Towards Construction of Integrable QFT with Bound States
(partly with D. Cadamuro, arXiv:1502.01313)

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Nonperturbative Quantum Field Theory

- Nonperturbative construction of QFT is hard.
- "What is Quantum Field Theory? I have no idea." (N. Seiberg)

Computing all correlation functions

- $F(x_1, \cdots, x_n) = \langle \Omega, \phi(x_1) \cdots \phi(x_n) \Omega \rangle$: Wightman functions
- Constructive QFT: $\mathcal{P}(\phi)_2$ models (Glimm-Jaffe), Sine-Gordon (Fröhlich-Seiler),...

Integrable QFT

- Factorizing S-matrix (Zamolodchikov-Zamolodchikov): Sine-Gordon, Sinh-Gordon, $O(N)$-σ models, Toda field theories...
- Compute form factors: $\langle p_1, \cdots, p_n | \phi(x) | q_1, \cdots q_m \rangle$ and expand $F(x_1, x_2) = \sum \int \langle \Omega, \phi(x_1) p_1, \cdots p_n \rangle \langle p_1, \cdots, p_n \, \phi(x_2) \Omega \rangle$.
- Problem: convergence of the expansion.

Alternative strategy

- Pointlike field are hard. Larger regions have better observables.
- **(left-)Wedge**: $W_{\mathrm{L}} := \{(t,x) : x < |t|\}$.

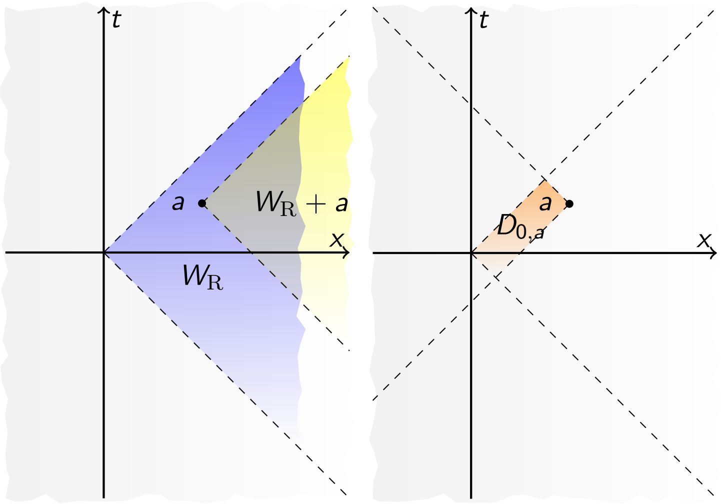
Wedge-local fields in integrable models (Schroer, Lechner)

- S: factorizing S-matrix.
- $z^\dagger, z$: Zamolodchikov-Faddeev algebra (creation and annihilation operators defined on S-**symmetric Fock space**).
- $\phi(f) = z^\dagger(f^+) + z(J_1 f^-)$, $\mathrm{supp}\, f \subset W_{\mathrm{L}}$, is localized in W_{L}.

The full QFT (without bound states)

- The observables $\mathcal{A}(W_{\mathrm{L}})$ in W_{L} are generated by $\phi(f)$.
- For diamonds $D_{a,b}$, define $\mathcal{A}(D_{a,b}) = \mathcal{A}(W_{\mathrm{L}} + a) \cap \mathcal{A}(W_{\mathrm{R}} + b)$.
- Examine the **boost operator** in order to show the existence of local operators (modular nuclearity (Buchholz, D'antoni, Longo, Lechner)).

Standard wedge and double cone



Factorizing S-matrix models (Schroer, Lechner)

- **Input**: meromorphic function $S : \mathbb{R} + i(0, \pi) \to \mathbb{C}$,

$$\overline{S(\theta)} = S(\theta)^{-1} = S(-\theta) = S(\theta + \pi i), \quad \theta \in \mathbb{R}.$$

- S-symmetric Fock space: $\mathcal{H}_1 = L^2(\mathbb{R}, d\theta)$, $\mathcal{H}_n = P_n \mathcal{H}_1^{\otimes n}$, where P_n is the projection onto S-**symmetric** functions:

$$\Psi_n(\theta_1, \cdots, \theta_n) = S(\theta_{k+1} - \theta_k)\Psi_n(\theta_1, \cdots, \theta_{k+1}, \theta_k, \cdots, \theta_n).$$

