Proofs in high energy physics

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Unreasonable effectiveness vs Crisis in particle physics?

- **Unreasonable effectiveness** of Mathematics in the Natural Sciences (Wigner 1960, actually mentions the (non-)unification of GR and QFT)
- **Crisis in particle physics**: no new particles in experiments, no supersymmetry, little progress in quantum gravity. (Cern article)

Supersymmetry: a "beautiful" mathematical extension of the usual symmetry, would have resolved the hierarchy problem, enabled grand unification, given candidates for dark matter. **No experimental evidence**.

The standard model is too good. Need no new theory, only a few problems left?

(except Quantum gravity)
(except that we don't have the standard model in a mathematical form)

Physics vs Mathematics

Physics: Construct a model. Make predictions. Compare them with experiments.
A theory is good if it makes good predictions. (M. Schwartz (2013): "the subject is a mess... QFT provides, first and foremost, a set of tools for practical calculations")

Mathematics/mathematical physics: Construct a model. Prove its properties. Relate different models.
A theory is good if the proofs are rigorous.
(or study toy models numerically, as long as it is remembered that they are toy models)

?????: Construct a model (or something that looks like a model). Claim its properties without proof. No comparison with experiments (or numerical studies).

Cf. Replication crisis

Physics vs Mathematics

(by Hrvoje Nikolić)

Physical language	Mathematical language
physics	applied mathematics
mathematics	applied mathematics
abstract nonsense	mathematics
conjecture	vague idea
theorem	conjecture
rigorous theorem	theorem
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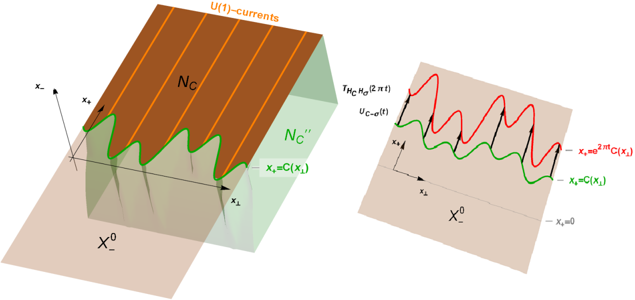
really?

Scale vs conformal in 2d

Claim: any 2d QFT with (1) unitarity, (2) Poincaré invariance (causality), (3) discrete spectrum in scaling dimension, (4) existence of scale current and (5) unbroken **scale invariance**, is **conformally invariant**. [1][2]
Argument: change the stress-enery tensor to the one having the canonical scaling. The rest is ok.

Counterexample [3]: Take the chiral field $J(x), [J(z), J(w)] = \delta'(z-w)$.
Let T be the usual stress-energy tensor. Take
$T_\kappa(z) = T(z) + \kappa(J'(z) + i\frac{z-1}{z+1}J(z))$. T_κ satisfies the Virasoro relations except $z = -1$. No other changes. T_κ does not give the correct conformal transformations.
Another example [4]: take the Virasoro net, $c > 1$ and its dual net. The stress-energy tensor T_c generate the correct translation-dilation group, but not the rotations for the dual net.

Quantum Null Energy Condition (QNEC)



Figures of [5]

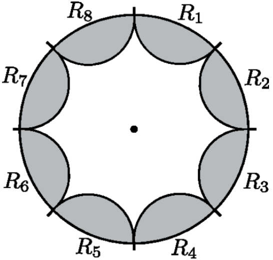
Claim of QNEC: $S_{\mathrm{rel}}(\psi|\Omega;\mathcal{A}(N''_{bC}))$ is convex in b.
[6] Theorem 1. Assume that $\mathcal{A}(N''_{aC}) \subset \mathcal{A}(N''_{bC})$ is a half-sided modular inclusion. Then ∂S can be calculated. Theorem 2. QNEC holds.

