Call1.

(1) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$\frac{e^x}{e} = \boxed{a} + \boxed{b}(x-1) + \frac{\boxed{c}}{\boxed{d}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

$\boxed{a}$: $\boxed{1 \checkmark}$ $\boxed{b}$: $\boxed{1 \checkmark}$ $\boxed{c}$: $\boxed{1 \checkmark}$ $\boxed{d}$: $\boxed{2 \checkmark}$

$$\sqrt{x} = \boxed{e} + \frac{\boxed{f}}{\boxed{g}}(x-1) + \frac{\boxed{h}}{\boxed{i}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

$\boxed{e}$: $\boxed{1 \checkmark}$ $\boxed{f}$: $\boxed{1 \checkmark}$ $\boxed{g}$: $\boxed{2 \checkmark}$ $\boxed{h}$: $\boxed{-1 \checkmark}$ $\boxed{i}$: $\boxed{8 \checkmark}$

$$\log x = \boxed{j} + \boxed{k}(x-1) + \frac{\boxed{l}}{\boxed{m}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

$\boxed{j}$: $\boxed{0 \checkmark}$ $\boxed{k}$: $\boxed{1 \checkmark}$ $\boxed{l}$: $\boxed{-1 \checkmark}$ $\boxed{m}$: $\boxed{2 \checkmark}$

For various $\alpha \in \mathbb{R}$, study the limit:

$$\lim_{x \to 1} \frac{\frac{e^x}{e} - 2\sqrt{x} - \alpha}{\log x - (x-1)}.$$

This limit converges for $\alpha = \boxed{n}$.

$\boxed{n}$: $\boxed{-1 \checkmark}$

In that case, the limit is $\frac{\boxed{o}}{\boxed{p}}$.

$\boxed{o}$: $\boxed{-3 \checkmark}$ $\boxed{p}$: $\boxed{2 \checkmark}$

> Use the Taylor formula $f(x) = f(1) + f'(1)(x-1) + \frac{1}{2!}f''(1)(x-1)^2 + o((x-1)^2)$ as $x \to 1$. To determine α, one only has to compare the numereator and the denominator and choose α in such a way that they have the same degree of infinitesimal (in this case, both of them should be of order x^2).

(2) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$\frac{e^x}{e} = \boxed{\text{a}} + \boxed{\text{b}}(x-1) + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

$\boxed{\text{a}}$: $\boxed{1 \checkmark}$ $\boxed{\text{b}}$: $\boxed{1 \checkmark}$ $\boxed{\text{c}}$: $\boxed{1 \checkmark}$ $\boxed{\text{d}}$: $\boxed{2 \checkmark}$

$$(x)^{\frac{1}{3}} = \boxed{\text{e}} + \frac{\boxed{\text{f}}}{\boxed{\text{g}}}(x-1) + \frac{\boxed{\text{h}}}{\boxed{\text{i}}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

$\boxed{\text{e}}$: $\boxed{1 \checkmark}$ $\boxed{\text{f}}$: $\boxed{1 \checkmark}$ $\boxed{\text{g}}$: $\boxed{3 \checkmark}$ $\boxed{\text{h}}$: $\boxed{-1 \checkmark}$ $\boxed{\text{i}}$: $\boxed{9 \checkmark}$

$$\log x = \boxed{\text{j}} + \boxed{\text{k}}(x-1) + \frac{\boxed{\text{l}}}{\boxed{\text{m}}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

$\boxed{\text{j}}$: $\boxed{0 \checkmark}$ $\boxed{\text{k}}$: $\boxed{1 \checkmark}$ $\boxed{\text{l}}$: $\boxed{-1 \checkmark}$ $\boxed{\text{m}}$: $\boxed{2 \checkmark}$

For various $\alpha \in \mathbb{R}$, study the limit:

$$\lim_{x \to 1} \frac{\frac{e^x}{e} - 3(x)^{\frac{1}{3}} - \alpha}{\log x - (x-1)}.$$

This limit converges for $\alpha = \boxed{\text{n}}$.

$\boxed{\text{n}}$: $\boxed{-2 \checkmark}$

In that case, the limit is $\frac{\boxed{\text{o}}}{\boxed{\text{p}}}$.

