Wightman fields for two-dimensional CFT
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1 Mathematical Quantum Field Theory

QFT: quantum physics (Hilbert space, operators) with infinite degrees of freedom. Axioms + examples
- Gårding-Wightman axioms: operator-valued distribution
- Araki-Haag-Kastler (operator-algebras)
- Osterwalder-Schrader (probability).

2. (1+1)-dim Conformal Field Theory

A special class of QFT
- fixed points of renormalization group / critical phenomena (cf. Osborne-Stottmeister)
- conformal ($\mathrm{Diff}(\mathbb{R}) \times \mathrm{Diff}(\mathbb{R})$) covariance.

$\Rightarrow$ chiral components (fixed points w.r.t. $\mathrm{Diff}(\mathbb{R}) \times 1$, $1 \times \mathrm{Diff}(\mathbb{R})$)

1-dim QFT on $\mathbb{R} \to S^1 = \mathbb{R} \cup \{\infty\}$

studied in { conformal nets, vertex operator algebras

+ charged fields (cf. Doplicher-Roberts reconstruction)

3 Wightman fields on S^1

$\mathcal{H}$: Hilbert space, $\mathcal{D} \subset \mathcal{H}$ dense subspace.

ϕ: operator-valued distribution on S^1.

$C^\infty(S^1) \ni f \mapsto \phi(f): \mathcal{D} \to \mathcal{D}$.

$\Omega \in \mathcal{D}$ "vacuum, $U: \mathrm{Diff}(S^1) \to PU(\mathcal{H})$ "symmetry"

satisfying Wightman axioms: locality if $\mathrm{supp} f$ and $\mathrm{supp}\, g$ are disjoint, $[\phi(f), \phi(g)] = 0$

covariance, positive energy...

4. Example: U(1)-current / Heisenberg alg / massless free field (cf. Dybalski-Morinelli)

Consider the Lie algebra spanned by $\{J_n, n \in \mathbb{Z}, C\}$ with $[J_m, J_n] = m\delta_{m+n,0} C$ central.

For any $\alpha \in \mathbb{R}$, there is a "Verma module" or Fock rep V_α with Ω_α. $J_n \Omega_\alpha = 0$ for $n > 0$, $J_0 = \alpha$, spanned by $J_{n_1} \cdots J_{n_k} \Omega_\alpha$

Take $\overline{V_0} = \mathcal{H}_0$. For $f(\theta) = \sum e^{in\theta} \hat{f}_n \in C^\infty(S^1)$

Put $J(f) = \sum \hat{f}_n J_n$ on V_0. J is a Wightman field on S^1

4. Charged fields

Put $\hat{\mathcal{H}} = \bigoplus_{\alpha\in\mathbb{R}} \mathcal{H}_\alpha$. $\mathcal{H}_\alpha = \overline{V_\alpha}$. $\hat{J}_n = \bigoplus_{\alpha\in\mathbb{R}} J_n$.

For $\beta\in\mathbb{R}$, put $C_\beta : \mathcal{H}_\alpha \to \mathcal{H}_{\alpha+\beta}$, $J_{-n_1}\cdots J_{-n_k}\Omega_\alpha \mapsto J_{-n_1}\cdots J_{-n_k}\Omega_{\alpha+\beta}$

$$E^+(\beta,z) = \prod_{j>0} \exp\left(-\frac{\beta \hat{J}_j}{j} z^{-j}\right)$$

$$E^-(\beta,z) = \prod_{j<0} \exp\left(-\frac{\beta \hat{J}_j}{j} z^{-j}\right)$$

formal series in z

$z\in\mathbb{C}$

S^1

Put $Y_\beta(z) = C_\beta E^-(\beta,z) E^+(\beta,z) z^{J_0\beta}$

$Y_\beta(z)$: "charged primary field"

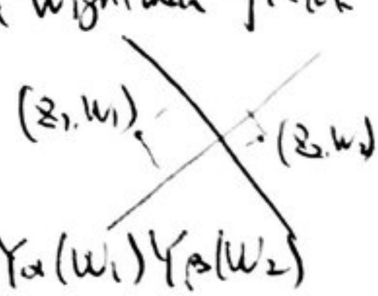
Braiding $Y_\alpha(z) Y_\beta(w) = e^{i\pi\alpha\beta} Y_\beta(w) Y_\alpha(z)$ if $\arg z > \arg w$

5. (1+1)-dim local Wightman field

Define $\hat{Y}_\alpha(z,w) = Y_\alpha(z) \otimes Y_\alpha(w)$ on $\hat{\mathcal{H}} \otimes \hat{\mathcal{H}}$

<u>Thm</u> (Adamo-Giorgetti-T.) $\hat{Y}_\alpha$ is a Wightman field

proof) We show locality.



$\arg z_1 > \arg z_2$, $\arg w_1 < \arg w_2$

$$\begin{aligned}
\hat{Y}_\alpha(z_1,w_1)\hat{Y}_\beta(z_2,w_2) &= Y_\alpha(z_1)Y_\beta(z_2) \otimes Y_\alpha(w_1)Y_\beta(w_2) \\
&= e^{i\pi\alpha\beta} Y_\beta(z_2)Y_\alpha(z_1) \otimes e^{-i\pi\alpha\beta} Y_\beta(w_2)Y_\alpha(w_1) \\
&= \hat{Y}_\beta(z_2,w_2)\hat{Y}_\alpha(z_1,w_2)
\end{aligned}$$

6. OS axioms

We can see $z, w \in \mathbb{C} = \mathbb{R}^2$. Put $J(z) = \sum J_n z^{-n}$.

$S_n(z_1, \dots, z_n) = \langle \Omega_0, J(z_1) \cdots J(z_n) \Omega_0 \rangle$.

One can show that this is convergent if $|z_1| > \cdots > |z_n|$

Glue them together to get Schwinger functions

Unitarity $\Rightarrow$ Reflection positivity. (Work in progress)