}$: $\boxed{3\ ✓}$ $\boxed{\text{c}}$: $\boxed{-4\ ✓}$ $\boxed{\text{d}}$: $\boxed{9\ ✓}$

The function $f(x)$ has one stationary point: $x = \boxed{\text{e}}$.

$\boxed{\text{e}}$: $\boxed{1\ ✓}$

Choose the behaviour of $f(x)$ in the interval $(-5, -3)$.

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(7) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us calculate the following integral.

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{x}{x^3 + x^2 + 4x + 4}dx.$$

Complete the formula

$$\frac{x}{x^3+x^2+4x+4} = \frac{\frac{\boxed{a}}{\boxed{b}}x + \frac{\boxed{c}}{\boxed{d}}}{x^2+\boxed{e}} + \frac{\frac{\boxed{f}}{\boxed{g}}}{x+\boxed{h}}.$$

$\boxed{a}$: $\boxed{1 \ \checkmark}$ $\boxed{b}$: $\boxed{5 \ \checkmark}$ $\boxed{c}$: $\boxed{4 \ \checkmark}$ $\boxed{d}$: $\boxed{5 \ \checkmark}$ $\boxed{e}$: $\boxed{4 \ \checkmark}$ $\boxed{f}$: $\boxed{-1 \ \checkmark}$ $\boxed{g}$: $\boxed{5 \ \checkmark}$ $\boxed{h}$: $\boxed{1 \ \checkmark}$

Choose a primitive of $\frac{x}{x^2+4}$.

- $\arctan(x+2)$
- $\frac{x}{2}\arctan(2x)$
- $x\arctan(x^2+4)$
- $\frac{1}{4}\log(x^2+4)$
- $\frac{1}{2}\log(x^2+4)$ ✓
- $\log(x(x^2+4))$
- $\frac{1}{4}\arcsin(x^2+4)$
- $\frac{1}{2}\arcsin(x^2+4)$
- $\arcsin(x(x^2+4))$

Choose a primitive of $\frac{1}{x^2+4}$.

- $\arctan(2x)$
- $\arctan(\frac{x}{2})$
- $\frac{1}{2}\arctan(\frac{x}{2})$ ✓
- $\frac{1}{4}\arctan(4x)$
- $\arcsin(2x)$
- $\arcsin(\frac{x}{2})$
- $\frac{1}{2}\arcsin(\frac{x}{2})$
- $\frac{1}{4}\arcsin(4x)$

By continuing, we get

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{x}{x^3+x^2+4x+4} = \frac{\boxed{i}}{\boxed{j}}\arctan\left(\frac{\boxed{k}}{\boxed{l}}\right) + \frac{\boxed{m}}{\boxed{n}}\log 3$$

.

$\boxed{i}$: $\boxed{4 \ \checkmark}$ $\boxed{j}$: $\boxed{5 \ \checkmark}$ $\boxed{k}$: $\boxed{1 \ \checkmark}$ $\boxed{l}$: $\boxed{4 \ \checkmark}$ $\boxed{m}$: $\boxed{-1 \ \checkmark}$ $\boxed{n}$: $\boxed{5 \ \checkmark}$

The partial fractions of $\frac{x}{x^3+x^2+4x+4}$ can be found first by factorizing the denominator $x^3+x^2+4x+4=(x+1)(x^2+4)$ and then by putting $\frac{x}{x^3+x^2+4x+4}=\frac{A}{x+1}+\frac{Bx+C}{x^2+4}$.
The indefinite integral of $\frac{x}{x^2+4}$ can be found by the substitution $u=x^2+4$, while one can use $\int\frac{1}{x^2+4}dx=\frac{1}{2}\int\frac{1}{(\frac{x}{2})^2+4}\frac{dx}{2}$ and the substitution $v=\frac{x}{2}$ and $\int\frac{1}{x^2+1}=\arctan x$.

