Towards integral perturbation of two-dimensional CFT

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This work is very much incomplete and in progress!

- (Glimm, Jaffe...) Start with the free field on the **Minkowski space**, take the interacting Hamiltonian, define the new dynamics and take a new representation of the algebras.
- (Barata, Jäkel, Mund) Start with the free field on the **de Sitter space**, take a new vacuum in the same Hilbert space, and let the modular groups generate the dynamics and a new Haag-Kastler net.
- There are many two-dimensional CFTs. You can put them on the de Sitter space easily. Can one **change the dynamics** to obtain a **Haag-Kastler net**?
- **Hopefully**. Take a primary field, add it to the Lorentz generators, let them generate a new dynamics.

- Two-dimensional non-chiral CFT.
- Charged primary fields in the CFT, the representation theory, the brading.
- Analysis of operators (incomplete!). Time-zero fields, generators of the Lorentz group.
- *W**-perturbation of the KMS state. Buchholz-Borchers axioms for the de Sitter space.
- Infinite volume (Minkowski) limit? Scaling limit (back to CFT)?
- Integrable perturbations of (rational) CFT?

- The Haag's theorem roughly says that, if your QFT on the Minkowski space is unitarily equivalent to the free field at time 0, then it is unitarily equivalent to the free field for all t. ⇒ need to change the representation
- An algebraic version (Weiner '11): if two Haag-Kastler nets on the Minkowski space (with the split property, the Bisognano-Wichmann property and the Haag duality) have the same wedge algebras at time 0, then the two nets are the same, including the vacuum.
- This can be circumvented on the de Sitter space (Barata-Jäkel-Mund '13). Keep the algebras at time 0, change the dynamics and the vacuum.
- Why not start with a 2d CFT?

- A general strategy (Jäkel-Mund '18).
- Fix a Haag-Kastler net on the de Sitter space (isotony, locality, Lorentz covariance, the vacuum, the KMS property for wedges (Buchholz-Borchers '99).
- On the same Hilbert space, construct a new representation of the Lorentz group whose restriction to rotations remains the same.
- Under certain conditions (finite speed of propergation, existence of vacuum), one can generate a new Haag-Kastler net: keep the algebras of wedges at time zero, and the rest is defined by covariance with respect to the new representation.
- Examples: $\mathcal{P}(\phi)_2$ -models.

CFT as a Haag-Kastler net on de Sitter space

- Any conformal (Möb × Möb-covariant in 2d) field theory extends to the Einstein cylinder (Guido-Longo '03)
- The de Sitter space is conformally equivalent to part of the cylinder.
- By composing these maps, any CFT can be considered as a QFT on the de Sitter space.
- Lorentz transformations are contained in the Möb × Möb group. Indeed, the spacelike rotations $r_t \times r_{-t}$ and Lorentz boosts $\delta_s \times \delta_{-s}$ generate a three-dimensional Lie group, the (2 + 1)-dimensional Lorentz group (consider the Lie algebra generated by $L_1 \otimes \mathbb{1} - \mathbb{1} \otimes L_{-1}, L_0 \otimes \mathbb{1} - \mathbb{1} \otimes L_0, L_{-1} \otimes \mathbb{1} - \mathbb{1} \otimes L_1$).
- These generators are smearing of the field L(z) ⊗ 1 − 1 ⊗ L(z⁻¹) by the combination of sin, cos, 1.

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Figure: The Minkowski space M_0 and the de Sitter space dS^2 conformally embedded in \mathbb{R}^2 . The cylinder is obtained by identifying the dotted lines. The dark grey region is a wedge W and the light grey region is a double cone.

Chiral components and primary fields in CFT

- Two-dimensional CFT has the following structure: there are chiral components that are QFT living on the left and right lightrays, and the whole CFT is an extension (in the sense close to that of Doplicher-Roberts).
- Chiral components are called conformal nets and there are many examples. They contain the Virasoro algebra $[L_m, L_n] = (m n)L_{m+n} + \frac{c}{12}m(m^2 1)\delta_{m+n}$, which extends the Möb group.
- The extension can be studied through the DHR representation theory.
- Extensions are generated by charged primary fields ψ, which satisfy
 [L_m, ψ_n] = ((d − 1)m − n)ψ_{m+n}, where d is called the conformal
 dimension of ψ.
- Primary fields satisfy the so-called braiding relations.
- Some fields in a CFT have very specific and explicit commutation relations.

- Consider the derivative of the massless scalar field.
- It decomposes into the left and right chiral components: the U(1)-current. $[J_m, J_n] = n\delta_{m+n}$.
- The current field is given by $J(z) = \sum_n z^{-n-1} J_n$.
- The U(1)-current net admits a family of representations \mathcal{H}_{α} parametrized by $\alpha \in \mathbb{R}$, associated with the field Y_{α} .
- There are two-dimensional extensions of the tensor product of U(1)-current nets, parametrized by α defined on ⊕_{n∈Z} H_{nα} ⊗ H_{nα}, (Morinelli-T.-Weiner '18).
- Each of these extensions is generated by the field $Y_{\alpha}\bar{Y}_{\alpha}$, the product of left and right charged fields.