$\boxed{\text{o}}$: $\boxed{-5 \checkmark}$ $\boxed{\text{p}}$: $\boxed{3 \checkmark}$

(3) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$\frac{e^x}{e} = \boxed{\text{a}} + \boxed{\text{b}}(x-1) + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

a: 1 ✓ b: 1 ✓ c: 1 ✓ d: 2 ✓

$$(x)^{\frac{1}{4}} = \boxed{\text{e}} + \frac{\boxed{\text{f}}}{\boxed{\text{g}}}(x-1) + \frac{\boxed{\text{h}}}{\boxed{\text{i}}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

e: 1 ✓ f: 1 ✓ g: 4 ✓ h: −3 ✓ i: 32 ✓

$$\log x = \boxed{\text{j}} + \boxed{\text{k}}(x-1) + \frac{\boxed{\text{l}}}{\boxed{\text{m}}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

j: 0 ✓ k: 1 ✓ l: −1 ✓ m: 2 ✓

For various $\alpha \in \mathbb{R}$, study the limit:

$$\lim_{x \to 1} \frac{\frac{e^x}{e} - 4(x)^{\frac{1}{4}} - \alpha}{\log x - (x-1)}.$$

This limit converges for $\alpha = \boxed{\text{n}}$.

n: −3 ✓

In that case, the limit is $\frac{\boxed{\text{o}}}{\boxed{\text{p}}}$.

o: −7 ✓ p: 4 ✓

(4) **Q1**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Complete the formulae.

$$\frac{e^x}{e} = \boxed{\text{a}} + \boxed{\text{b}}(x-1) + \frac{\boxed{\text{c}}}{\boxed{\text{d}}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

a: 1 ✓ b: 1 ✓ c: 1 ✓ d: 2 ✓

$$(x)^{\frac{1}{5}} = \boxed{\text{e}} + \frac{\boxed{\text{f}}}{\boxed{\text{g}}}(x-1) + \frac{\boxed{\text{h}}}{\boxed{\text{i}}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

[e]: [1 ✓] [f]: [1 ✓] [g]: [5 ✓] [h]: [−2 ✓] [i]: [25 ✓]

$$\log x = \boxed{\text{j}} + \boxed{\text{k}}(x-1) + \frac{\boxed{\text{l}}}{\boxed{\text{m}}}(x-1)^2 + o((x-1)^2) \text{ as } x \to 1.$$

[j]: [0 ✓] [k]: [1 ✓] [l]: [−1 ✓] [m]: [2 ✓]

For various $\alpha \in \mathbb{R}$, study the limit:

$$\lim_{x\to 1} \frac{\frac{e^x}{e} - 5(x)^{\frac{1}{5}} - \alpha}{\log x - (x-1)}.$$

This limit converges for $\alpha = \boxed{\text{n}}$.

[n]: [−4 ✓]

In that case, the limit is $\frac{\boxed{\text{o}}}{\boxed{\text{p}}}$.

[o]: [−9 ✓] [p]: [5 ✓]

(5) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} x^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{(n!)^2}{(2n)!} x^n = \frac{\boxed{\text{a}}}{\boxed{\text{b}}} + \frac{\boxed{\text{c}}}{\boxed{\text{d}}} i$.

[a]: [5 ✓] [b]: [6 ✓] [c]: [1 ✓] [d]: [2 ✓]

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{(n!)^2}{(2n)!}|x|^n$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{\text{e}}}{\boxed{\text{f}}}|x|$$

[e]: [1 ✓] [f]: [4 ✓]

Therefore, by the ratio test, the series converges absolutely for

- all x.
- $-\frac{1}{4} < x < \frac{1}{4}$.
- $-\frac{1}{2} < x < \frac{1}{2}$.

- $x = 0$.
- $-1 < x < 1$.
- $-2 < x < 2$.
- $-4 < x < 4$. ✓
- none of x.

For the case $x = -4$, the series
- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = -\frac{1}{2}$, the series
- converges absolutely. ✓
- converges but not absolutely.
- diverges.

