Local energy bounds and strong locality in chiral CFT.
joint with S. Carpi, M. Weiner

0. QEI: conditions on E. Energy bounds: conditions by E. Cf. Sanders
Some applications of QEI. Hollands

1. Mathematical QFT.

Wightman, Haag-Kastler, Osterwalder-Schrader, VOA...

$\boxed{\text{Wightman}}$ operator-valued distribution. $\mathcal{S}(\mathbb{R}^{d+1}) \ni f \mapsto \phi(f) \curvearrowright \mathcal{D} \subset \mathcal{H}$
unbounded on $\mathcal{D}$.

Locality: If supp f and supp g are spacelike, then $[\phi(f), \phi(g)] = 0$.

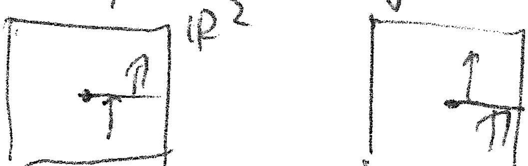
$\boxed{\text{Haag-Kastler}}$ net of von Neumann algs. $\mathbb{R}^{d+1} \supset O \mapsto \mathcal{A}(O) \curvearrowright \mathcal{H}$.

Locality: If O_1 and O_2 spacelike, then $[\mathcal{A}(O_1), \mathcal{A}(O_2)] = \{0\}$.

W $\to$ HK. $\mathcal{A}(O) = \{e^{i\phi(f)} : \text{supp} f \subset O\}''$.

Does W locality imply HK locality? "strong locality".

2. Strong locality.



Nelson's counterexample.
$\partial x, \partial y$ on smooth functions commute, but not strongly.

Linear energy bounds (Nelson, Glimm-Jaffe, ~~Fröh~~ Driessler-Fröhlich)

If the Hamiltonian H satisfies $\|\phi(f)v\| \le C_f \|(H+1)v\|$,
then $\phi(f)$ and $\phi(g)$ commute strongly (cf. commutator theorem).

For some fields, linear energy bounds fail. Alternative?

3. 2d Conformal field theory.

A 2d CFT contains "chiral components", living on the lightrays.
$\Rightarrow$ Wightman fields on $\mathbb{R} \subset S^1 \subset \mathbb{C}$. Use $z \in S^1$.

Lüscher-Mack theorem: the stress-energy tensor $L(z)$ satisfies
the Virasoro alg. $[L(z), L(w)] = 2\partial_w \delta(z-w) + \delta(z-w)\partial_w L(w) + \frac{c}{12}\partial_w^3 \delta(z-w)$

There are good "primary" field ϕ (diff covariant) with conformal dim

$[L(z), \phi(w)] = d\,\partial_w \delta(z-w)\phi(w) + \delta(z-w)\partial_w \phi(w)$.

Obs: If $d > 2$, ϕ never satisfies linear energy bounds.

4. Local energy bounds.

For primary fields ϕ, $[L(f), \phi(f^{d-1})] = 0$.

If $f \geq 0$, $L(f) + r_f \mathbb{1}$ is positive self-adjoint (Fewster-Hollands).

Thm Assume that $\|\phi(f^{d-1})v\| \leq C_f \|(L(f)+r_f\mathbb{1})^{d-1} v\| \cdots \circledast$.
for positive f. Then $\phi(f^{d-1})$ and $\phi(g^{d-1})$ commute strongly.

proof). Apply the Driessler-Fröhlich theorem with $(L(f)+L(g)+r_{f,g}\mathbb{1})^{d-1}$.

We call $\circledast$ local energy bounds. How to prove it?

Thm If a primary field satisfies $\|\phi_0 v\| \leq C\|(L_0+\mathbb{1})^{d-1}v\| \cdots (\star)$
$\phi_0 = \int \phi(z) z^d dz$. Then ϕ satisfies $\circledast$.

proof) $U(\gamma)\phi(f)U(\gamma) = \phi\left((\cancel{\ldots})^{d-1} f\circ\gamma^{-1}\right)$

Cor If a unitary VOA is generated by primary fields satisfying $(\star)$, then one has a conformal net (c.f Carpi-Kawahigashi-Longo-Weiner)

5. Example. The W_3-algebra. Generated by $L(z), \phi(z)$ (Zamolodchikov)

$[L(z), L(w)] = 2\partial_w\delta(z-w)L(w) + \delta(z-w)\partial_w L(w) + \frac{c}{12}\partial_w^3\delta(z-w)$.

$[L(z), \phi(w)] = 3\partial_w\delta(z-w)\phi(w) + \delta(z-w)\partial_w\phi(w)$ (primary, $d=3$).

$$
\begin{aligned}
[\phi(z), \phi(w)] = {} & \frac{c}{3\cdot 5!}\partial_w^5\delta(z-w) + \frac{1}{3}\partial_w^3\delta(z-w)L(w) + \frac{1}{2}\partial_w^2\delta(z-w)\partial_w L(w) \\
& + \partial_w\delta(z-w)\left(\frac{3}{10} + 2b^2\Lambda(w)\right) \\
& + \delta(z-w)\left(\frac{1}{15}\partial_w^3 L(w) + b^2\partial_w\Lambda(w)\right).
\end{aligned}
$$

where $b^2 = \frac{16}{22+5c}$, $\Lambda(z) = :L(z)^2: - \frac{3}{10}\partial_z^2 L(z)$.

This admits a unitary rep for $c \geq 2$, using a free field rep.

We can prove $(\star)$; so there is a conformal net for each $c \geq 2$.

6. Outlook. VOA $\Longleftrightarrow$ Wightman $\overset{?}{\Longleftrightarrow}$ conformal net.

Raymond-T.-Tener