Call2.

(1) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$e^x = \boxed{a} + \boxed{b}x + \frac{\boxed{c}}{\boxed{d}}x^2 + \frac{\boxed{e}}{\boxed{f}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{a}$: $\boxed{1 \ \checkmark}$ $\boxed{b}$: $\boxed{1 \ \checkmark}$ $\boxed{c}$: $\boxed{1 \ \checkmark}$ $\boxed{d}$: $\boxed{2 \ \checkmark}$ $\boxed{e}$: $\boxed{1 \ \checkmark}$ $\boxed{f}$: $\boxed{6 \ \checkmark}$

$$x\sqrt{1+x} = \boxed{g} + \boxed{h}x + \frac{\boxed{i}}{\boxed{j}}x^2 + \frac{\boxed{k}}{\boxed{l}}x^3 \text{ as } x \to 0.$$

$\boxed{g}$: $\boxed{0 \ \checkmark}$ $\boxed{h}$: $\boxed{1 \ \checkmark}$ $\boxed{i}$: $\boxed{1 \ \checkmark}$ $\boxed{j}$: $\boxed{2 \ \checkmark}$ $\boxed{k}$: $\boxed{-1 \ \checkmark}$ $\boxed{l}$: $\boxed{8 \ \checkmark}$

$$x\sin(x^2) = \boxed{m} + \boxed{n}x + \boxed{o}x^2 + \boxed{p}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{m}$: $\boxed{0 \ \checkmark}$ $\boxed{n}$: $\boxed{0 \ \checkmark}$ $\boxed{o}$: $\boxed{0 \ \checkmark}$ $\boxed{p}$: $\boxed{1 \ \checkmark}$

For various $\alpha \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{e^x - x\sqrt{1+x} - 1 + \alpha x^2}{x\sin(x^2)}.$$

This limit converges for $\alpha = \boxed{q}$.

$\boxed{q}$: $\boxed{0 \ \checkmark}$

In that case, the limit is $\frac{\boxed{r}}{\boxed{s}}$.

$\boxed{r}$: $\boxed{7 \ \checkmark}$ $\boxed{s}$: $\boxed{24 \ \checkmark}$

Use the Taylor formula $f(x) = f(0)+f'(0)x+\frac{1}{2!}f''(0)x^2+\frac{1}{3!}f^{(3)}(0)x^3 + o(x^3)$ as $x \to 0$. For a product $f(x)g(x)$, the Taylor formula can be obtained by multiplying the corresponding expansions and pick the order smaller than or equal to x^3. Composed functions such as $g(x^2)$ can be expanded by substituting $y = x^2$ in the Taylor formula for $g(y)$.
To determine α, one only has to compare the numereator and the denominator and choose α in such a way that they have the same degree of infinitesimal (in this case, both of them should be of order x^2).

(2) **Q1**
If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$e^{-x} = \boxed{a} + \boxed{b}x + \frac{\boxed{c}}{\boxed{d}}x^2 + \frac{\boxed{e}}{\boxed{f}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{a}$: $\boxed{1 \ \checkmark}$ $\boxed{b}$: $\boxed{-1 \ \checkmark}$ $\boxed{c}$: $\boxed{1 \ \checkmark}$ $\boxed{d}$: $\boxed{2 \ \checkmark}$ $\boxed{e}$: $\boxed{-1 \ \checkmark}$ $\boxed{f}$: $\boxed{6 \ \checkmark}$

$$x\sqrt{1-x} = \boxed{g} + \boxed{h}x + \frac{\boxed{i}}{\boxed{j}}x^2 + \frac{\boxed{k}}{\boxed{l}}x^3 \text{ as } x \to 0.$$

$\boxed{g}$: $\boxed{0 \ \checkmark}$ $\boxed{h}$: $\boxed{1 \ \checkmark}$ $\boxed{i}$: $\boxed{-1 \ \checkmark}$ $\boxed{j}$: $\boxed{2 \ \checkmark}$ $\boxed{k}$: $\boxed{-1 \ \checkmark}$ $\boxed{l}$: $\boxed{8 \ \checkmark}$

$$x\sin(x^2) = \boxed{m} + \boxed{n}x + \boxed{o}x^2 + \boxed{p}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{m}$: $\boxed{0 \ \checkmark}$ $\boxed{n}$: $\boxed{0 \ \checkmark}$ $\boxed{o}$: $\boxed{0 \ \checkmark}$ $\boxed{p}$: $\boxed{1 \ \checkmark}$

For various $\alpha \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{e^{-x} + x\sqrt{1-x} - 1 + \alpha x^2}{x\sin(x^2)}.$$

This limit converges for $\alpha = \boxed{\text{q}}$.