> The partial sum means the following finite sum: $\sum_{n=0}^{2} a_n = a_0 + a_1 + a_2$, so one just has to apply $n = 0, 1, 2$ in the concrete series and sum the numbers up. Notice that $i^2 = -1$.
> To apply the ratio test, one considers $R = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$. Note that $(n+1)! = (n+1)n!$ etc.
> If this limit $R < 1$, then the series converges absolutely (for such x), while if $R > 1$ the series diverges.
> If $R = 1$, one needs to study the convergence with other criteria. In this case, if $x = 4$, then $\frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{(2n+2)(2n+1)}(-4) = \frac{-(2n+2)}{2n+1} < -1$, and a_n is not converging to 0. Therefore, the series is divergent.

(6) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{(n!)^3}{(3n)!} x^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{(n!)^3}{(3n)!} x^n = \frac{\boxed{\text{a}}}{\boxed{\text{b}}} + \frac{\boxed{\text{c}}}{\boxed{\text{d}}} i$.

a: 89 ✓ b: 90 ✓ c: 1 ✓ d: 6 ✓

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{(n!)^3}{(3n)!}|x|^n$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{e}}{\boxed{f}}|x|$$

e: 1 ✓ f: 27 ✓

Therefore, by the ratio test, the series converges absolutely for

- all x.
- $-27 < x < 27$. ✓
- $-9 < x < 9$.
- $-1 < x < 1$.
- $x = 0$.
- $-\frac{1}{9} < x < \frac{1}{9}$.
- $-1/27 < x < 1/27$.
- none of x.

For the case $x = -9$, the series

- converges absolutely. ✓
- converges but not absolutely.
- diverges.

For the case $x = 27$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(7) **Q2**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us study the following series $\sum_{n=0}^{\infty} \frac{(n!)(2n)!}{(3n)!}x^n$, with various x.

This series makes sense also for $x \in \mathbb{C}$. For $x = i$, calculate the partial sum $\sum_{n=0}^{2} \frac{(n!)(2n)!}{(3n)!}x^n = \frac{\boxed{a}}{\boxed{b}} + \frac{\boxed{c}}{\boxed{d}}i$.

a: 14 ✓ b: 15 ✓ c: 1 ✓ d: 3 ✓

In order to use the ratio test for $x \in \mathbb{R}$, we put $a_n = \frac{(n!)(2n)!}{(3n)!} x^n |x|^n$. Complete the formula.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{\boxed{\text{e}}}{\boxed{\text{f}}}|x|$$

$\boxed{\text{e}}$: $\boxed{4 \quad \checkmark}$ $\boxed{\text{f}}$: $\boxed{27 \quad \checkmark}$

Therefore, by the ratio test, the series converges absolutely for

- all x.
- $-1/27 < x < 1/27$.
- $-4/27 < x < 4/27$.
- $x = 0$.
- $-1 < x < 1$.
- $27/4 < x < 27/4$. ✓
- $-27 < x < 27$.
- none of x.

For the case $x = -7$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

For the case $x = \frac{27}{4}$, the series

- converges absolutely.
- converges but not absolutely.
- diverges. ✓

(8) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us consider the following function

$$f(x) = \log\left(\frac{x^2 - 3}{x - 2}\right).$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- $-e$ ✓
- -2 ✓

- -1
- 0
- 1
- 2 ✓
- e

Choose all asymptotes of $f(x)$.

- $y = -e$
- $y = -2$
- $y = 0$
- $y = 2$
- $y = e$
- $x = -2$
- $x = -\sqrt{3}$ ✓
- $x = -\sqrt{2}$
- $x = -1$
- $x = 0$
- $x = 1$
- $x = \sqrt{2}$
- $x = \sqrt{3}$ ✓
- $x = 2$ ✓
- $y = x$
- $y = -x$

One has

$$f'(0) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$: $\boxed{1\ ✓}$ $\boxed{\text{b}}$: $\boxed{2\ ✓}$

The function $f(x)$ has two stationary points: $x = \boxed{\text{c}}, \boxed{\text{d}}$.

$\boxed{\text{c}}$: $\boxed{1\ ✓}$ $\boxed{\text{d}}$: $\boxed{3\ ✓}$

Choose the behaviour of $f(x)$ in the interval $(2, 4)$.

