BSc Engineering Sciences – A. Y. 2017/18

Written exam of the course Mathematical Analysis 2

July 9, 2018

Last name: .... First name: ....

Matriculation: ....

Solve the following problems, motivating in detail the answers.

1. Find the Taylor series expansion, with initial point $x_0 = 0$, of the function

$$f(x) := \frac{2x}{2x^2 - 3x + 1},$$

determine its radius of convergence r, and study the convergence for $x = \pm r$.

Solution.

Matriculation: ....

2. Find the extremal values of the function $f(x, y) = e^{x^2+y}$ on the circle $x^2 + y^2 = 1$.
Solution.

Matriculation: ....

3. Determine whether the following vector field on $\mathbb{R}^2$

$$\mathbb{f}(x,y) = (-y\sin x \cdot \cos(y\cos x),\ \cos x \cdot \cos(y\cos x))$$

is a gradient of some scalar field. If so, find one of these scalar fields φ such that $\mathbb{f}(x,y) = \nabla\varphi(x,y)$.

Solution.

Matriculation: ....

4. Compute the following double integral

$$\iint_T \frac{y}{(x^2+y^2)^2}\,dxdy,$$

where T is the quadrilateral with vertices $(1,0)$, $(1,\sqrt{3})$, $(3,3\sqrt{3})$, $(3,0)$.
Solution.

Matriculation: ....

5. Let $\mathbb{F}(x,y,z) = ((x-y+z)e^{x^2+y^2+z^2},\ (x+y+z)e^{x^2+y^2+z^2},\ (-x+y-z)e^{x^2+y^2+z^2})$ be a vector field on $\mathbb{R}^3$, C be the circle

$$C = \{(x,y,z) : x^2+y^2 = 1, z = 0\}.$$

Compute the line integral

$$\int_C \mathbb{F} \cdot d\boldsymbol{\alpha},$$

where $\boldsymbol{\alpha}$ is a parametrization of C going counterclockwise.
Solution.