- There is a primary field $Y_{\alpha}(z)$ acting on $\bigoplus_n \mathcal{H}_{n\alpha}$ and $Y_{\alpha}(z)$ shifts the charge by α .
- $E^{\pm}(\alpha, z) = \exp\left(\mp \sum_{n>0} \frac{\alpha J_{\pm n}}{n} z^{\mp n}\right)$
- $Y_{\alpha}(z) = c_{\alpha}E^{-}(z)E^{+}(z)z^{\alpha(0)}$, where c_{α} is the unitary shift $\mathcal{H}_{\beta} \to \mathcal{H}_{\alpha+\beta}$, $\alpha(0)$ on \mathcal{H}_{β} gives $\alpha \cdot \beta$.
- The commutation relation is given by Y_α(z)Y_β(ζ) = e^{iαβ}Y_β(ζ)Y_α(z) if arg z > arg ζ.
- The factor $e^{i\alpha\beta}$ is called the braiding: $Y_{\alpha}(z)$, defined as a charged field of the U(1)-current algebra, is not local.

- Consider the Hilbert space $\bigoplus_n \mathcal{H}_{n\alpha} \otimes \mathcal{H}_{n\alpha}$ and the charged fields acting on the left and right components, respectively: $Y_{\alpha}(z), \bar{Y}_{\alpha}(\zeta)$.
- The product $Y_{\alpha}(z)\bar{Y}_{\alpha}(\zeta)$ can be considered as a two-dimensional field.
- As the braiding relation holds, this field is local in the two-dimensional sense: for the spacelike separation, we have z₁ > z₂, ζ₁ < ζ₂.
- It is also relatively local with respect to the currents (both left and right).

Time-zero fields

- We would like to take the restriction of $Y_{\alpha}(z)\overline{Y}_{\alpha}(\zeta)$ to the time-zero plane $z = \frac{1}{\zeta}$, define the field $\varphi(z)$, and construct the new generator from the field $L(z) \otimes 1 1 \otimes L(z) + \varphi(z)$. Does this make sense?
- Consider the Fourier components of the charged fields $Y_{\alpha,n}, \bar{Y}_{\alpha,n}$.
- The restriction of the two-dimensional field to the time-zero circle corresponds to the Fourier components ∑_n Y_{α,n} Y_{α,n+m} for m ∈ Z.
- Cf. the normal product $\sum_{n} Y_{\alpha,n} \overline{Y}_{\alpha,m-n}$, defined on finite energy vectors because of the positive energy condition.
- Estimates $||Y_{\alpha,-n}\Omega||^2 = \binom{2d+n-1}{n} \sim n^{2d-1}$ (Carpi-Kawahigashi-Longo-Weiner '18).
- For d < ¹/₄, the Fourier components of the time-zero field makes sense as operators, because they can be applied to Ω ⊗ Ω (and other vectors).

New Lorentz generators

Lemma

With $\lambda \in \mathbb{R}$, the Lorentz relations are **formally** satisfied for

$$L_{1} \otimes \mathbb{1} - \mathbb{1} \otimes L_{-1} + i\lambda \left(\sum_{k} Y_{\alpha,k} \bar{Y}_{\alpha,-1+k} + \text{h.c.} \right)$$
$$L_{0} \otimes \mathbb{1} - \mathbb{1} \otimes L_{0}$$
$$L_{-1} \otimes \mathbb{1} - \mathbb{1} \otimes L_{1} - i\lambda \left(\sum_{k} Y_{\alpha,k} \bar{Y}_{\alpha,k+1} + \text{h.c.} \right)$$

Proof.

As $Y_{\alpha}(z)$ is primary, it holds that $[L_m, Y_{\alpha,n}] = ((d-1)m - n)Y_{\alpha,m+n}$,

$$[L_m \otimes \mathbb{1} - \mathbb{1} \otimes L_{-m}, \sum_k Y_{\alpha,k} \bar{Y}_{\alpha,-n+k}] = ((2d-1)m - n) \sum_k Y_{\alpha,k} \bar{Y}_{\alpha,-m-n+k}$$

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- Are new Lorentz generators self-adjoint on a nice domain?
- Do the Lorentz relations hold as operators?
- Do the Lorentz relations extend to a group representation?
- Is there a rotation-invariant KMS state?

- Is there a Lagrangian for this model?
- What about other CFT with charged fields?
- Cf. "integrable perturbation" by Zamolodchikov.
- The Minkowski limit? The S-matrix?
- Classifying possible interactions? "Relevant fields"?