$\boxed{\text{q}}$: $\boxed{0\ \checkmark}$

In that case, the limit is $\dfrac{\boxed{\text{r}}}{\boxed{\text{s}}}$.

$\boxed{\text{r}}$: $\boxed{-7\ \checkmark}$ $\boxed{\text{s}}$: $\boxed{24\ \checkmark}$

(3) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\dfrac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$e^x = \boxed{\text{a}} + \boxed{\text{b}}x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}x^2 + \frac{\boxed{\text{e}}}{\boxed{\text{f}}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{\text{a}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{b}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{c}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{d}}$: $\boxed{2\ \checkmark}$ $\boxed{\text{e}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{f}}$: $\boxed{6\ \checkmark}$

$$x\sqrt{1+x} = \boxed{\text{g}} + \boxed{\text{h}}x + \frac{\boxed{\text{i}}}{\boxed{\text{j}}}x^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}x^3 \text{ as } x \to 0.$$

$\boxed{\text{g}}$: $\boxed{0\ \checkmark}$ $\boxed{\text{h}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{i}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{j}}$: $\boxed{2\ \checkmark}$ $\boxed{\text{k}}$: $\boxed{-1\ \checkmark}$ $\boxed{\text{l}}$: $\boxed{8\ \checkmark}$

$$x\sin(2x^2) = \boxed{\text{m}} + \boxed{\text{n}}x + \boxed{\text{o}}x^2 + \boxed{\text{p}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{\text{m}}$: $\boxed{0\ \checkmark}$ $\boxed{\text{n}}$: $\boxed{0\ \checkmark}$ $\boxed{\text{o}}$: $\boxed{0\ \checkmark}$ $\boxed{\text{p}}$: $\boxed{2\ \checkmark}$

For various $\alpha \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{e^x - x\sqrt{1+x} + \alpha}{x\sin(2x^2)}.$$

This limit converges for $\alpha = \boxed{\text{q}}$.

$\boxed{\text{q}}$: $\boxed{-1\ \checkmark}$

In that case, the limit is $\dfrac{\boxed{\text{r}}}{\boxed{\text{s}}}$.

$\boxed{\text{r}}$: $\boxed{7\ \checkmark}$ $\boxed{\text{s}}$: $\boxed{48\ \checkmark}$

(4) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$e^{-x} = \boxed{\text{a}} + \boxed{\text{b}}x + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}x^2 + \frac{\boxed{\text{e}}}{\boxed{\text{f}}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{\text{a}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{-1 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{d}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{e}}$: $\boxed{-1 \ \checkmark}$ $\boxed{\text{f}}$: $\boxed{6 \ \checkmark}$

$$x\sqrt{1-x} = \boxed{\text{g}} + \boxed{\text{h}}x + \frac{\boxed{\text{i}}}{\boxed{\text{j}}}x^2 + \frac{\boxed{\text{k}}}{\boxed{\text{l}}}x^3 \text{ as } x \to 0.$$

$\boxed{\text{g}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{h}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{i}}$: $\boxed{-1 \ \checkmark}$ $\boxed{\text{j}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{k}}$: $\boxed{1 \ \checkmark}$ $\boxed{\text{l}}$: $\boxed{8 \ \checkmark}$

$$x\sin(2x^2) = \boxed{\text{m}} + \boxed{\text{n}}x + \boxed{\text{o}}x^2 + \boxed{\text{p}}x^3 + o(x^3) \text{ as } x \to 0.$$

$\boxed{\text{m}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{n}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{o}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{p}}$: $\boxed{2 \ \checkmark}$

For various $\alpha \in \mathbb{R}$, study the limit:

$$\lim_{x\to 0} \frac{e^{-x} + x\sqrt{1-x} + \alpha}{x\sin(2x^2)}.$$

This limit converges for $\alpha = \boxed{\text{q}}$.