- monotonically decreasing
- monotonically increasing
- neither decreasing nor increasing ✓

To determine the natural domain of a function, it is enough to observe the components. For example, $\log y$ is defined only for $y > 0$, $\frac{1}{y-a}$ is defined only for $y \neq a$, etc. It is enought to exclude all such points where the composed function is not defined. In this case, $\frac{x^2-3}{x-2} > 0$.
There can be asymptotes for $x \to \pm\infty$, and for $x \to a$, where a is a boundary of the domain. In this case, one should check $x \to 2, \pm\sqrt{3}$. All of them are asymptotes.
For the derivative, the chain rule $(f(g(x)))' = g'(x)f'(g(x))$ is useful. In this case, $f(x) = \log(\frac{x^2-3}{x-2})$, $f'(x) = \frac{x-2}{x^2-3} \cdot \frac{2x(x-2)-(x^2-3)}{(x-2)^2} = -\frac{x^2-4x+3}{(x^2-3)(x-2)}$.
If $f'(x_0) = 0$, x_0 is called a stationary point.
If $f'(x) \geq 0$ (≤ 0) in one interval, then $f(x)$ is monotonically increasing (decreasing) there.

(9) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us consider the following function

$$f(x) = \log\left(\frac{x^2-5}{x-3}\right).$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- -3 ✓
- -2
- -1
- 0
- 1
- 2
- 3 ✓

Choose all asymptotes of $f(x)$.

- $y = -e$
- $y = -2$

- $y = 0$
- $y = 2$
- $y = e$
- $x = -3$
- $x = -\sqrt{5}$ ✓
- $x = -\sqrt{3}$
- $x = -1$
- $x = 0$
- $x = 1$
- $x = \sqrt{3}$
- $x = \sqrt{5}$ ✓
- $x = 3$ ✓
- $y = x$
- $y = -x$

One has

$$f'(0) = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$: $\boxed{1 \ ✓}$ $\boxed{\text{b}}$: $\boxed{3 \ ✓}$

The function $f(x)$ has two stationary points: $x = \boxed{\text{c}}, \boxed{\text{d}}$.

$\boxed{\text{c}}$: $\boxed{1 \ ✓}$ $\boxed{\text{d}}$: $\boxed{5 \ ✓}$

Choose the behaviour of $f(x)$ in the interval $(4, 5)$.

- monotonically decreasing ✓
- monotonically increasing
- neither decreasing nor increasing

(10) **Q3**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us consider the following function

$$f(x) = \log\left(\frac{x^2 - 7}{x - 4}\right).$$

The function $f(x)$ is not defined on the whole real line $\mathbb{R}$. Choose all the points that are **not** in the natural domain of $f(x)$.

- -3 ✓
- -2

- -1
- 0
- 1
- 2
- 3 ✓

Choose all asymptotes of $f(x)$.

- $y=-e$
- $y=-2$
- $y=0$
- $y=2$
- $y=e$
- $x=-3$
- $x=-\sqrt{7}$ ✓
- $x=-\sqrt{3}$
- $x=-1$
- $x=0$
- $x=1$
- $x=\sqrt{3}$
- $x=\sqrt{7}$ ✓
- $x=3$ ✓
- $y=x$
- $y=-x$

One has

$$f'(0)=\frac{\boxed{\text{a}}}{\boxed{\text{b}}}.$$

$\boxed{\text{a}}$: $\boxed{1\ ✓}$ $\boxed{\text{b}}$: $\boxed{4\ ✓}$

The function $f(x)$ has two stationary points: $x=\boxed{\text{c}},\boxed{\text{d}}$.

$\boxed{\text{c}}$: $\boxed{1\ ✓}$ $\boxed{\text{d}}$: $\boxed{7\ ✓}$

Choose the behaviour of $f(x)$ in the interval $(0,1)$.