(8) **Q4**
If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us calculate the following integral.

$$\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{2x}{x^3+x^2+4x+4}dx.$$

Complete the formula

$$\frac{2x}{x^3+x^2+4x+4}=\frac{\frac{\boxed{\text{a}}}{\boxed{\text{b}}}x+\frac{\boxed{\text{c}}}{\boxed{\text{d}}}}{x^2+\boxed{\text{e}}}+\frac{\frac{\boxed{\text{f}}}{\boxed{\text{g}}}}{x+\boxed{\text{h}}}.$$

$\boxed{\text{a}}$: $\boxed{2\ \checkmark}$ $\boxed{\text{b}}$: $\boxed{5\ \checkmark}$ $\boxed{\text{c}}$: $\boxed{8\ \checkmark}$ $\boxed{\text{d}}$: $\boxed{5\ \checkmark}$ $\boxed{\text{e}}$: $\boxed{4\ \checkmark}$ $\boxed{\text{f}}$: $\boxed{-2\ \checkmark}$ $\boxed{\text{g}}$: $\boxed{5\ \checkmark}$ $\boxed{\text{h}}$: $\boxed{1\ \checkmark}$

Choose a primitive of $\frac{x}{x^2+4}$.

- $\arctan(x+2)$
- $\frac{x}{2}\arctan(2x)$
- $x\arctan(x^2+4)$
- $\frac{1}{4}\log(x^2+4)$
- $\frac{1}{2}\log(x^2+4)$ ✓
- $\log(x(x^2+4))$
- $\frac{1}{4}\arcsin(x^2+4)$
- $\frac{1}{2}\arcsin(x^2+4)$
- $\arcsin(x(x^2+4))$

Choose a primitive of $\frac{1}{x^2+4}$.

- $\arctan(2x)$

- $\arctan(\frac{x}{2})$
- $\frac{1}{2}\arctan(\frac{x}{2})$ ✓
- $\frac{1}{4}\arctan(4x)$
- $\arcsin(2x)$
- $\arcsin(\frac{x}{2})$
- $\frac{1}{2}\arcsin(\frac{x}{2})$
- $\frac{1}{4}\arcsin(4x)$

By continuing, we get

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{2x}{x^3+x^2+4x+4} = \frac{\boxed{\text{i}}}{\boxed{\text{j}}}\arctan\left(\frac{\boxed{\text{k}}}{\boxed{\text{l}}}\right) + \frac{\boxed{\text{m}}}{\boxed{\text{n}}}\log 3$$

.

$\boxed{\text{i}}$: $\boxed{8 \ \checkmark}$ $\boxed{\text{j}}$: $\boxed{5 \ \checkmark}$ $\boxed{\text{k}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{l}}$: $\boxed{4 \ \checkmark}$ $\boxed{\text{m}}$: $\boxed{-2 \ \checkmark}$ $\boxed{\text{n}}$: $\boxed{5 \ \checkmark}$

(9) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Choose the general solution of the following differential equation.

$$y'(x) = 2x\cos(x^2)e^y$$

.

- $y(x) = \log(\sin x^2 + C)$
- $y(x) = \log(x\sin(x^2)) + C$
- $y(x) = \log(Cx\sin(x^2))$
- $y(x) = -\log(-\sin(x^2) + C)$ ✓
- $y(x) = -\log(-x\sin(x^2)) + C$
- $y(x) = -\log(-Cx\sin(x^2))$
- $y(x) = \log(\cos(x^2) + C)$
- $y(x) = \log(x\cos(x^2)) + C$
- $y(x) = \log(Cx\cos(x^2))$
- $y(x) = -\log(\cos(x^2) + C)$
- $y(x) = -\log(x\cos(x^2)) + C$
- $y(x) = -\log(Cx\cos(x^2))$

Determine $C = \boxed{\text{a}}$ with the initial condition $y(0) = \log(0.2)$

$\boxed{\text{a}}$: $\boxed{5 \ \checkmark}$

Choose the general solution of the following differential equation.

$$y''(x) + 4y'(x) + 3y(x) = 0$$

.