$\boxed{\text{q}}$: $\boxed{-1 \ \checkmark}$

In that case, the limit is $\frac{\boxed{\text{r}}}{\boxed{\text{s}}}$.

$\boxed{\text{r}}$: $\boxed{-7 \ \checkmark}$ $\boxed{\text{s}}$: $\boxed{48 \ \checkmark}$

(5) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{2^n+1}{3^n+2} x^{2x}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$, calculate the partial sum $\sum_{n=0}^{2} \frac{2^n+1}{3^n+2} x^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} i$.

$\boxed{a}$: $\boxed{7 \checkmark}$ $\boxed{b}$: $\boxed{33 \checkmark}$ $\boxed{c}$: $\boxed{3 \checkmark}$ $\boxed{d}$: $\boxed{5 \checkmark}$

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{2^n+1}{3^n+2} x^{2n}$. Complete the formula.

$$\lim_{n \to \infty} (a_n)^{\frac{1}{n}} = \frac{\boxed{e}}{\boxed{f}} |x|^{\boxed{g}}$$

$\boxed{e}$: $\boxed{2 \checkmark}$ $\boxed{f}$: $\boxed{3 \checkmark}$ $\boxed{g}$: $\boxed{2 \checkmark}$

Therefore, by the root test, the series converges absolutely for

- all x.
- $-\sqrt{3} < x < \sqrt{3}$.
- $-2 < x < 2$.
- $-\frac{3}{2} < x < \frac{3}{2}$.
- $-(\frac{3}{2})^{\frac{1}{2}} < x < (\frac{3}{2})^{\frac{1}{2}}$. ✓
- $-(\frac{2}{3})^{\frac{1}{2}} < x < (\frac{2}{3})^{\frac{1}{2}}$.
- $-\frac{2}{3} < x < \frac{2}{3}$.
- $x = 0$.
- none of x.

For the case $x = -2$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = -(\frac{3}{2})^{\frac{1}{2}}$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

> The partial sum means the following finite sum: $\sum_{n=0}^{2} a_n = a_0+a_1+a_2$, so one just has to apply $n = 0, 1, 2$ in the concrete series and sum the numbers up. Notice that $i^2 = -1$.
> To apply the root test, one considers $R = \lim_{n\to\infty} |a_n|^{\frac{1}{n}}$. Note that $(n^2+1)^{\frac{1}{n}} \to 2$ etc.
> If this limit $R < 1$, then the series converges absolutely (for such x), while if $R > 1$ the series diverges.
> If $R = 1$, one needs to study the convergence with other criteria. In this case, if $x = -(\frac{3}{2})^{\frac{1}{2}}$, then $a_n = \frac{2^n+1}{3^n+2}(\frac{3}{2})^n \to 1$, and a_n is not convergent to 0. Therefore, the series is divergent.

(6) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{2^n+1}{3^n+2} x^{2x}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$, calculate the partial sum $\sum_{n=0}^{2} \frac{2^n+1}{3^n+2} x^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} i$.

$\boxed{a}$: $\boxed{7\ \checkmark}$ $\boxed{b}$: $\boxed{33\ \checkmark}$ $\boxed{c}$: $\boxed{-3\ \checkmark}$ $\boxed{d}$: $\boxed{5\ \checkmark}$

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{2^n+1}{3^n+2} x^{2n}$. Complete the formula.

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \frac{\boxed{e}}{\boxed{f}} |x|^{\boxed{g}}$$

$\boxed{e}$: $\boxed{2\ \checkmark}$ $\boxed{f}$: $\boxed{3\ \checkmark}$ $\boxed{g}$: $\boxed{2\ \checkmark}$

Therefore, by the root test, the series converges absolutely for

- all x.
- $-\sqrt{3} < x < \sqrt{3}$.
- $-2 < x < 2$.
- $-\frac{3}{2} < x < \frac{3}{2}$.