- Zamolodchikov-Faddeev algebra: S-symmetrized **creation and annihilation operators**

$$z^\dagger(\xi) = Pa^\dagger(\xi)P, \quad z(\xi) = Pa(\xi)P, \quad P = \bigoplus_n P_n,$$

$$(a^\dagger(\xi)\Psi_n)(\theta_1, \cdots, \theta_{n+1}) = \xi(\theta_1)\Psi_n(\theta_2, \cdots, \theta_{n+1}).$$

If S has **no pole**, then $\phi(f) = z^\dagger(f^+) + z(J_1 f^-)$ and the reflected field $J\phi(g)J$ commute (Lechner '04).

The bound state operator

S: two-particle S-matrix, **simple poles** at $\frac{\pi i}{3}, \frac{2\pi i}{3}$, bootstrap equation

$$S(\theta) = S\left(\theta + \frac{\pi i}{3}\right) S\left(\theta - \frac{\pi i}{3}\right).$$

P_n: S-symmetrization, $\mathcal{H} = \bigoplus P_n \mathcal{H}_1^{\otimes n}$, $\mathcal{H}_1 = L^2(\mathbb{R})$,

$$\mathrm{Dom}(\chi_1(f)) := H^2\left(-\tfrac{\pi}{3}, 0\right)$$
$$(\chi_1(f))\xi(\theta) := \sqrt{2\pi|R|} f^+\left(\theta + \tfrac{\pi i}{3}\right) \xi\left(\theta - \tfrac{\pi i}{3}\right),$$
$$R = \operatorname*{Res}_{\zeta = \frac{2\pi i}{3}} S(\zeta),$$

where $H^2(\alpha, \beta)$ is the space of analytic functions in $\mathbb{R} + i(\alpha, \beta)$ such that $\xi(\cdot - \gamma i)$ is uniformly bounded in L^2-norm, $\gamma \in (\alpha, \beta)$, and f^+ is analytic.

$$\chi_n(f) = n P_n \left(\chi_1(f) \otimes I \otimes \cdots \otimes I\right) P_n,$$
$$\chi(f) := \bigoplus \chi_n(f).$$

Wedge-local fields and weak commutativity

The CPT operator: $(J_n\Psi_n)(\theta_1,\cdots,\theta_n) = \overline{\Psi_n(\theta_n,\cdots,\theta_1)}$.
New field:

$$\widetilde{\phi}(f) := \phi(f) + \chi(f) = z^\dagger(f^+) + \chi(f) + z(J_1 f^-),$$

and the **reflected field**: $\widetilde{\phi}'(g) := J\widetilde{\phi}(g^j)J$, $g^j(x) = \overline{g(-x)}$.

Theorem (Cadamuro-T. arXiv:1502.01313)

For real $f, g, \mathrm{supp}\, f \subset W_\mathrm{L}, \mathrm{supp}\, g \subset W_\mathrm{R}$, *then*

$$\langle\widetilde{\phi}(f)\Phi, \widetilde{\phi}'(g)\Psi\rangle = \langle\widetilde{\phi}'(g)\Phi, \widetilde{\phi}(f)\Psi\rangle, \;\; \Phi,\Psi \in \mathrm{Dom}(\widetilde{\phi}(f)) \cap \mathrm{Dom}(\widetilde{\phi}'(g)).$$

Strong commutativity remains open.
Modular nuclearity can be proved if one assumes strong commutativity:
One has to estimate the complex boost $\Psi_n(\underline{\theta} - \frac{\pi i}{2})$, which follows from the definition of the domain of $\chi(f)$.

Summary

- input: two-particle factorizing S-matrix with **poles**
- **new field** $\widetilde{\phi}(f) = \phi(f) + \chi(f)$
- weak commutativity
- modular nuclearity (by assuming strong commutation)
- some features of $\widetilde{\phi}(f)$: no polynomial Reeh-Schlieder property, no energy bound, non-temperateness

Open problems

- **strong commutativity**
- non-scalar models (Sine-Gordon, Z(N)-Ising...): weakly commuting fields are available. Strong commutativity and modular nuclearity are more difficult.