- $y(x) = C_1 \sin(-3x) + C_2 \cos(-x)$
- $y(x) = C_1 \cos(-3x) + C_2 \sin(-x)$
- $y(x) = C_1 \exp(-3x) + C_2 \exp(-x)$ ✓
- $y(x) = C_1 \sin(3x) + C_2 \cos(x)$
- $y(x) = C_1 \cos(3x) + C_2 \sin(x)$
- $y(x) = C_1 \exp(3x) + C_2 \exp(x)$
- $y(x) = C_1 \exp(3x) + C_2 \exp(x) \cos(x)$
- $y(x) = C_1 \exp(-3x) + C_2 \exp(-x) \cos(x)$
- $y(x) = C_1 \exp(3x) \sin(x) + C_2 \cos(-x)$

Let us consider the following six cases and choose the one where $y(10)$ is the largest among the six:

- $C_1 = 5, C_2 = 0$
- $C_1 = 4, C_2 = 1$
- $C_1 = 3, C_2 = 2$
- $C_1 = 2, C_2 = 3$
- $C_1 = 1, C_2 = 4$
- $C_1 = 0, C_2 = 5$ ✓

The equation $y'(x) = 2x\cos(x^2)e^y$ is separable, hence one obtains the relation $-e^{-y} = \int e^{-y}dy = \int 2x\cos(x^2)dx + C = \sin(x^2) + C$, or $e^{-y} = -\sin(x^2) + C$, or $y = -\log(-\sin(x^2) + C)$.

The second-order differential equation $y'' + ay' + by = 0$ can be solved as follows: put $z^2 + az + b = 0$, and solve this equation. If this has two real solutions z_1, z_2, then the general solution is $y = C_1 e^{z_1 x} + C_2 e^{z_2 x}$. If it has two complex solutions $z_1 \pm iz_2$, then $y = C_1 e^{z_1 x}\sin(z_2 x) + C_2 e^{z_1 x}\cos(z_2 x)$.

The constant can be obtaind by substituting the initial condition.

The function $y(x) = C_1 \exp(-3x) + C_2 \exp(-x)$ is a sum of two exponential functions, and $\exp(-3x)$ decreases much faster than $\exp(-x)$. One can verify, using $e \sim 2.7$, that at $x = 10$ the first term is much smaller than the second, and hence the value is the largest if C_2 is the largest.

(10) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Choose the general solution of the following differential equation.

$$y'(x) = 2x\sin(x^2)e^y$$

.

- $y(x) = \log(-\sin x^2 + C)$
- $y(x) = \log(-x\sin(x^2)) + C$
- $y(x) = \log(-Cx\sin(x^2))$
- $y(x) = -\log(-\sin(x^2) + C)$
- $y(x) = -\log(-x\sin(x^2)) + C$
- $y(x) = -\log(-Cx\sin(x^2))$
- $y(x) = \log(\cos(x^2) + C)$
- $y(x) = \log(x\cos(x^2)) + C$
- $y(x) = \log(Cx\cos(x^2))$
- $y(x) = -\log(\cos(x^2) + C)$ ✓
- $y(x) = -\log(x\cos(x^2)) + C$
- $y(x) = -\log(Cx\cos(x^2))$

Determine $C = \boxed{\text{a}}$ with the initial condition $y(0) = \log(0.2)$

$\boxed{\text{a}}$: $\boxed{4 \quad ✓}$

Choose the general solution of the following differential equation.

$$y''(x) - 4y'(x) + 3y(x) = 0$$

.

- $y(x) = C_1\sin(-3x) + C_2\cos(-x)$
- $y(x) = C_1\cos(-3x) + C_2\sin(-x)$
- $y(x) = C_1\exp(-3x) + C_2\exp(-x)$
- $y(x) = C_1\sin(3x) + C_2\cos(x)$
- $y(x) = C_1\cos(3x) + C_2\sin(x)$
- $y(x) = C_1\exp(3x) + C_2\exp(x)$ ✓
- $y(x) = C_1\exp(3x) + C_2\exp(x)\cos(x)$
- $y(x) = C_1\exp(-3x) + C_2\exp(-x)\cos(x)$
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- $C_1 = 1, C_2 = 4$
- $C_1 = 0, C_2 = 5$