- monotonically decreasing
- monotonically increasing ✓
- neither decreasing nor increasing

(11) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us calculate the following integral.

$$\int_{-1}^{\sqrt{2}} (4-x^2)^{\frac{1}{2}}\,dx.$$

With the change of variable $2\sin t = x$ (where $-\pi < t < \pi$), we get

$$\int_{-1}^{\sqrt{2}} (4-x^2)^{\frac{1}{2}}\,dx = \int_{X}^{Y} \boxed{\text{c}}\,dt.$$

Fill in the blanks, $X = \frac{\pi}{\boxed{\text{a}}}$, $Y = \frac{\pi}{\boxed{\text{b}}}$.

$\boxed{\text{a}}$: $\boxed{-6\ \checkmark}$ $\boxed{\text{b}}$: $\boxed{4\ \checkmark}$

Choose the function $\boxed{\text{c}}$ after the change of variables.

- t^2
- $1/(2-t^2)$
- $2\arccos t$
- $4\cos t$
- $2\cos 2t$
- $4\cos^2 t$ ✓
- $1/(2\cos t)$

By continuing, we get $\int_{-1}^{\sqrt{2}} (4-x^2)^{\frac{1}{2}}\,dx = \frac{\sqrt{\boxed{\text{d}}}}{\boxed{\text{e}}} + \frac{\boxed{\text{f}}}{\boxed{\text{g}}}\pi + \boxed{\text{h}}$.

$\boxed{\text{d}}$: $\boxed{3\ \checkmark}$ $\boxed{\text{e}}$: $\boxed{2\ \checkmark}$ $\boxed{\text{f}}$: $\boxed{5\ \checkmark}$ $\boxed{\text{g}}$: $\boxed{6\ \checkmark}$ $\boxed{\text{h}}$: $\boxed{1\ \checkmark}$

> By the change of variables $x = 2\sin t$, one has $\frac{dx}{dt} = 2\cos t$ and hence with $2\sin(-\frac{\pi}{6}) = -1, 2\sin\frac{\pi}{4} = \sqrt{2}$, $\int_{-1}^{\sqrt{2}}(4-x^2)^{\frac{1}{2}}dx = \int_{-\frac{\pi}{6}}^{\frac{\pi}{4}} 4\cos^2 t dt$, then use $\cos^2 t = \frac{1+\cos(2t)}{2}$, $\int \cos(2t)dt = \frac{\sin(2t}{2} + C$.

(12) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us calculate the following integral.

$$\int_1^{\sqrt{2}} (4-x^2)^{\frac{1}{2}}\,dx.$$

With the change of variable $2\sin t = x$ (where $-\pi < t < \pi$), we get

$$\int_1^{\sqrt{2}} (4-x^2)^{\frac{1}{2}}\,dx = \int_X^Y \boxed{\text{c}}\,dt.$$

Fill in the blanks, $X = \frac{\pi}{\boxed{\text{a}}}$, $Y = \frac{\pi}{\boxed{\text{b}}}$.

$\boxed{\text{a}}$: $\boxed{6\ \checkmark}$ $\boxed{\text{b}}$: $\boxed{4\ \checkmark}$

Choose the function $\boxed{\text{c}}$ after the change of variables.

- t^2
- $1/(2-t^2)$
- $2\arccos t$
- $4\cos t$
- $2\cos 2t$
- $4\cos^2 t$ ✓
- $1/(2\cos t)$

By continuing, we get $\int_1^{\sqrt{2}} (4-x^2)^{\frac{1}{2}}\,dx = \frac{\boxed{\text{d}}}{\boxed{\text{e}}}\pi - \frac{\sqrt{\boxed{\text{f}}}}{\boxed{\text{g}}} + \boxed{\text{h}}$.

$\boxed{\text{d}}$: $\boxed{1\ \checkmark}$ $\boxed{\text{e}}$: $\boxed{6\ \checkmark}$ $\boxed{\text{f}}$: $\boxed{3\ \checkmark}$ $\boxed{\text{g}}$: $\boxed{2\ \checkmark}$ $\boxed{\text{h}}$: $\boxed{1\ \checkmark}$

(13) **Q4**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{\text{a}}}{\boxed{\text{b}}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Let us calculate the following integral.

$$\int_{-1}^{1} (4-x^2)^{\frac{1}{2}}\,dx.$$

With the change of variable $2\sin t = x$ (where $-\pi < t < \pi$), we get

$$\int_{-1}^{1} (4-x^2)^{\frac{1}{2}}\,dx = \int_X^Y \boxed{\text{c}}\,dt.$$

Fill in the blanks, $X = \frac{\pi}{\boxed{\text{a}}}$, $Y = \frac{\pi}{\boxed{\text{b}}}$.

a: -6 ✓ b: 6 ✓

Choose the function c after the change of variables.

