Let x be an operator localized in $\mathcal{A}_r$. With the modular involution S for $\mathcal{A}_r$, we have $Sx\Omega = x^*\Omega$. x^* is also localized in $\mathcal{A}_r$. On the other hand, with $S = J\Delta^{\frac{1}{2}}$, J implements the CPT with π-rotation around 1-axis (the Bisognano-Wichmann property) and Jx^*J is localized in $\mathcal{A}_l$ ($= \mathcal{A}_r'$ if we assume the Haag duality). The above equation is equivalent to

$$\Delta^{\frac{1}{2}} x \Delta^{-\frac{1}{2}} \Omega = Jx^* J\Omega. \tag{1}$$

Now, assume [1] the decomposition $\mathcal{H} = \mathcal{H}_l \otimes \mathcal{H}_r, \mathcal{B}(\mathcal{H}) \cong \mathcal{A}_l \otimes \mathcal{A}_r$ and accordingly $\Delta^{\frac{1}{2}} = \exp(-\pi K_l)\exp(-\pi K_r)$. Let x be an element localized in a double cone (a compact set) in the right half-space. Especially, $x \in \mathcal{A}_r$. The decomposition above would imply [2].

$$\Delta^{\frac{1}{2}} x \Delta^{-\frac{1}{2}} = \exp(-\pi K_r)\exp(-\pi K_l) x \exp(\pi K_l)\exp(\pi K_r) = \exp(-\pi K_r) x \exp(\pi K_r)$$

which is still localized in $\mathcal{A}_r$. On the other hand, Jx^*J is localized in a certain double cone, since J implements CPT plus a rotation. Therefore, there is a large wedge (in Minkowski space) where both $\Delta^{\frac{1}{2}} x \Delta^{-\frac{1}{2}}$ and Jx^*J are localized.

With the help of the Reeh-Schlieder property (i.e. Ω is separating), it follows [3] from (1) that $\Delta^{\frac{1}{2}} x \Delta^{-\frac{1}{2}} = Jx^*J$ and this must be localized both in $\mathcal{A}_l, \mathcal{A}_r$, which is a contradiction.

References

[Wit18] Edward Witten. Notes on some entanglement properties of quantum field theory. 2018. `https://arxiv.org/abs/1803.04993`.

[1] We know that this is mathematically false, but we are trying to show that this is not a healthy argument even at the physicists' level, by finding a contradiction without using "type III algebra" or "outerness of the modular automorphisms". **Only these assumptions in this paragraph and careless treatment of unbounded operators in the next paragraph (see the the footnote) are not rigorous. Consequently, any contradiction is due to the assumptions or arguments here.**

[2] A similar formal operation is done in e.g. [Wit18, (5.10)].

[3] Mathematically, this part is invalid, because $\Delta^{\frac{1}{2}} x \Delta^{-\frac{1}{2}}$ is a possibly unbounded operator. But this level of operation is common in physicists' arguments and indeed is used in [Wit18, Section 2.4].