# Mathematical Analysis I: Lecture 1

Lecturer: Yoh Tanimoto

21/09/2020

# Basic information

- Lecturer: Yoh Tanimoto (hoyt@mat.uniroma2.it, or you can write to me in chat of MS teams)
- Mon 11:30-13:15, Wed 11:30-13:15, Thu 9:30-11:15, Fri 14:00-15:45 (exercises)
- "Calculus" Vol. I by Tom M. Apostol, Wiley, (but we follow the chapters in a different order).
- Lecture notes:
  `http://www.mat.uniroma2.it/~tanimoto/teaching/2020MA1/2020MA1.pdf`
- Slides:
  `http://www.mat.uniroma2.it/~tanimoto/teaching/2020MA1/`
- Exercises:
  `http://www.mat.uniroma2.it/~tanimoto/teaching/2020MA1/2020MA1ex.pdf`
- Office hours: to be determined

# How to study mathematics

In mathematics, we assume the basic properties (axioms) of mathematical objects (e.g. numbers, functions) and derive their relations (theorems). It is very impotant that you learn the definitions of new concepts (limit, derivative, integral...) and apply it to concrete examples.

Try

- to keep track of what we are talking about: if you see a symbol, it has been defined somewhere before.
- to do exercises: we apply what we leared in a general form to concrete cases.
- not only to do computations, but also to understand the concepts.

# How to study mathematics

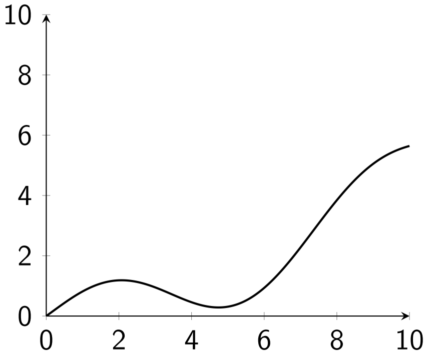
We will have an office hour for questions and exercises (you can ask what you did not understand during the lecture, or we can solve exercises together). **Preference for day and time?**

**Exam**: written test (exercises) + oral verification. In oral verification, we will check your understanding of concepts, whether you can apply computational methods, and so on.

# What is analysis and why study it

In a real-world science, it is crucial to study **quantitative aspects** of the subject. When a quantity **changes** by time, one can study its change in a short time ($\Rightarrow$ differentiation) and then sum it up ($\Rightarrow$ integration). Another important problem is optimization: maximizing benefit or minimizing cost.

- Mechanics, the equation of motion $F(x,t) = m\frac{d^2x}{dt^2} (= ma)$
- Economics
- Electrodynamics, thermodynamics, fluid mechanics (Mathematical Analysis II)
- Epidemiology (the SIR model $\frac{dS}{dt} = -\frac{\beta S(t) I(t)}{N}, \frac{dI}{dt} = \frac{\beta S(t) I(t)}{N} - \gamma I(t), \frac{dR}{dt} = \gamma I(t)$)



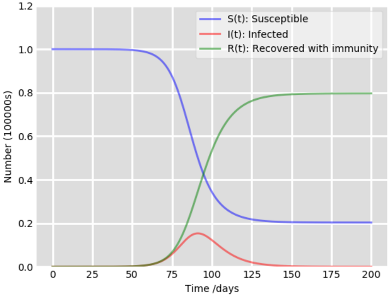




Figure: Left: A graph can be used to study changing quantities. Right:the SIR model. We will learn differential equations in the last part of the course.

# Mathematical symbols

We use **symbols** for general mathematical objects. Before using a symbol, we **declare** what kind of object it is. For example,

- $x, y, z, a, b, c$ often for numbers (but be careful of the declaration)
- $A$, $B$, $C$ sometimes for "sets"
- $f, g, h, F, G, H$ often for "functions" (which we will study later)

A symbol might be "recycled", that is, can be declared to be something different (unfortunately, we have only $26 \times 2$ alphabets).
Symbols are very useful because we can express general properties of certain mathematical objects at the same time, without specifying them every time.

# Integers and rational numbers

We assume that we know

- **integers**: $0, 1, 2, 3, \cdots, 100, 101, \cdots, 492837498\cdots, -1, -2, -3, \cdots$
- **rational numbers**: $\frac{1}{2}, \frac{2}{3}, \cdots, \frac{23}{62518}, -\frac{3028746}{26543}, \cdots$ (integers are also rational numbers)
- calculations between them (sum, difference, product, division, order)

# Integers and rational numbers

On rational numbers, we have the set of operations $+$ (summation), $\cdot$ (product): For $x, y, z$ rational numbers (**declaration**), $x + y$ and $x \cdot y$ are again rational numbers and they satisfy

- (commutativity) $x + y = y + x, x \cdot y = y \cdot x$
- (associativity) $(x + y) + z = x + (y + z), (x \cdot y) \cdot z = x \cdot (y \cdot z)$
- (distributive law) $(x + y) \cdot z = x \cdot z + y \cdot z$
- (zero and unity) There are special distinct rational numbers, called 0 and 1, such that $x + 0 = x$ and $x \cdot 0 = 0$. And $x \cdot 1 = x$.
- (negative) There is a only one rational number, which we call $-x$, such that $x + (-x) = 0$.
- (inverse) If $x \neq 0$, there is only one rational number, which we call $x^{-1}$, such that $x \cdot x^{-1} = 1$.

We often simply write $xy$ for $x \cdot y$ and $x - y$ for $x + (-y)$, $\frac{x}{y}$ for $xy^{-1}$.
**Exercise:** Take concrete rational numbers and check these properties!

Other properties of rational numbers can be **derived** from these. Indeed, we can prove the following

## Theorem

*Let $a, b, c, d$ be rational numbers.*

- *if $a + b = a + c$, then $b = c$.*
- $-(-a) = a$.
- $a(b - c) = ab - ac$.
- $a \cdot 0 = 0 \cdot a = 0$.
- *if $ab = ac$ and $a \neq 0$, then $b = c$.*
- *if $a \neq 0$, then $a^{-1} \neq 0$ and $(a^{-1})^{-1} = a$.*
- *if $ab = 0$, then $a = 0$ or $b = 0$.*
- $(-a)b = -(ab)$ *and* $(-a)(-b) = ab$.
- *if $b \neq 0, d \neq 0$, then $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$.*
- *if $b \neq 0, d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.*
- *if $a \neq 0, b \neq 0$, then $(\frac{a}{b})^{-1} = \frac{b}{a}$.*

## Proof.

We only prove a few of them and **leave the rest as exercises**.
Let us assume that $a + b = a + c$. Then, we take $-a$ and

$$\begin{aligned}(a + b) + (-a) &= a + (b + (-a)) \text{ (associativity)}\\ &= a + ((-a) + b) \text{ (commutativity)}\\ &= (a + (-a)) + b \text{ (associativity)}\\ &= 0 + b \text{ (definition of 0)}\\ &= b \text{ (property of 0)}\end{aligned}$$

Similarly, $(a + c) + (-a) = c$. But as $a + b = a + c$, we have $b = (a + b) + (-a) = (a + c) + (-a) = c$.
Next, we have $(-a) + a = a + (-a) = 0$ by commutativity. By definition of $-(-a)$, $a = -(-a)$.
If $ab = 0$ and $a \neq 0$, then we can take $a^{-1}$ and $0 = a^{-1}0 = a^{-1}ab = 1 \cdot b = b$.

□

# A geometric representation of integers and rational numbers