- t^2
- $1/(2-t^2)$
- $2\arccos t$
- $4\cos t$
- $2\cos 2t$
- $4\cos^2 t$ ✓
- $1/(2\cos t)$

By continuing, we get $\int_{-1}^{1}(4-x^2)^{\frac{1}{2}}dx = \frac{\boxed{d}}{\boxed{e}}\pi + \sqrt{\boxed{f}}$.

d: 2 ✓ e: 3 ✓ f: 3 ✓

(14) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Choose the general solution of the following differential equation.

$$y'(x) = xe^{x^2}y$$

.

- $y(x) = \exp(x^2)/2 + C$
- $y(x) = \exp(x^2 + C)/2$
- $y(x) = \exp(\exp(x^2)/2 + C)$ ✓
- $y(x) = \exp(\exp(x^2)/2) + C$
- $y(x) = \log(\exp(x^2) + C)$
- $y(x) = \log\exp(x^2 + C)$
- $y(x) = (\exp(x^2 + C))^{\frac{1}{2}}$
- $y(x) = \exp(x^2)^{\frac{1}{2}} + C$

Determine $C = \frac{\boxed{a}}{\boxed{b}}$ with the initial condition $y(0) = e^{-1}$

a: -3 ✓ b: 2 ✓

Choose the general solution of the following differential equation.

$$y''(x) - 4y'(x) + 8y = 0$$

.

- $y(x) = C_1\exp(-4x) + C_2\exp(-2x)$

- $y(x) = C_1 \exp(2x) + C_2 \exp(-4x)$
- $y(x) = C_1 \sin(2x) + C_2 \cos(2x)$
- $y(x) = C_1 \exp(2x) + C_2 \exp(4x) \cos(2x)$
- $y(x) = C_1 \exp(-2x) + C_2 \exp(-2x) \cos(2x)$
- $y(x) = C_1 \exp(2x) \sin(2x) + C_2 \cos(2x)$
- $y(x) = C_1 \exp(2x) \sin(2x) + C_2 \exp(2x) \cos(2x)$ ✓
- $y(x) = C_1 \exp(-4x) \sin(4x) + C_2 \exp(-2x) \cos(-2x)$

Determine $C_1 = \boxed{c}, C_2 = \boxed{d}$ with the initial condition $y(0) = 2, y'(0) = 2$

$\boxed{a}$: $\boxed{-1 \quad ✓}$ $\boxed{b}$: $\boxed{2 \quad ✓}$

The equation $y'(x) = xe^{x^2}y$ is separable, hence one obtains the relation $\int \frac{1}{y}dy = \int xe^{x^2}dx + C$, or $\log y = \frac{e^{x^2}}{2} + C$, or $y = \exp(\frac{e^{x^2}}{2} + C)$.
The second-order differential equation $y'' + ay' + by = 0$ can be solved as follows: put $z^2 + az + b = 0$, and solve this equation. If this has two real solutions z_1, z_2, then the general solution is $y = C_1e^{z_1x} + C_2e^{z_2x}$. If it has two complex solutions $z_1 \pm iz_2$, then $y = C_1e^{z_1x}\sin(z_2x) + C_2e^{z_1x}\cos(z_2x)$.
The constant can be obtaind by substituting the initial condition.

(15) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly 0 or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Choose the general solution of the following differential equation.

$$y'(x) = x \cos x^2 y$$

.