- $-(\frac{3}{2})^{\frac{1}{2}} < x < (\frac{3}{2})^{\frac{1}{2}}$. ✓
- $-(\frac{2}{3})^{\frac{1}{2}} < x < (\frac{2}{3})^{\frac{1}{2}}$.
- $-\frac{2}{3} < x < \frac{2}{3}$.
- $x = 0$.
- none of x.

For the case $x = -1$, the series

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = -(\frac{3}{2})^{\frac{1}{2}}$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(7) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n+2}{2^n+1} x^{2x}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$, calculate the partial sum $\sum_{n=0}^{2} \frac{3^n+2}{2^n+1} x^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} i$.

$\boxed{a}$: $\boxed{-7 \ ✓}$ $\boxed{b}$: $\boxed{10 \ ✓}$ $\boxed{c}$: $\boxed{5 \ ✓}$ $\boxed{d}$: $\boxed{3 \ ✓}$

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n+2}{2^n+1} x^{2n}$. Complete the formula.

$$\lim_{n \to \infty} (a_n)^{\frac{1}{n}} = \frac{\boxed{e}}{\boxed{f}} |x|^{\boxed{g}}$$

$\boxed{e}$: $\boxed{3 \ ✓}$ $\boxed{f}$: $\boxed{2 \ ✓}$ $\boxed{g}$: $\boxed{2 \ ✓}$

Therefore, by the root test, the series converges absolutely for

- all x.
- $-\sqrt{3} < x < \sqrt{3}$.
- $-2 < x < 2$.
- $-\frac{3}{2} < x < \frac{3}{2}$.
- $-(\frac{3}{2})^{\frac{1}{2}} < x < (\frac{3}{2})^{\frac{1}{2}}$.

- $-(\frac{2}{3})^{\frac{1}{2}} < x < (\frac{2}{3})^{\frac{1}{2}}$. ✓
- $-\frac{2}{3} < x < \frac{2}{3}$.
- $x = 0$.
- none of x.

For the case $x = -2$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = -(\frac{2}{3})^{\frac{1}{2}}$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(8) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{3^n+2}{2^n+1} x^{2x}$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$, calculate the partial sum $\sum_{n=0}^{2} \frac{3^n+2}{2^n+1} x^{2n} = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}} i$.

$\boxed{a}$: $\boxed{-7 \ ✓}$ $\boxed{b}$: $\boxed{10 \ ✓}$ $\boxed{c}$: $\boxed{-5 \ ✓}$ $\boxed{d}$: $\boxed{3 \ ✓}$

In order to use the root test for $x \in \mathbb{R}$, we put $a_n = \frac{3^n+2}{2^n+1} x^{2n}$. Complete the formula.

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \frac{\boxed{e}}{\boxed{f}} |x|^{\boxed{g}}$$

$\boxed{e}$: $\boxed{3 \ ✓}$ $\boxed{f}$: $\boxed{2 \ ✓}$ $\boxed{g}$: $\boxed{2 \ ✓}$

Therefore, by the root test, the series converges absolutely for
- all x.
- $-\sqrt{3} < x < \sqrt{3}$.
- $-2 < x < 2$.
- $-\frac{3}{2} < x < \frac{3}{2}$.
- $-(\frac{3}{2})^{\frac{1}{2}} < x < (\frac{3}{2})^{\frac{1}{2}}$.
- $-(\frac{2}{3})^{\frac{1}{2}} < x < (\frac{2}{3})^{\frac{1}{2}}$. ✓

- $-\frac{2}{3} < x < \frac{2}{3}$.
- $x = 0$.
- none of x.

For the case $x = -1$, the series
- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = -(\frac{2}{3})^{\frac{1}{2}}$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(9) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us consider the following function

$$f(x) = \frac{e^{x^2}}{|x-1|-1}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.
- $-e$
- -2
- -1
- 0 ✓
- 1
- 2 ✓
- e

Choose all asymptotes of $f(x)$.
- $y = -e$
- $y = -2$
- $y = 0$
- $y = 2$
- $y = e$
- $x = -2$
- $x = -\sqrt{2}$

- $x = -1$
- $x = 0$ ✓
- $x = 1$
- $x = \sqrt{2}$
- $x = 2$ ✓
- $y = x$
- $y = -x$

The function $f(x)$ has three stationary points: $x = \pm\sqrt{\frac{\boxed{\text{c}}}{\boxed{\text{d}}}}, \boxed{\text{e}} + \sqrt{\frac{\boxed{\text{f}}}{\boxed{\text{g}}}}$.

