BSc Engineering Sciences – A. Y. 2018/19

Written exam of the course Mathematical Analysis 2

September 12, 2019

Last name: ..... First name: .....
Matriculation: .....

Solve the following problems, motivating in detail the answers.

1. Find the Taylor series expansion, with initial point $x_0 = 1$, of the function

$$f(x) := (x-1)\log(x^2 - 2x + 2),$$

find its radius of convergence r, and study the convergence for $x = 1 \pm r$.
Solution.

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2. Find all the stationary points of the following scalar field, defined on $\mathbb{R}^2$,

$$f(x, y) = 2x^3 - 2x^2y - x + y^2$$

and classify them into relative minima, maxima and saddle points.
Solution.

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3. Let C be the curve $\{(x, y) : x^2 + 4y^2 = 4, 0 \leq x\}$ in $\mathbb{R}^2$. Find a parametrization $\boldsymbol{\alpha}(t)$ of C starting at $(0, -1)$ and ending at $(0, 1)$, and compute the line integral

$$\int_C \boldsymbol{f} \cdot d\boldsymbol{\alpha},$$

where $\boldsymbol{f}(x, y) = (y + 1, x)$ is a vector field in $\mathbb{R}^2$.
Solution.

Matriculation: .....

4. Compute the integral

$$\iiint_T x^4 dxdydz\,,$$

with

$$T := \{(x, y, z) \in \mathbb{R}^3 \,:\, x^2 + z^2 \leq 1,\, 1 \leq y \leq 2 - x^2 - z^2\}.$$

Solution.

Matriculation: .....

5. Let $\boldsymbol{F}(x, y, z) = (0, xyz, x)$ be a vector field on $\mathbb{R}^3$ and

$$S = \{(x, y, z) : x^2 + y^2 + z^2 \leq 4,\ y = z,\ x \geq 0\}$$

be a surface in $\mathbb{R}^3$. Compute the surface integral

$$\iint_S \boldsymbol{F} \cdot \boldsymbol{n}\, dS,$$

where $\boldsymbol{n}$ is a unit normal vector on S with positive z-component.
Solution.