BSc Engineering Sciences – A. Y. 2017/18

Written exam of the course Mathematical Analysis 2

January 29, 2018

Last name: .... First name: ....

Matriculation: ....

Solve the following problems, motivating in detail the answers.

1. Determine the values of $\alpha \in \mathbb{R}$ for which the following improper integral is convergent, and compute it for $\alpha = \frac{1}{2}$:

$$\int_0^{\pi/4} \frac{\cos x}{(\cos^2 x - \sin^2 x)^\alpha} dx.$$

Solution.

Matriculation: ....

2.

(1) Find all the stationary points of the following scalar field, defined on $\mathbb{R}^2$,

$$f(x, y) = e^{x+y}(x^2 + xy)$$

and classify them into relative minima, maxima and saddle points.

(2) Compute the derivative of the following function on $\mathbb{R}$:

$$f(t) = (1 + \cosh t)^{1+\cosh t}.$$

Solution.

Matriculation: ....

3. Determine whether the following vector field on $\mathbb{R}^2$

$$\mathbb{f}(x,y) = (e^y \cos(xe^y), xe^y \cos(xe^y))$$

is a gradient of some scalar field. If so, find one of these scalar fields φ such that $\mathbb{f}(x,y) = \nabla\varphi(x,y)$.

Solution.

Matriculation: ....

4. Compute

$$\iiint\limits_{S} (x^2 + y^2 - \arctan z)\, dxdydz,$$

where

$$S = \{(x, y, z) \in \mathbb{R}^3 \,:\, x^2 + y^2 \leq 1,\, 0 \leq z \leq 1\}.$$

Solution.

Matriculation: ....

5. Let $\mathbb{F}(x, y, z) = (x^3, y^3, z^3)$ be a vector field on $\mathbb{R}^3$, S be the sphere

$$\{(x, y, z) : x^2 + y^2 + z^2 = a^2\},$$

where $a > 0$ and $\mathbb{n}$ the outgoing normal unit vector on S at each point of S.

Compute the surface integral

$$\iint_S \mathbb{F} \cdot \mathbb{n} \, dS.$$

Solution.