BSc Engineering Sciences – A. Y. 2017/18

Written exam (call II) of the course Mathematical Analysis 2

February 21, 2018

Last name: .... First name: ....

Matriculation: ....

Solve the following problems, **motivating in detail the answers**.
You may also use the other side of the sheets if necessary. **Papers for draft will not be evaluated.**

1. Find a power series expression for the solution $y(x)$ of the differential equation

$$xy'' + (1 + x)y' + 2y = 0$$

such that $y(0) = 1$, $y'(0) = -2$, and determine its radius of convergence.
Solution.

1. continued.

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2. Let C be the curve in $\mathbb{R}^2$ defined by

$$C = \{(x, y) : 2(x + y - 1)^2 + (x - y)^2 = 8\}.$$

Find the points on C which are the nearest and the farthest from the origin $(0, 0)$.
Solution.

2. continued.

3.

(1) Let $c > 0$. Find the solution $f(x, t)$ of the partial differential equation

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}$$

with the initial condition $f(x, 0) = e^{-x^2}$, $\frac{\partial f}{\partial t}(x, 0) = \frac{x}{(x^2+1)^2}$.

(2) Let $h(s)$ be a twice continuously differentiable function of $s \in \mathbb{R}$, $\boldsymbol{\alpha} = (\alpha_x, \alpha_y, \alpha_z) \in \mathbb{R}^3$ such that $\|\boldsymbol{\alpha}\|^2 = 1$. Let us define a function g of $(x, y, z, t) \in \mathbb{R}^4$ by

$$g(x, y, z, t) = h(x\alpha_x + y\alpha_y + z\alpha_z - ct).$$

Prove that g satisfies the following partial differential equation (3d wave equation):

$$\frac{\partial^2 g}{\partial t^2} = c^2 \left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2} \right)$$

Solution.

3. continued.

Matriculation: ....

4. Compute the integral

$$\iiint_T dxdydz\, z\sqrt{x^2+y^2}$$

where T is the region bounded by the cylinder $x^2 - 2x + y^2 = 0$, the sphere $x^2 + y^2 + z^2 = 4$ and the plane $z = 0$.

4. continued.

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5. Let $\mathbb{F}(x, y, z) = (-y^3 e^z + y^2 e^{xy^2},\ x^3 \cos z + 2xye^{xy^2},\ xyz)$ be a vector field on $\mathbb{R}^3$, C be the circle

$$C = \{(x, y, z) : x^2 + y^2 = a^2, z = 0\},$$

where $a > 0$.

Compute the line integral

$$\int_C \mathbb{F} \cdot d\boldsymbol{\alpha},$$

where $\boldsymbol{\alpha}$ is a parametrization of C going counterclockwise.

Solution.

5. continued.