$\boxed{\text{c}}$: $\boxed{1\ ✓}$ $\boxed{\text{d}}$: $\boxed{2\ ✓}$ $\boxed{\text{e}}$: $\boxed{1\ ✓}$ $\boxed{\text{f}}$: $\boxed{3\ ✓}$ $\boxed{\text{g}}$: $\boxed{2\ ✓}$

The $f(x)$ is not differentiable at $x = 1$. The point $x = 1$ is

- a local minimum
- a local maximum
- neither a local minimum nor a local maximum ✓

Choose the behaviour of $f(x)$ in the interval $(2, 4)$.

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

To determine the natural domain of a function, it is enough to observe the components. For example, $\log y$ is defined only for $y > 0$, $\frac{1}{y-a}$ is defined only for $y \neq a$, etc. It is enought to exclude all such points where the composed function is not defined. In this case, $|x-1|-1 \neq 0$, hence $x \neq 0, 2$.
There can be asymptotes for $x \to \pm\infty$, and for $x \to a$, where a is a boundary of the domain. In this case, one should check $x \to 0, 2$. All of them are asymptotes. On the other hand, as $x \to \pm\infty$, the function diverges because $e^{x^2} > e^{|}x|$ and the exponential function grows much faster than any polynomial.
To find stationary points, we need to compute the derivative and solve $f'(x) = 0$. In this case, we need to split the cases into $x - 1 > 0$ or $x - 1 < 0$. Respectively, we have $f'(x) = \frac{e^{x^2}(2x(x-2)-1)}{(x-1)^2}$ and $f'(x) = \frac{e^{x^2}(2x^2-1)}{x^2}$. From each of the equations $f'(x) = 0$ we obtain two solutions, but they must satisfy $x - 1 > 0, x - 1 < 0$ respectively.
If $f'(x) \geq 0$ (≤ 0) in one interval, then $f(x)$ is monotonically increasing (decreasing) there.

(10) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us consider the following function

$$f(x) = \frac{e^{(x-1)^2}}{|x-2|-1}.$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- $-e$
- -3
- -2
- -1

- 0
- 1 ✓
- 2
- 3 ✓
- e

Choose all asymptotes of $f(x)$.

- $y = -e$
- $y = -2$
- $y = 0$
- $y = 2$
- $y = e$
- $x = -3$
- $x = -2$
- $x = -\sqrt{2}$
- $x = -1$
- $x = 0$
- $x = 1$ ✓
- $x = \sqrt{2}$
- $x = 2$
- $x = 3$ ✓
- $y = x$
- $y = -x$

The function $f(x)$ has three stationary points: $x = \boxed{c} \pm \sqrt{\frac{\boxed{d}}{\boxed{e}}}, \boxed{f} + \sqrt{\frac{\boxed{g}}{\boxed{h}}}$.

$\boxed{c}$: $\boxed{1 \checkmark}$ $\boxed{d}$: $\boxed{1 \checkmark}$ $\boxed{e}$: $\boxed{2 \checkmark}$ $\boxed{f}$: $\boxed{2 \checkmark}$ $\boxed{g}$: $\boxed{3 \checkmark}$ $\boxed{h}$: $\boxed{2 \checkmark}$

The $f(x)$ is not differentiable at $x = 2$. The point $x = 2$ is

- a local minimum
- a local maximum
- neither a local minimum nor a local maximum ✓

Choose the behaviour of $f(x)$ in the interval $(4, 5)$.

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(11) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us calculate the following integral.

$$\int_1^2 x^5 \exp(x^2) dx.$$

Choose a primitive of $x \exp(x^2)$.