Proof of HSMI in AQFT missing

(Typically it follows if the inclusion is standard, but often such inclusions are not standard)

Absense of global symmetry in quantum gravity

[7] (to be fair, this paper does not claim that it is rigorous. But its accompanying press release says literally that AdS/CFT correspondence has rigorously defined the holographic principle)



(Figure from [7])

Claim: no global symmetry in Quantum gravity
(1) Assume AdS/CFT (2) take an operator x in bulk (3) cut the symmetry in the boundary "using the split property" (4) x commutes with any of the small pieces (5) $x = c\mathbb{1}$
With the same proof, x **would commute with anything in CFT??**

More

[8] Abstract: We argue that the following three statements cannot all be true: (i) Hawking radiation is in a pure state, (ii) the information carried by the radiation is emitted from the region near the horizon, with low energy effective field theory valid beyond some microscopic distance from the horizon, and (iii) the infalling observer encounters nothing unusual at the horizon.
Argument: **by contradiction**

[9] Abstract: **We show** that for any such IQFT there are infinitely many integrable deformations generated by scalar local fields, which are in one-to-one correspondence with the local integrals of motion;
Text: **We now want to show** that every field generates an integrable deformation of a given IQFT... With this reasoning, **we conjecture** that the IM of the deformed theory generally still commute with each other.

Quantum Yang-Mills in 4d

[10] 1982-1989 Series of papers, total $\geq$ 500 pages
Defines Yang-Mills theory on lattices. Define counterterms on finer scales. Renormalize. Claims the **UV stability** (the boundedness of the partition functions).

- starts with simple models
- applies the results to more complicated models
- applies the results from older papers
- some parts are extremely sketchy

(these papers are about the real world physics, but exemplify the consequence of papers written not very precisely)

More recent attempts [11]

Remedy?

What should we do in science when experimental input is missing?

Observation: there is often a problem when the claim is qualitative.
Quantum information, statistical physics: considerably many people care about rigor.

Proposal: Don't make qualitative claim with non rigorous arguments.
Don't do proof by contradiction.

Rigorously constructed interacting QFT

Some say QFT does not have a rigorous definition, what does it mean? Cf. Summers '12

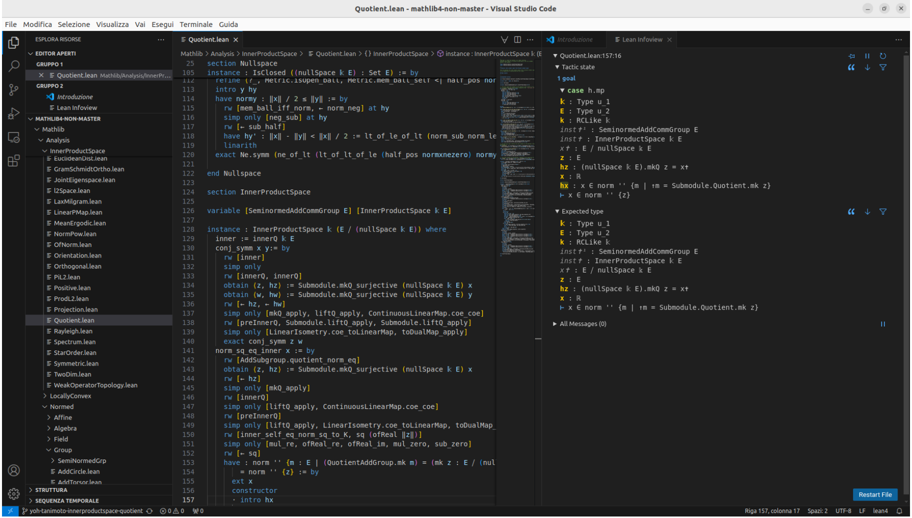
QFT in Gårding-Wightman/Osterwalder-Schrader/Araki-Haag-Kastler axioms

- **2d** $P(\phi)_2$, exponential, Yukawa, Federbush, Gross-Neveu, Sine-Gordon, Thirring, Abelian Higgs, massive Ising, some factorizing S-matrix, CFT (minimal models, WZW models, Narain, Virasoro ($c \geq 1$), $\mathcal{W}_3$-algebra...), thermal completion, ground state completion, dual nets
- **3d** ϕ^4_3, Yukawa, Abelian Higgs
- **4d**

Axioms are fine (asking only fundamental properties (locality, symmetry, positive energy, vacuum), except that they are "too weak").

The main difficulty is 4d.

Formalisation (mathlib in Lean 4)



latest developments: weak operator topology, continuous functional calculus, Riesz-Markov-Kakutani theorem...

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