- $y(x) = \sin(x^2)/2 + C$
- $y(x) = \sin(x^2 + C)/2$
- $y(x) = \cos(\sin(x^2) + C)$
- $y(x) = \cos(\sin(x^2 + C)$
- $y(x) = \exp(\sin(x^2)/2 + C)$ ✓

- $y(x) = \exp(\cos(x^2)/2 + C)$
- $y(x) = (\sin(x^2 + C))^{\frac{1}{2}}$
- $y(x) = \sin(x^2)^{\frac{1}{2}} + C$

Determine $C = \frac{\boxed{a}}{\boxed{b}}$ with the initial condition $y(\sqrt{\frac{\pi}{2}}) = e$

$\boxed{a}$: $\boxed{1\ \checkmark}$ $\boxed{b}$: $\boxed{2\ \checkmark}$

Choose the general solution of the following differential equation.

$$y''(x) + 4y'(x) + 8y = 0$$

.

- $y(x) = C_1 \exp(-4x) + C_2 \exp(-2x)$
- $y(x) = C_1 \exp(2x) + C_2 \exp(-4x)$
- $y(x) = C_1 \sin(2x) + C_2 \cos(2x)$
- $y(x) = C_1 \exp(-2x) \sin(2x) + C_2 \exp(-2x) \cos(2x)$ ✓
- $y(x) = C_1 \exp(-4x) \sin(4x) + C_2 \exp(-2x) \cos(-2x)$
- $y(x) = C_1 \exp(-2x) + C_2 \exp(4x) \cos(2x)$
- $y(x) = C_1 \exp(-2x) + C_2 \exp(-2x) \cos(2x)$
- $y(x) = C_1 \exp(-2x) \sin(2x) + C_2 \cos(2x)$

Determine $C_1 = \boxed{c}, C_2 = \boxed{d}$ with the initial condition $y(0) = 5, y'(0) = 4$

$\boxed{a}$: $\boxed{7\ \checkmark}$ $\boxed{b}$: $\boxed{5\ \checkmark}$

(16) **Q5**

If not specified otherwise, fill in the blanks with **integers (possibly** 0 **or negative)**. A fraction should be **reduced** (for example, $\frac{1}{2}$ is accepted but not $\frac{2}{4}$), and if it is negative and the answer boxes (such as $\frac{\boxed{a}}{\boxed{b}}$) have ambiguity, the negative sign should be put on the numerator (for example $\frac{-1}{2}$ is accepted but $\frac{1}{-2}$ is not).

Choose the general solution of the following differential equation.

$$y'(x) = x \sin x^2 y$$

.

- $y(x) = \cos(x^2)/2 + C$
- $y(x) = \sin(x^2 + C)/2$
- $y(x) = \cos(\sin(x^2) + C)$
- $y(x) = \cos(\sin(x^2 + C)$
- $y(x) = \exp(\sin(x^2)/2 + C)$
- $y(x) = \exp(\cos(x^2)/2 + C)$ ✓
- $y(x) = (\sin(x^2 + C))^{\frac{1}{2}}$

- $y(x) = \cos(x^2)^{\frac{1}{2}} + C$

Determine $C = \frac{\boxed{\text{a}}}{\boxed{\text{b}}}$ with the initial condition $y(0) = 1$

$\boxed{\text{a}}$: $\boxed{-1 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{2 \ \checkmark}$

Choose the general solution of the following differential equation.

$$y''(x) + 4y'(x) + 8y = 0$$

.

- $y(x) = C_1 \exp(-4x) + C_2 \exp(-2x)$
- $y(x) = C_1 \exp(2x) + C_2 \exp(-4x)$
- $y(x) = C_1 \sin(2x) + C_2 \cos(2x)$
- $y(x) = C_1 \exp(-2x) \sin(2x) + C_2 \exp(-2x) \cos(2x)$ ✓
- $y(x) = C_1 \exp(-4x) \sin(4x) + C_2 \exp(-2x) \cos(-2x)$
- $y(x) = C_1 \exp(-2x) + C_2 \exp(4x) \cos(2x)$
- $y(x) = C_1 \exp(-2x) + C_2 \exp(-2x) \cos(2x)$
- $y(x) = C_1 \exp(-2x) \sin(2x) + C_2 \cos(2x)$

Determine $C_1 = \boxed{\text{c}}, C_2 = \boxed{\text{d}}$ with the initial condition $y(0) = 4, y'(0) = -4$

$\boxed{\text{a}}$: $\boxed{2 \ \checkmark}$ $\boxed{\text{b}}$: $\boxed{4 \ \checkmark}$