- $x^2 \exp(x^2)$
- $2x^2 \exp(x^2)$
- $x^2 \exp(x^2)/2$
- $\exp(x^2/2)/2$
- $\exp(x^2)/2$ ✓
- $\exp(2x^2)/2$
- $\exp(x^3/3)$
- $\exp(x^3)/3$
- $x^2 \exp(x^3)/3$

Using the above primitive of $x \exp(x^2)$, by integration by parts, we have the following.

$$\int_1^2 x^5 \exp(x^2) dx = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}[x^{\boxed{\text{c}}} \exp(x^2)]_1^2 - \boxed{\text{d}} \int_1^2 x^{\boxed{\text{e}}} \exp(x^2) dx.$$

$\boxed{\text{a}}$: $\boxed{1 \ ✓}$ $\boxed{\text{b}}$: $\boxed{2 \ ✓}$ $\boxed{\text{c}}$: $\boxed{4 \ ✓}$ $\boxed{\text{d}}$: $\boxed{2 \ ✓}$ $\boxed{\text{e}}$: $\boxed{3 \ ✓}$

By continuing, we get $\int_1^2 x^5 \exp(x^2) dx = \frac{\boxed{\text{f}}}{\boxed{\text{g}}} e + \boxed{\text{h}} e^{\boxed{\text{i}}}$.

$\boxed{\text{f}}$: $\boxed{-1 \ ✓}$ $\boxed{\text{g}}$: $\boxed{2 \ ✓}$ $\boxed{\text{h}}$: $\boxed{5 \ ✓}$ $\boxed{\text{i}}$: $\boxed{4 \ ✓}$

> We should use integration by parts $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$. In this case, $f(x) = \frac{x^4}{2}$, $g'(x) = 2xe^{x^2}$ and $g(x) = e^{x^2}$. The remaining integral can be computed in a similar way.

(12) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us calculate the following integral.

$$\int_0^3 x^5 \exp(x^2)dx.$$

Choose a primitive of $x\exp(x^2)$.

- $x^2\exp(x^2)$
- $2x^2\exp(x^2)$
- $x^2\exp(x^2)/2$
- $\exp(x^2/2)/2$
- $\exp(x^2)/2$ ✓
- $\exp(2x^2)/2$
- $\exp(x^3/3)$
- $\exp(x^3)/3$
- $x^2\exp(x^3)/3$

Using the above primitive of $x\exp(x^2)$, by integration by parts, we have the following.

$$\int_0^3 x^5 \exp(x^2)dx = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}[x^{\boxed{\text{c}}}\exp(x^2)]_0^3 - \boxed{\text{d}}\int_0^3 x^{\boxed{\text{e}}}\exp(x^2)dx.$$

a: 1 ✓ b: 2 ✓ c: 4 ✓ d: 2 ✓ e: 3 ✓

By continuing, we get $\int_0^3 x^5\exp(x^2)dx = \boxed{\text{f}} + \frac{\boxed{\text{g}}}{\boxed{\text{h}}}e^{\boxed{\text{i}}}$.

f: −1 ✓ g: 65 ✓ h: 2 ✓ i: 9 ✓

(13) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the convergence of the following improper integral

$$\int_0^\infty \frac{x\log(x+1)}{x^\alpha(x+\frac{1}{2})}dx$$

.

Complete the formula.

$$\frac{x\log(x+1)}{x+\frac{1}{2}} = \boxed{\text{a}} + \boxed{\text{b}}x + \boxed{\text{c}}x^2 + o(x^2) \text{ as } x \to 0.$$

$\boxed{\text{a}}$: $\boxed{0\ \checkmark}$ $\boxed{\text{b}}$: $\boxed{0\ \checkmark}$ $\boxed{\text{c}}$: $\boxed{2\ \checkmark}$

Let us take an intermediate point $x = 1$. The integral $\int_1^\infty \frac{x\log(x+1)}{x^\alpha(x+\frac{1}{2})}dx$ converges for

- all α.
- $\alpha > 3$
- $\alpha < 3$
- $\alpha < 2$
- $\alpha > 2$
- $\alpha < 1$
- $\alpha > 1$ ✓
- $\alpha < 0$
- $\alpha > 0$
- none of α.

On the other hand, the integral $\int_0^1 \frac{x\log(x+1)}{x^\alpha(x+\frac{1}{2})}dx$ converges for

- all α.
- $\alpha > 3$
- $\alpha < 3$ ✓
- $\alpha < 2$
- $\alpha > 2$
- $\alpha < 1$
- $\alpha > 1$
- $\alpha < 0$
- $\alpha > 0$
- none of α.

