BSc Engineering Sciences – A. Y. 2017/18

Written exam of the course Mathematical Analysis 2

June 20, 2018

Last name: .... First name: ....

Matriculation: ....

Solve the following problems, motivating in detail the answers.

1. Study the pointwise and uniform convergence, as $n \to \infty$, of the following sequence of functions:

$$f_n(x) := \sin(x + 2\pi\sqrt{n^2+1}), \qquad x \in \mathbb{R},\, n \in \mathbb{N}.$$

Solution.

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2.

(1) Find all the stationary points of the following scalar field, defined on $\mathbb{R}^2$,

$$f(x,y) = e^y(x^2 - 2xy + 3)$$

and classify them into relative minima, maxima and saddle points.

(2) Compute the derivative of the following function of $t \in \mathbb{R}$:

$$g(t) = \int_0^{\sin t} \cos e^s ds$$

Solution.

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3.

(1) Find the solution $f(x, y)$ of the partial differential equation

$$3\frac{\partial f}{\partial x} + 5\frac{\partial f}{\partial y} = 0$$

with the initial condition $f(x, 0) = \cos(x^2)$.

(2) Let $k > 0$ be a constant and $g(x, t) = \frac{1}{\sqrt{t}} \exp\left(-\frac{kx^2}{t}\right)$. Determine the value of k for which $g(x, t), t > 0, x \in \mathbb{R}$ satisfies the following partial differential equation (1d heat equation):

$$\frac{\partial g}{\partial t} = \frac{\partial^2 g}{\partial x^2}$$

Solution.

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4. Compute the integral

$$\iiint_T \frac{z}{1+x^2+y^2}\,dxdydz,$$

where $T = \{(x,y,z) \in \mathbb{R}^3 \,:\, x^2+y^2 \le (z-2)^2,\ 0 \le z \le 1\}$.
Solution.

Matriculation: ....

5. Let $\mathbb{F}(x, y, z) = (x(y^2 + 1), y(z^2 + 1), z(x^2 + 1))$ be a vector field on $\mathbb{R}^3$, S be the sphere

$$\{(x, y, z) : x^2 + y^2 + z^2 = 1\}$$

and let $\mathbb{n}$ be the outgoing normal unit vector on S at each point of S. Compute the surface integral

$$\iint_S \mathbb{F} \cdot \mathbb{n} \, dS.$$

Solution.