For $\alpha = 2$, l'integrale $\int_0^\infty \frac{x\log(x+1)}{x^\alpha(x+\frac{1}{2})}dx$

- converges assolumtamente ✓
- converges but not absolutely
- does not converge

For $\alpha = 1$, the integral $\int_0^\infty \frac{x\log(x+1)}{x^\alpha(x+\frac{1}{2})}dx$

- converges assolumtamente
- converges but not absolutely
- does not converge ✓

On the other hand, for $\alpha = 1$, the integral $\int_0^\infty \frac{x\sin x}{x^\alpha(x+\frac{1}{2})}dx$

- converges assolumtamente
- converges but not absolutely ✓
- does not converge

An improper integral $\int_a^b f(x)dx$ is defined by $\lim_{\alpha\to a}\int_\alpha^c f(x)dx + \lim_{\beta\to b}\int_c^\beta f(x)dx$, if $a=-\infty, b=\infty$ or $f(x)$ is not bounded.
We know that $\int_0^1 x^\alpha dx$ is convergent if and only if $\alpha > -1$ and $\int_1^\infty x^\alpha dx$ is convergent if and only if $\alpha < -1$. Furthermore, we can compare $f(x)$ as $x\to 0$ and $x\to\infty$ with x^α. If $\frac{f(x)}{x^\alpha}$ is bounded and if $\int_a^b x^\alpha dx$ is convergent, then so is $\int_a^b f(x)dx$. We should check this condition as $x\to 0$ and $x\to\infty$. For $x\to 0$ we can use the Taylor formula.
The integral $\int_0^\infty \frac{x\sin x}{x(x+\frac{1}{2})}dx$ is oscillating. We can compare it with $\int x\sin x dx$ which is convergent but not absolutely convergent (see the lecture notes).

(14) **Q5**
If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the convergence of the following improper integral

$$\int_0^\infty \frac{x^2\log(x+1)}{x^\alpha(x+\frac{1}{3})^2}dx$$

.

Complete the formula.

$$\frac{x^2\log(x+1)}{(x+\frac{1}{3})^2} = \boxed{\text{a}} + \boxed{\text{b}}x + \boxed{\text{c}}x^2 + \boxed{\text{d}}x^3 + o(x^3) \text{ as } x\to 0.$$

$\boxed{\text{a}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{0 \ \checkmark}$ $\boxed{\text{c}}$: $\boxed{9 \ \checkmark}$

Let us take an intermediate point $x=1$. The integral $\int_1^\infty \frac{x^2\log(x+1)}{x^\alpha(x+\frac{1}{3})^2}dx$ converges for

- all α.
- $\alpha > 4$
- $\alpha < 4$
- $\alpha > 3$
- $\alpha < 3$

- $\alpha < 2$
- $\alpha > 2$
- $\alpha < 1$
- $\alpha > 1$ ✓
- $\alpha < 0$
- $\alpha > 0$
- none of α.

On the other hand, the integral $\int_0^1 \frac{x^2 \log(x+1)}{x^\alpha (x+\frac{1}{3})^2} dx$ converges for

- all α.
- $\alpha > 4$
- $\alpha < 4$ ✓
- $\alpha > 3$
- $\alpha < 3$
- $\alpha < 2$
- $\alpha > 2$
- $\alpha < 1$
- $\alpha > 1$
- $\alpha < 0$
- $\alpha > 0$
- none of α.

For $\alpha = 0$, l'integrale $\int_0^\infty \frac{x^2 \log(x+1)}{x^\alpha (x+\frac{1}{3})^2} dx$

- converges assolumtamente
- converges but not absolutely
- does not converge ✓

For $\alpha = 1$, the integral $\int_0^\infty \frac{x^2 \log(x+1)}{x^\alpha (x+\frac{1}{3})^2} dx$

- converges assolumtamente
- converges but not absolutely
- does not converge ✓

On the other hand, for $\alpha = 1$, the integral $\int_0^\infty \frac{x^2 \sin(x)}{x^\alpha (x+\frac{1}{3})^2} dx$

- converges assolumtamente
- converges but not absolutely ✓
- does not converge